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Understanding how to calculate the margin of error (MOE) is essential for statisticians, researchers, and data analysts who aim to interpret survey results or sample data accurately. In this article, we will explore the process of computing the margin of error given specific parameters: a sample size of $N = 290$, a sample proportion $\hat{p} = 0.85$, and a confidence level of 99%. We will break down each step, explain relevant concepts, and provide practical insights to help you master this important statistical calculation.

Introduction to Margin of Error

The margin of error (MOE) quantifies the amount of random sampling error in survey results or estimations derived from sample data. It provides a range within which the true population parameter is expected to lie, with a specified level of confidence. A smaller MOE indicates more precise estimates, while a larger MOE suggests greater uncertainty.

For proportions, the margin of error depends on:

- The sample size (N)
- The sample proportion ($\hat{p}$)
- The confidence level (e.g., 90%, 95%, 99%)

Understanding these factors enables us to determine how reliable the sample estimate is as an approximation of the population parameter.

Key Concepts and Definitions

Sample Proportion ($\hat{p}$)

- Represents the proportion of the sample exhibiting a particular characteristic.
- In our case, $\hat{p} = 0.85$, indicating 85% of the sample has the attribute of interest.

Sample Size (N)

- The number of observations or individuals in the sample.
- Here, $N = 290$.

Confidence Level

- The probability that the true population parameter falls within the calculated confidence interval.
- At a 99% confidence level, we are 99% confident that the interval contains the true proportion.

Z-Score for Confidence Level

- The critical value from the standard normal distribution corresponding to the confidence level.
- For 99%, the Z-score is approximately 2.576.

Calculating the Margin of Error for a Population Proportion

The formula to compute the margin of error for a population proportion is:

$$MOE = Z_{\alpha/2} \times \sqrt{\frac{p \times (1 - p)}{N}}$$

Where:

- $Z_{\alpha/2}$ is the Z-score for the desired confidence level.
- p is the sample proportion.
- N is the sample size.

Let's examine each component in detail.

Step 1: Determine the Z-score for 99% Confidence Level

- For a 99% confidence level, the area in each tail is 0.005 (since $1 - 0.99 = 0.01$, split equally).
- The Z-score corresponding to this level is approximately 2.576.

Step 2: Calculate the Standard Error (SE)

- The standard error reflects the variability in the sample proportion:

$$SE = \sqrt{\frac{p \times (1 - p)}{N}}$$

Using the provided values:

- $p = 0.85$
- $N = 290$

Calculate numerator:

$$p \times (1 - p) = 0.85 \times (1 - 0.85) = 0.85 \times 0.15 = 0.1275$$

\]

Calculate the standard error:

\[

$$SE = \sqrt{\frac{0.1275}{290}} \approx \sqrt{0.0004397} \approx 0.02096$$

\]

Step 3: Compute the Margin of Error

- Multiply the Z-score by the standard error:

\[

$$MOE = 2.576 \times 0.02096 \approx 0.0541$$

\]

Rounding to appropriate decimal places (commonly two decimal places for MOE):

\[

$$MOE \approx 0.05$$

\]

Interpreting the Result

The margin of error at a 99% confidence level, given the sample data, is approximately 0.05 or 5%. This means:

- The true population proportion is likely within $\pm 5\%$ of the sample proportion of 85%.
- The confidence interval can be expressed as:

\[

$$0.85 \pm 0.05$$

\]

or

\[

$$[0.80, 0.90]$$

\]

with 99% confidence.

Practical Implications of the Margin of Error

Understanding the margin of error helps in making informed decisions based on survey data. For example:

- If a political poll estimates that 85% of voters favor a candidate, with a margin of error of $\pm 5\%$, the

actual support in the population could range from 80% to 90%.

- The higher the confidence level (like 99%), the larger the margin of error, reflecting increased certainty but reduced precision.
- Conversely, increasing the sample size reduces the margin of error, leading to more precise estimates.

Factors Affecting Margin of Error

Several factors influence the size of the margin of error:

- **Sample Size:** Larger samples decrease the MOE, increasing precision.
- **Sample Proportion:** Proportions near 0.5 yield larger MOEs than proportions near 0 or 1.
- **Confidence Level:** Higher confidence levels result in larger Z-scores, increasing the MOE.

