If The Gradient Of The Tangent To [tex]y = \sqrt{x} [/tex]is [tex] \frac{1}{6} [/tex]at Point A, Find

If The Gradient Of The Tangent To \(y = \sqrt{x} \) is \(\frac{1}{6} \) at Point A, Find

Understanding how to analyze the gradient of a tangent to a curve is a fundamental aspect of differential calculus. When given a function such as \(y = \sqrt{x} \), determining the point at which the tangent's gradient equals a specific value involves applying derivative rules, solving equations, and interpreting the geometric significance of the results. In this comprehensive guide, we will explore the problem step-by-step, elucidate the concepts involved, and provide insights into related topics for a deeper understanding.

Overview of the Problem

The problem states:

> If the gradient of the tangent to \(y = \sqrt{x} \) is \(\frac{1}{6} \) at Point A, find the coordinates of Point A.

This involves several key concepts:
- Differentiation to find the gradient function
- Setting the gradient equal to the given value
- Solving for the x-coordinate
- Finding the y-coordinate using the original function

By understanding each component, students and enthusiasts can systematically approach similar problems involving derivatives and tangent lines.

Understanding the Function $y = \sqrt{x}$

Before delving into derivatives, it is essential to understand the behavior and properties of the function:

- The function $y = \sqrt{x}$ is defined for $x \geq 0$.
- It is a monotonically increasing function.
- The graph is a curve starting at the origin $(0, 0)$, gradually flattening as x increases.

Graphically, the tangent lines touch the curve at specific points, and their slopes (gradients) indicate how steep those tangents are at those points.

Calculating the Gradient of the Tangent: Derivative of $y = \sqrt{x}$

The first step is to find the derivative $\frac{dy}{dx}$, which represents the gradient of the tangent to the curve at any point x.

Differentiation of $y = \sqrt{x}$

Since $y = \sqrt{x}$, we can rewrite it as:

$$
y = x^{1/2}
$$

Applying the power rule:

$$
\frac{dy}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2} \times \frac{1}{\sqrt{x}} = \frac{1}{2 \sqrt{x}}
$$

This derivative gives us the gradient of the tangent at any point x.

Finding the Point Where the Gradient is $\frac{1}{6}$

Given that:

$$
\frac{dy}{dx} = \frac{1}{6}
$$

we set the derivative equal to this value:

$$
\frac{1}{2 \sqrt{x}} = \frac{1}{6}
$$

Solving for \(x \)

Cross-multiplied:

\[

6 \times \frac{1}{2 \sqrt{x}} = 1

\]

which simplifies to:

\[

\frac{6}{2 \sqrt{x}} = 1

\]

\[

\frac{3}{\sqrt{x}} = 1

\]

Multiplying both sides by \(\sqrt{x} \):

\[

3 = \sqrt{x}

\]

Squaring both sides:

\[

(3)^2 = (\sqrt{x})^2

\]

\[

$$
9 = x
$$

Thus, the x-coordinate of Point A is:

$$
x = 9
$$

Finding the Corresponding y-Coordinate at Point A

Now, substitute $x = 9$ into the original function:

$$
y = \sqrt{9} = 3
$$

Therefore, Point A has coordinates:

$$
(9, 3)
$$

Summary of Results

- The point where the tangent to $y = \sqrt{x}$ has a gradient of $\frac{1}{6}$ is at $(9, 3)$.
- The process involved differentiating the function, equating the derivative to the given gradient, solving for x, and then finding y.

Extended Discussion: Tangent Line Equation at Point A

Understanding the point is only part of the problem. To deepen comprehension, let's determine the equation of the tangent line at point A.

Equation of the Tangent Line

The general form of a tangent line at (x_0, y_0) with gradient m:

$$
y - y_0 = m (x - x_0)
$$

Substituting:
- $x_0 = 9$
- $y_0 = 3$
- $m = \frac{1}{6}$

we get:

$$
y - 3 = \frac{1}{6} (x - 9)
$$

Simplify:

$$
y = \frac{1}{6} x - \frac{9}{6} + 3
$$

$$
y = \frac{1}{6} x - \frac{3}{2} + 3
$$

$$
y = \frac{1}{6} x + \frac{3}{2}
$$

This is the equation of the tangent line at Point A.

Applications and Related Concepts

Understanding how to find points where the derivative takes specific values is crucial in various fields:

1. **Optimization Problems:** Finding maximum or minimum values of functions by analyzing derivatives.

2. **Curve Sketching:** Determining points of inflection, increasing/decreasing intervals, and concavity.

3. **Physics:** Calculating velocity and acceleration where the position function's derivative equals specific values.

4. **Economics:** Marginal analysis where derivatives represent rates of change of costs, revenues, or profits.

Common Mistakes to Avoid

While solving derivative-based problems, be cautious of:

- **Incorrect Differentiation:** Remember to apply the power rule correctly, especially with fractional exponents.

