If The Half-life Of A Radioactive Element Is 18 Days, What Percentage Of The Original Sample Would Be

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Understanding the concept of half-life is fundamental in nuclear chemistry and physics. When dealing with radioactive decay, the half-life refers to the time required for half of the radioactive nuclei in a sample to decay. In this article, we explore what happens to a sample of a radioactive element with a known half-life of 18 days over various periods, focusing on determining the remaining percentage of the original sample after specific durations. Through this analysis, readers will grasp the mathematical principles underlying radioactive decay and their practical implications.

Introduction to Radioactive Decay and Half-life

What Is Radioactive Decay?

Radioactive decay is a spontaneous process by which unstable atomic nuclei lose energy by emitting radiation in the form of alpha particles, beta particles, or gamma rays. This process transforms the original nucleus into a different element or isotope, often resulting in a more stable configuration.

The Concept of Half-life

The half-life (denoted as $T_{1/2}$) is a characteristic property of each radioactive isotope. It indicates the time it takes for half of the nuclei present in a sample to decay. This decay process follows an exponential pattern, meaning the quantity of the remaining radioactive material decreases by half during each successive half-life period.

Mathematical Foundation of Radioactive Decay

The Decay Formula

The amount of radioactive substance remaining after a certain time can be calculated using the formula:

$$ N(t) = N_0 \times \left(\frac{1}{2} \right)^{\frac{t}{T_{1/2}}} $$

Where:
- $N(t)$ is the quantity remaining after time t,
- N_0 is the initial quantity,
- $T_{1/2}$ is the half-life,
- t is the elapsed time.

Understanding the Formula

This exponential decay formula indicates that after each half-life period, the remaining quantity is halved. The ratio $\frac{t}{T_{1/2}}$ determines how many half-lives have elapsed. The larger this ratio, the smaller the remaining fraction of the original sample.

Calculating Remaining Percentage After a Given Period

Basic Approach

To find what percentage of the original sample remains after a specific time, we:

1. Determine how many half-lives have passed.
2. Calculate the remaining fraction using the decay formula.
3. Convert this fraction into a percentage.

Step-by-step Calculation Example

Suppose we want to find out how much of the original sample remains after 54 days for an isotope with an 18-day half-life:

1. Calculate the number of half-lives:

$$ \text{Number of half-lives} = \frac{54 \text{ days}}{18 \text{ days}} = 3 $$

2. Determine the remaining fraction:

$$ N(54) = N_0 \times \left(\frac{1}{2} \right)^3 = N_0 \times \frac{1}{8} $$

3. Convert to percentage:

$$ \frac{1}{8} \times 100\% = 12.5\% $$

Thus, after 54 days, 12.5% of the original sample remains.

Application: Percentage Remaining After Specific Timeframes

After One Half-life (18 days)

- Remaining percentage: 50%

After Two Half-lives (36 days)

- Remaining percentage: 25%

After Three Half-lives (54 days)

- Remaining percentage: 12.5%

After Four Half-lives (72 days)

- Remaining percentage: 6.25%

Implications in Real-world Scenarios

Radioactive Dating

Geologists and archaeologists use the concept of half-life to date ancient artifacts and fossils. Knowing the half-life of a radioactive isotope allows scientists to estimate the age of a sample by measuring its remaining radioactivity.

Medical Applications

Radioisotopes with specific half-lives are used in medical imaging and treatments. For example, isotopes with short half-lives are preferred for diagnostic scans because they decay quickly, minimizing radiation exposure.

Waste Management

Nuclear waste management relies on understanding the decay rates of various isotopes to determine the duration for which waste must be stored safely.

Practical Calculation: What Percentage Would Be Left After 18 Days?

Given Data

- Half-life, $T_{1/2} = 18$ days
- Time elapsed, $t = 18$ days

Calculation

Number of half-lives:

$$\frac{t}{T_{1/2}} = \frac{18}{18} = 1$$

Remaining fraction:

$$N(t) = N_0 \times \left(\frac{1}{2} \right)^1 = N_0 \times \frac{1}{2}$$

Remaining percentage:

$$\frac{1}{2} \times 100\% = 50\%$$

Answer: After 18 days, 50% of the original sample remains.

Extending the Concept: Decay Over Longer Periods

After 36 Days (Two Half-lives)

- Remaining percentage: 25%

After 54 Days (Three Half-lives)

- Remaining percentage: 12.5%

After 72 Days (Four Half-lives)

- Remaining percentage: 6.25%

This pattern emphasizes the exponential nature of radioactive decay and how rapidly the quantity diminishes over successive half-lives.

Limitations and Considerations

Assumptions in Calculations

- Radioactive decay follows an ideal exponential pattern.
- The decay process is random and independent for each nucleus.
- External factors (e.g., environmental influences) do not affect the decay rate.

Real-world Variations

While the mathematical model provides an accurate average, individual nuclei decay unpredictably. The half-life is a statistical measure rather than an exact predictor for any single atom.