Additional Considerations

- **Sample Size Constraints:** When planning a survey, researchers must balance the desired precision against resources available.
- **Finite Population Correction:** For small populations, adjustments may be necessary, but with $N=290$, the correction is minimal.
- **Assumptions:** The calculations assume a simple random sample and independence of observations.

Summary

- Given a sample size of 290 and a sample proportion of 0.85 at a 99% confidence level, the margin of error is approximately 0.05 or 5%.
- This calculation provides a range within which the true population proportion is likely to fall.
- Increasing sample size or adjusting confidence levels can modify the margin of error.

Conclusion

Calculating the margin of error is a fundamental skill in statistics, enabling accurate interpretation of sample data and effective communication of uncertainty. By understanding the inputs—sample size, sample proportion, and confidence level—and applying the appropriate formula, analysts can confidently quantify the reliability of their estimates. Whether for academic research, public opinion polling, or market analysis, mastering MOE calculations ensures informed decision-making grounded in statistical principles.

Keywords: margin of error, sample proportion, confidence level, Z-score, standard error, survey statistics, population estimate, statistical analysis

Frequently Asked Questions

What is the formula to calculate the margin of error for a proportion at a given confidence level?

The margin of error (ME) for a proportion is calculated using the formula $ME = Z \sqrt{(\hat{p}(1 - \hat{p}))/n}$, where Z is the z-score for the confidence level, $\hat{p}$ is the sample proportion, and n is the sample size.

What is the z-score corresponding to a 99% confidence level?

The z-score for a 99% confidence level is approximately 2.576.

Given $N=290$ and $\hat{p}=0.85$, how do you compute the standard error for the margin of error?

The standard error is computed as $\sqrt{(\hat{p}(1 - \hat{p}))/n} = \sqrt{(0.85 \cdot 0.15)/290}$.

What is the calculated standard error for the given data?

Standard error = $\sqrt{(0.85 \cdot 0.15)/290} \approx \sqrt{0.1275/290} \approx \sqrt{0.0004397} \approx 0.02097$.

How do you find the margin of error at a 99% confidence level with the given data?

Multiply the z-score (2.576) by the standard error: $ME = 2.576 \cdot 0.02097 \approx 0.0541$.

What is the rounded margin of error at a 99% confidence level for the given data?

The rounded margin of error is approximately 0.054.

Why is it important to round the margin of error to three decimal places?

Rounding to three decimal places ensures clarity and consistency in reporting statistical results, especially in survey and research contexts.

Additional Resources

Margin of Error Calculation for a Proportion at 99% Confidence Level: An Expert Breakdown

Introduction: Understanding the Significance of Margin of Error in Statistical Analysis

In the realm of statistics, particularly in survey analysis and polling, the margin of error (MOE) serves as a critical indicator of the accuracy and reliability of an estimate derived from a sample. Whether you're a data analyst, researcher, or a curious reader, grasping how to compute the margin of error at various confidence levels is essential for interpreting results accurately.

Imagine you're analyzing a large population with $N = 290$ individuals, and your sample proportion, $\hat{p}$ (p-hat), stands at 0.85. The question arises: What is the margin of error at a 99% confidence level? This article offers a comprehensive, step-by-step guide to answering this question, breaking down the process as if reviewing a sophisticated product—meticulously, clearly, and thoroughly.

Understanding the Core Components

Before diving into calculations, it's vital to understand the key components involved:

Population Size (N)

- $N = 290$ indicates the total size of the population being studied. This is a finite population, which influences the calculation of the margin of error via the finite population correction (FPC).

Sample Proportion ($\hat{p}$)

- $\hat{p} = 0.85$ signifies that in your sample, 85% of the respondents or observations possess a particular characteristic of interest. This proportion is essential because the variability of the estimate depends on the proportion itself.

Confidence Level

- 99% Confidence Level reflects the degree of certainty that the true population parameter falls within the calculated margin of error. The higher the confidence level, the wider the margin—aiming to ensure the true value is captured with high probability.

Step-by-Step Calculation of the Margin of Error

Calculating the margin of error for a proportion in a finite population involves several steps and formulas. Let's explore each in detail.

1. Recognize the Appropriate Formula

For proportions, the general formula for the margin of error at a given confidence level is:

$$\text{MOE} = Z_{\alpha/2} \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} \times \text{FPC}$$

Where:

- $Z_{\alpha/2}$ is the Z-score corresponding to the desired confidence level.
- $\hat{p}$ is the sample proportion.
- n is the sample size.
- FPC is the finite population correction factor, used when the sample size is large relative to the population.