- **Not Solving Fully:** Ensure to solve for all variables step-by-step, avoiding skipped steps that can lead to errors.

- **Domain Restrictions:** The original function $y = \sqrt{x}$ is only defined for $x \geq 0$, so solutions should respect this domain.

- **Sign Errors:** Be mindful of signs when rearranging equations, especially with negative exponents or subtracting terms.

Additional Practice Problems

To strengthen understanding, consider practicing the following:

1. Find the point on $y = x^{3/2}$ where the tangent has a gradient of 2.
2. Determine the point(s) on $y = \sin x$ where the tangent line is horizontal (gradient = 0).
3. For the curve $y = \frac{1}{x}$, find all points where the tangent has a gradient of -1.

Conclusion

In summary, analyzing the gradient of the tangent to the curve $y = \sqrt{x}$ at a specific point involves:

- Deriving the gradient function using differentiation
- Setting this derivative equal to the given gradient
- Solving for the corresponding x-coordinate
- Calculating the y-coordinate from the original function

This process exemplifies fundamental calculus techniques that are widely applicable across mathematics and science disciplines. Mastery of these concepts enhances problem-solving skills and deepens understanding of the geometric interpretation of derivatives.

Should you wish to explore further, delving into the topics of second derivatives, curvature, and the relationships between derivatives and the shape of curves will provide a more comprehensive grasp of calculus principles.

Key Takeaways:

- The derivative of $y = \sqrt{x}$ is $\frac{dy}{dx} = \frac{1}{2 \sqrt{x}}$.
- Setting the derivative equal to $\frac{1}{6}$ yields $x = 9$.
- The corresponding y-coordinate is 3.
- The tangent line at this point is $y = \frac{1}{6} x + \frac{3}{2}$.

By understanding each step and practicing similar problems, learners can confidently analyze tangents, gradients, and their implications in various contexts.

Frequently Asked Questions

What is the derivative of $y = \sqrt{x}$?

The derivative of $y = \sqrt{x}$ is $y' = 1 / (2\sqrt{x})$.

How do you find the gradient of the tangent to $y = \sqrt{x}$ at a point?

The gradient of the tangent at a point $x = a$ is given by $y' = 1 / (2\sqrt{a})$.

Given the gradient of the tangent is 1/6, how do you find the x-coordinate of point A?

Set $1 / (2\sqrt{a}) = 1/6$ and solve for a: $2\sqrt{a} = 6$, so $\sqrt{a} = 3$, thus $a = 9$.

What is the y-coordinate of point A on y = √x when x = 9?

When x = 9, y = √9 = 3.

What are the coordinates of point A where the tangent has a gradient of 1/6?

Point A is at (9, 3) on the curve y = √x.

How do you verify the gradient at point A?

Calculate y' at x = 9: y' = 1 / (2√9) = 1 / (2 3) = 1/6, confirming the gradient.

What is the significance of the gradient being 1/6 at point A?

It indicates the slope of the tangent line to the curve y = √x at point A is 1/6.

Can you find the equation of the tangent line at point A?

Yes. With point (9, 3) and slope 1/6, the tangent line is y - 3 = (1/6)(x - 9).

How do you interpret the problem 'Find' at the end of the question?

It asks for the coordinates of point A where the tangent to y = √x has a gradient of 1/6.

Additional Resources

Mathematics Analysis: Understanding the Gradient of the Tangent to \(y = \sqrt{x} \) at a Specific Point

---

Introduction

When exploring the fascinating realm of calculus, the concept of tangent lines and their gradients (or slopes) plays a pivotal role in understanding the behavior of curves. The function $y = \sqrt{x}$, a fundamental example of a square root function, offers an insightful case study in how the slope of a tangent line varies with x. This article delves into a specific problem: If the gradient of the tangent to $y = \sqrt{x}$ at a certain point A is $\frac{1}{6}$, find the coordinates of point A.

Through a comprehensive exploration, we will uncover the calculus principles involved, perform detailed calculations, and interpret the significance of the results. This approach aims to provide clarity, depth, and practical understanding for students, educators, and enthusiasts alike.

Understanding the Function $y = \sqrt{x}$

Basic Properties and Graphical Behavior

The function $y = \sqrt{x}$ is defined for $x \geq 0$, producing a graph that starts at the origin $(0, 0)$ and gradually increases, becoming less steep as x grows large. Its shape is a classic example of a concave-down curve, illustrating how the rate of change diminishes with larger x.

- Domain: $x \geq 0$
- Range: $y \geq 0$
- Continuity and differentiability: The function is continuous and differentiable for all $x > 0$. At $x=0$, the derivative exists but approaches infinity from the left, which is acceptable since the domain excludes negative x.

Deriving the Gradient Function

Step 1: Express the derivative

The gradient of the tangent to the curve at any point x is given by the derivative $\frac{dy}{dx}$.