Summary and Key Takeaways

- The half-life of a radioactive element is the time it takes for half of the sample to decay.
- For an isotope with a half-life of 18 days, 50% remains after 18 days, 25% after 36 days, 12.5% after 54 days, and so on.
- The decay follows an exponential pattern, making the remaining percentage decrease rapidly with each half-life.
- Understanding this process is crucial in various scientific and practical applications, including dating, medicine, and waste management.

Conclusion

Knowing the half-life of a radioactive isotope allows scientists and engineers to predict the remaining quantity over time accurately. In the specific case of an 18-day half-life, the percentage of the original sample remaining after any given period can be calculated precisely using the exponential decay formula. This knowledge underpins many critical technologies and scientific methods, demonstrating the importance of understanding radioactive decay processes.

Frequently Asked Questions

If the half-life of a radioactive element is 18 days, what percentage of the original sample remains

after 36 days?

25% of the original sample remains after 36 days because it undergoes two half-lives (36 ÷ 18 = 2), halving twice: 100% → 50% → 25%.

How much of a radioactive sample remains after 54 days if the half-life is 18 days?

Remaining sample is 12.5% because 54 days is three half-lives (54 ÷ 18 = 3), and after three halvings: 100% → 50% → 25% → 12.5%.

What is the decay formula used to determine the remaining percentage after a certain time?

The remaining amount is given by $N = N_0 \times (1/2)^{t / T_{1/2}}$, where N_0 is the initial amount, t is time elapsed, and $T_{1/2}$ is the half-life.

If only 25% of the original radioactive sample remains, how many days have passed?

Six days have passed, which is one-third of a half-life period, but more precisely, after two half-lives (36 days), 25% remains. For shorter periods, use the decay formula accordingly.

Can you calculate the remaining percentage after 72 days for this element?

Yes, after 72 days (4 half-lives), remaining percentage is 6.25% (100% → 50% → 25% → 12.5% → 6.25%).

Why is understanding the half-life important in radioactive decay?

It helps determine how long it takes for a radioactive substance to decay to a certain level, which is crucial for medical, environmental, and dating applications.

Is the decay process linear or exponential over time, and why?

The decay process is exponential because the amount decreases by half every half-life period, not by a fixed amount each interval.

How would you find the remaining percentage after an

arbitrary number of days if the half-life is known?

Use the decay formula: Remaining percentage = $(1/2)^{t / T_{1/2}} \times 100\%$, where t is the elapsed time.

If an initial sample is 100 grams, what is the remaining amount after 54 days?

The remaining amount is 12.5 grams, since 54 days equals three half-lives ($54 \div 18 = 3$), and after three halvings: 100g → 50g → 25g → 12.5g.

Additional Resources

Understanding Radioactive Decay and Half-Life: A Comprehensive Analysis

Radioactivity is a fascinating and fundamental aspect of nuclear physics, with profound applications ranging from medical imaging to radiometric dating. Central to understanding radioactive decay is the concept of half-life, which defines the time it takes for half of a given sample of a radioactive isotope to decay. In this detailed review, we will explore the question: If the half-life of a radioactive element is 18 days, what percentage of the original sample would remain after a certain period? We will delve into the principles of decay, mathematical modeling, real-world applications, and nuances that influence these calculations, providing a thorough understanding of the topic.

Fundamentals of Radioactive Decay and Half-Life

What Is Radioactive Decay?

Radioactive decay is the spontaneous transformation of an unstable atomic nucleus into a more stable configuration, often accompanied by the emission of radiation such as alpha particles, beta particles, or gamma rays. This process is inherently random for individual atoms but exhibits a predictable statistical pattern across large populations.

Defining Half-Life

The half-life (denoted as $t_{1/2}$) is a characteristic property of each radioactive isotope. It is the duration required for half of the atoms in a sample to decay. For example:

- If you start with 100 grams of a substance with a half-life of 18 days,
- After 18 days, approximately 50 grams remain.
- After another 18 days (total 36 days), about 25 grams remain,
- And so forth, following an exponential decay pattern.

This concept simplifies the measurement and prediction of radioactive decay over time.

The Mathematical Model of Radioactive Decay

The Exponential Decay Law

Radioactive decay follows an exponential law, mathematically expressed as:

$$
N(t) = N_0 \times e^{-\lambda t}
$$

Where:
- $N(t)$ = quantity of remaining radioactive material at time t,
- N_0 = initial quantity of the material,
- λ = decay constant (probability per unit time that a nucleus will decay),
- t = elapsed time.

This equation highlights that the amount of remaining material decreases exponentially with time.

Connecting Half-Life to Decay Constant

The decay constant λ relates directly to the half-life:

$$
t_{1/2} = \frac{\ln 2}{\lambda}
$$

Rearranged, the decay constant can be expressed as:

$$
\lambda = \frac{\ln 2}{t_{1/2}}
$$

Given the half-life, one can compute λ, which is essential for making decay predictions.