Note: Since the problem provides only the population size and the proportion, but not the sample size n , we will assume the sample size is the entire population ($n = N = 290$), a common scenario in census-like studies. If the sample size is different, the calculation adjusts accordingly.

2. Determine the Z-Score for a 99% Confidence Level

The Z-score corresponding to a 99% confidence level is found from the standard normal distribution. It captures how many standard deviations away from the mean we need to go to encompass 99% of the distribution.

From Z-tables or statistical software:

$$Z_{0.005} \approx 2.576$$

Explanation: Because 99% confidence implies 1% significance level split equally in the two tails (0.5% in each tail), we look up the Z-score for 0.995 cumulative probability.

3. Calculate the Standard Error (SE)

The standard error for a proportion, assuming the entire population, is:

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Plugging in the known values:

$$SE = \sqrt{\frac{0.85 \times (1 - 0.85)}{290}} = \sqrt{\frac{0.85 \times 0.15}{290}}$$

Calculating numerator:

$$0.85 \times 0.15 = 0.1275$$

Dividing by the population size:

$$\frac{0.1275}{290} \approx 0.0004397$$

Taking the square root:

$$SE \approx \sqrt{0.0004397} \approx 0.02097$$

4. Incorporate the Finite Population Correction (FPC)

Since the sample size $n = 290$ equals the population $N = 290$, this indicates a census, meaning every individual is surveyed. In such cases, the standard error collapses to zero because there's no sampling variability—the estimate is exact.

However, if the sample were smaller than the population, the FPC would be:

$$FPC = \sqrt{\frac{N - n}{N - 1}}$$

In this particular case:

$$FPC = \sqrt{\frac{290 - 290}{289}} = 0$$

which would make the margin of error zero, indicating perfect certainty.

But, for demonstration and typical survey scenarios, let's assume the sample size is smaller, say $n = 100$. If you wish, you can adapt the calculation to that.

5. Final Calculation of the Margin of Error

Assuming $n = 100$ for illustrative purposes, the FPC becomes:

$$\begin{aligned} \backslash \\ \text{FPC} &= \sqrt{\frac{290 - 100}{290 - 1}} = \sqrt{\frac{190}{289}} \approx \sqrt{0.657} \approx 0.811 \\ \backslash \end{aligned}$$

Now, the margin of error:

$$\begin{aligned} \backslash \\ \text{MOE} &= Z_{0.005} \times \text{SE} \times \text{FPC} \\ \backslash \end{aligned}$$

Substituting values:

$$\begin{aligned} \backslash \\ \text{MOE} &= 2.576 \times 0.02097 \times 0.811 \approx 2.576 \times 0.01702 \approx 0.0439 \\ \backslash \end{aligned}$$

Interpretation: The margin of error is approximately 0.0439 or about 4.39% at a 99% confidence level, given the sample size of 100.

Summary & Practical Recommendations

- For the specific scenario where the entire population is sampled ($n = N = 290$):
The margin of error at 99% confidence effectively becomes zero, indicating perfect knowledge of the population proportion.
- For typical sample surveys where $n < N$:
Use the formula incorporating the finite population correction to accurately estimate the margin of error.
- Key takeaways:
 - The Z-score for 99% confidence is approximately 2.576.
 - Variability depends heavily on the sample proportion; values near 0.5 produce the largest MOE.
 - Larger samples reduce the margin of error, increasing estimate precision.

- Finite population correction becomes significant when the sample size is a substantial fraction of the population (usually over 5%).

Conclusion: The Power of Precise Margin of Error Calculation

Understanding and calculating the margin of error is essential for interpreting survey results responsibly. In scenarios with large populations and high proportions like $\hat{p} = 0.85$, the margin of error at a high confidence level such as 99% can be relatively small, provided the sample size is adequate.

By meticulously applying the formulas, considering the specifics of your population and sample size, and understanding the influence of the confidence level, you can confidently report findings with clarity and statistical rigor.

Whether you're analyzing a small community or a large demographic, mastering the margin of error calculation empowers you to communicate results transparently and accurately—an indispensable skill in data-driven decision-making.

Note: Adjust the sample size n in the calculation as per your actual data to get the precise margin of error relevant to your study.

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