Given:

$$y = \sqrt{x} = x^{1/2}$$

Applying the power rule:

$$\frac{dy}{dx} = \frac{1}{2} x^{-1/2} = \frac{1}{2 \sqrt{x}}$$

This derivative function tells us how the slope varies with x.

Step 2: Interpreting the derivative

- For any $x > 0$, the gradient is positive, indicating an increasing function.
- As $x \to 0^+$, $\frac{dy}{dx} \to \infty$, meaning the tangent becomes vertical.
- As $x \to \infty$, $\frac{dy}{dx} \to 0$, indicating the curve flattens out.

The Core Problem: Finding the Point A

Given: The gradient of the tangent at point A is $\frac{1}{6}$.

Objective: Find the coordinates (x, y) of point A.

Step-by-Step Solution

Step 1: Set the derivative equal to $\frac{1}{6}$

Using the derivative expression:

$$ \frac{dy}{dx} = \frac{1}{2 \sqrt{x}} $$

Set this equal to $\frac{1}{6}$:

$$ \frac{1}{2 \sqrt{x}} = \frac{1}{6} $$

Step 2: Solve for x

Cross-multiplied:

$$ 6 \times \frac{1}{2 \sqrt{x}} = 1 $$

$$ \frac{6}{2 \sqrt{x}} = 1 $$

$$ \frac{3}{\sqrt{x}} = 1 $$

Multiplying both sides by $\sqrt{x}$:

$$ 3 = \sqrt{x} $$

Squaring both sides:

$$ (3)^2 = x $$

$$ x = 9 $$

Step 3: Find the y-coordinate

Recall:

$$ y = \sqrt{x} $$

Substitute $x = 9$:

$$ y = \sqrt{9} = 3 $$

Result:

$$
\boxed{
\textbf{Point } A = (9, 3)
}
$$

This is the point on the curve where the tangent has a gradient of $\frac{1}{6}$.

Additional Insights and Context

The Significance of the Gradient

Understanding the gradient at a point on a curve provides insight into the curve’s behavior at that point:

- Steepness: At $x=9$, the tangent line has a gentle slope of $\frac{1}{6}$, indicating a mild incline.
- Rate of change: The derivative’s value indicates how quickly y is increasing relative to x.

Visual Interpretation

Imagine drawing the curve $y = \sqrt{x}$. At $x=9$, the tangent line touches the curve smoothly, with a shallow incline. Knowing the point allows us to sketch or analyze the tangent line explicitly, which can be useful in optimization problems, modeling, and understanding the local behavior of the function.

Extending the Analysis: Equation of the Tangent Line at A

For further application, it's often useful to find the equation of the tangent line at the point A.

- Point: $(9, 3)$
- Gradient: $m = \frac{1}{6}$

Using the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

$$
y - 3 = \frac{1}{6}(x - 9)
$$

Simplify:

$$
y = \frac{1}{6}x - \frac{9}{6} + 3
$$

$$
y = \frac{1}{6}x - 1.5 + 3
$$

$$
y = \frac{1}{6}x + 1.5
$$

This line is the tangent to the curve at $(9, 3)$.

Broader Applications and Real-World Contexts

Understanding how the gradient varies along the curve $y = \sqrt{x}$ has practical significance in various fields:

- Physics: The slope relates to velocity in certain kinematic models.
- Economics: Curves like $y = \sqrt{x}$ model diminishing returns.
- Engineering: Slope analysis informs structural design where gradual changes are critical.

Summary and Key Takeaways

- The derivative $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$ describes how the slope varies along $y = \sqrt{x}$.
- Given a gradient of $\frac{1}{6}$, solving $\frac{1}{2 \sqrt{x}} = \frac{1}{6}$ yields $x = 9$.
- The corresponding y-coordinate is $y=3$.
- The point A where the tangent has the specified gradient is $(9, 3)$.
- The tangent line at A has the equation $y = \frac{1}{6}x + 1.5$.

This detailed exploration emphasizes the interconnectedness of derivatives, tangent lines, and the geometric interpretation of functions—core concepts that underpin much of calculus and mathematical modeling.

Final Thoughts

Mastering the process of linking a gradient value to a specific point on a curve exemplifies the power of calculus in analyzing and interpreting the behavior of mathematical functions. Whether for academic pursuits or practical applications, such skills enhance our ability to navigate complex systems, optimize solutions, and deepen our understanding of the mathematical landscape.

If The Gradient Of The Tangent To Tex Y Sqrt X Tex Is Tex Frac 1 6 Tex At Point A Find

If The Gradient Of The Tangent To Tex Y Sqrt X Tex Is Tex Frac 1 6 Tex At Point A Find

Related Articles

- I Need Help With This Riddle Please.Women Gathered To Protest Against The Shortage Of This Item. Lack
- Jounal Entry For Nicole's Necklaces Purchased A Diamond Necklace Two Years Ago For $700. However, Due
- For A School Project, Rachel Is Supposed To Make A Flag Thats Similar To The One Shown, But Bigger In

Back to Home