Calculating the Remaining Percentage After a Given Time

General Formula

The percentage of the original sample remaining after time t can be derived from the decay law:

$$
\frac{N(t)}{N_0} = e^{-\lambda t}
$$

Expressed as a percentage:

$$
\text{Remaining Percentage} = 100 \times e^{-\lambda t}
$$

Applying the Formula for a Half-Life of 18 Days

Given:
- $t_{1/2} = 18$ days,

Calculate λ:

$$
\lambda = \frac{\ln 2}{18} \approx \frac{0.6931}{18} \approx 0.0385 \ \text{per day}
$$

Now, to determine what percentage of the original sample remains after a specific period, say:

- 18 days (one half-life),
- 36 days (two half-lives),
- 54 days (three half-lives),
- Any arbitrary time t.

Let's analyze these scenarios:

After One Half-Life (18 Days)

Using the decay law:

$$
\frac{N(18)}{N_0} = e^{-\lambda \times 18} = e^{-(0.0385) \times 18} = e^{-\ln 2} = \frac{1}{2}
$$

Result: 50% of the original sample remains, which aligns with the definition of half-life.

After Two Half-Lives (36 Days)

$$
\frac{N(36)}{N_0} = e^{-\lambda \times 36} = e^{-2 \times \ln 2} = (e^{-\ln 2})^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}
$$

Result: 25% remains.

After Three Half-Lives (54 Days)

$$
\frac{N(54)}{N_0} = \left(\frac{1}{2}\right)^3 = \frac{1}{8}
$$

Result: 12.5% remains.

General Case: After n Half-Lives

$$
\text{Remaining Percentage} = \left(\frac{1}{2}\right)^n \times 100\%

$$
\text{}
$$

Where:

$$
n = \frac{t}{t_{1/2}}
$$

This simplifies calculations for any period.

Practical Examples and Implications

Example 1: Remaining Percentage after 25 Days

Calculate n:

$$
n = \frac{25}{18} \approx 1.39
$$

Remaining fraction:

$$
\left(\frac{1}{2}\right)^{1.39} \approx e^{-\ln 2 \times 1.39} \approx e^{-0.6931 \times 1.39} \approx e^{-0.963} \approx 0.382
$$

Corresponding percentage:

$$
38.2\%
$$

Interpretation: Approximately 38.2% of the original sample remains after 25 days.

Example 2: Remaining Percentage after 50 Days

Calculate n:

$$

$$
n = \frac{50}{18} \approx 2.78
$$

Remaining fraction:

$$
\left(\frac{1}{2}\right)^{2.78} \approx e^{-0.6931 \times 2.78} \approx e^{-1.926} \approx 0.146
$$

Percentage:

$$
14.6\%
$$

Interpretation: About 14.6% of the original sample remains after 50 days.

Real-World Applications and Considerations

Radiometric Dating

Understanding the half-life is crucial for dating geological and archaeological samples. For example, Carbon-14, with a half-life of approximately 5730 years, allows scientists to estimate the age of ancient artifacts. In contrast, isotopes with shorter half-lives, like the hypothetical element with an 18-day half-life, are used in medical diagnostics and treatment.

Medical Imaging and Treatment

Radioisotopes with known half-lives are used in medical imaging (e.g., Technetium-99m with a half-life of about 6 hours) due to their predictable decay rates, which optimize imaging quality while minimizing radiation exposure.

Radioactive Waste Management

Knowing the decay profile of isotopes helps in designing storage and disposal strategies, ensuring safety as the radioactivity diminishes over time.

Nuances and Factors Affecting Decay Calculations

While the mathematical model provides a solid foundation, real-world scenarios may introduce complexities:

- Decay chains: Some isotopes decay into other radioactive isotopes, which may also be radioactive, affecting the overall activity over time.
- Environmental factors: External conditions like temperature, pressure, and chemical environment generally do not influence decay rates significantly for most isotopes.
- Detection efficiency: In practical measurements, detecting decay events involves instrumentation that may have varying efficiency, influencing the interpretation of remaining quantities.
- Sample purity and contamination: Impurities can affect decay measurements or the interpretation of data.

Summary and Final Thoughts

To succinctly answer the core question: If the half-life of a radioactive element is 18 days, what percentage of the original sample would be? — the answer depends on the elapsed time:

- After 1 half-life (18 days): 50% remains.
- After 2 half-lives (36 days): 25% remains.
- After 3 half-lives (54 days): 12.5% remains.
- After any arbitrary time t:

$$
\text{Remaining Percentage} = 100 \times \left(\frac{1}{2}\right)^{t/18}
$$

or equivalently,

$$
= 100 \times e^{-\lambda t}
$$

with

$$
\lambda = \frac{\ln 2}{18} \approx 0.0385 \, \text{per day}
$$

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This exponential decay model is robust and fundamental
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