If The Length Of Two Sides Of A Triangle Are 10cm And 17cm, What Is The Range Of Possible Lengths For

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Understanding the possible lengths of the third side of a triangle when two sides are known is a fundamental concept in geometry. This question explores the principles behind the triangle inequality theorem and how it determines the feasible range for the third side's length. Whether you're a student preparing for exams or simply interested in geometric properties, this comprehensive guide will provide clarity, detailed explanations, and practical examples to help you grasp the concept thoroughly.

Introduction to Triangle Inequality Theorem

The triangle inequality theorem is a critical rule in geometry that states:

In any triangle, the length of each side must be less than the sum of the lengths of the other two sides and greater than their difference.

This theorem ensures that a set of three lengths can form a triangle only if they satisfy certain conditions.

Mathematically, for three sides a, b, and c:

- $a < b + c$
- $b < a + c$
- $c < a + b$

and

- $a > |b - c|$
- $b > |a - c|$
- $c > |a - b|$

For the problem at hand, since two sides are known, the third side's possible lengths are derived from these inequalities.

Problem Statement Revisited

Given:

- Side 1 (a) = 10 cm
- Side 2 (b) = 17 cm

Find:

- The range of possible lengths for the third side (c), denoted as c, in centimeters, such that the three sides can form a valid triangle.

Applying the Triangle Inequality to the Given Sides

To find the possible values of c, apply the triangle inequality theorem:

1. $c < a + b$
2. $c > |a - b|$

Let's compute these:

- Sum of known sides: $a + b = 10 + 17 = 27$ cm
- Difference of known sides: $|a - b| = |10 - 17| = 7$ cm

Therefore, the third side c must satisfy:

$$7 < c < 27$$

This inequality indicates that the third side must be greater than 7 cm but less than 27 cm for the three lengths to form a valid triangle.

Understanding the Range of the Third Side

The key takeaway from the inequalities is that:

- The minimum length of the third side is just over 7 cm.
- The maximum length of the third side is just under 27 cm.

This range ensures the sides satisfy the triangle inequality theorem, enabling the formation of a valid triangle.

Visualizing the Range

Imagine a number line:

- Starting just above 7 cm (e.g., 7.0001 cm), the third side can be any length up to just below 27 cm (e.g., 26.9999 cm).
- At exactly 7 cm or 27 cm, the figure degenerates into a straight line rather than a triangle, which is invalid in Euclidean geometry.

Examples of Possible Lengths for the Third Side

Let's consider some specific values within the range:

Possible c (cm)	Valid?	Explanation
8	Yes	$8 > 7$ and $8 < 27$
15	Yes	$15 > 7$ and $15 < 27$
26	Yes	$26 > 7$ and $26 < 27$
27	No	Equal to the sum of other sides; degenerates into a line
7	No	Equal to the difference; no longer forms a triangle

Special Note:
- The triangle is degenerate if the third side equals exactly 7 cm or 27 cm, which is generally not considered a valid triangle in classical geometry.

Implications and Practical Applications

Understanding the range of possible third side lengths is not just an academic exercise; it has practical applications in various fields:

- Engineering and Construction: Ensuring structural components can fit together correctly.
- Navigation and Surveying: Calculating feasible distances when triangulating positions.
- Computer Graphics: Generating valid triangles for mesh creation and rendering.
- Education: Enhancing comprehension of geometric properties and problem-solving skills.

Advanced Considerations

While the basic triangle inequality provides a clear range, real-world scenarios might involve additional constraints:

- Material limitations: In manufacturing, the length of a side might be restricted by material sizes.
- Design specifications: Certain applications may require the third side to fall within a specific sub-range.
- Coordinate geometry: When sides are represented in coordinate systems, the problem involves calculating distances using the distance formula.

Step-by-Step Solution Summary

To summarize, when two sides of a triangle are given as 10 cm and 17 cm:

1. Calculate the sum: $10 + 17 = 27$ cm
2. Calculate the difference: $|10 - 17| = 7$ cm
3. Apply the triangle inequality:

$$
7 < c < 27
$$

4. Final answer:

The length of the third side must be greater than 7 cm and less than 27 cm.

Conclusion

Determining the range of possible lengths for the third side of a triangle when two sides are known involves applying the triangle inequality theorem. The key is understanding that the third side must be strictly greater than the absolute difference of the two known sides and strictly less than their sum.

In this specific case, with sides measuring 10 cm and 17 cm, the third side's length can be any value in the open interval:

$$
(7\, \text{cm},\, 27\, \text{cm})
$$

This fundamental concept helps in solving various geometric problems, designing structures, and understanding the properties of triangles more deeply.

FAQs

1. **Can the third side be exactly 7 cm or 27 cm?**
   No, because that would result in a degenerate triangle, which in classical geometry is not considered a valid triangle.

2. **What happens if the third side is less than 7 cm?**
   The three sides cannot form a triangle because the triangle inequality is violated.

3. **What about if the third side is more than 27 cm?**
   Again, the triangle inequality is violated, and a triangle cannot be formed.

4. **How is this concept useful in real-world applications?**
   It helps in designing structures, navigation, computer graphics, and understanding geometric constraints in various engineering problems.

By mastering the principles outlined above, you can confidently determine the feasible range for the third side of any triangle given two sides, enhancing your problem-solving toolkit in geometry.

Frequently Asked Questions

If the lengths of two sides of a triangle are 10cm and 17cm, what is the range of possible lengths for the third side?

The third side must be greater than the difference of the two known sides (17cm - 10cm = 7cm) and less than their sum (10cm + 17cm = 27cm). Therefore, the third side length must be between 7cm and 27cm, exclusive.

Why does the third side of a triangle with sides 10cm and 17cm need to be between 7cm and 27cm?

Because of the triangle inequality theorem, the sum of the lengths of any two sides must be greater than the third side. Applying this to the unknown side, it must be greater than 17cm - 10cm = 7cm and less than 27cm.

Can the third side of the triangle be exactly 7cm or 27cm?

No, the third side cannot be exactly 7cm or 27cm because the triangle inequality requires the sum to be strictly greater than the third side, so the length must be between 7cm and 27cm, not including the endpoints.

If the third side is 15cm, does it form a valid triangle with sides 10cm and 17cm?

Yes, since 15cm is greater than 7cm and less than 27cm, it satisfies the triangle inequality, and the three sides can form a valid triangle.

What is the significance of the triangle inequality theorem in determining the possible third side lengths?

The theorem ensures that the sum of any two sides must be greater than the third, which restricts the possible lengths of the third side based on the lengths of the known sides.

How do you calculate the minimum and maximum possible lengths of the third side in this triangle?

Subtract the known sides to find the minimum (17cm - 10cm = 7cm) and add the sides for the maximum (10cm + 17cm = 27cm). The third side must be greater than 7cm and less than 27cm.

Is a third side length of 5cm possible for this triangle?

No, because 5cm is less than the minimum length of 7cm required by the triangle inequality, so it cannot form a valid triangle with sides 10cm and 17cm.

How does the triangle inequality theorem help in real-world applications like construction or design?

It helps ensure structural stability by confirming that side lengths will form a proper triangle, preventing construction errors and ensuring safety and integrity of designs involving triangular components.

Additional Resources

Understanding the Range of Possible Lengths for the Third Side of a Triangle with Two Known Sides (10cm and 17cm)

When dealing with triangles in geometry, one of the fundamental questions often posed involves determining the possible lengths of a third side given two sides. This problem not only encapsulates core principles of the triangle inequality theorem but also provides insight into how geometric constraints influence possible measurements. In this detailed review, we will explore the question: If the length of two sides of a triangle are 10 cm and 17 cm, what is the range of possible lengths for the third side?

Throughout this discussion, we will delve into the underlying principles, work through mathematical derivations, examine real-world applications, and clarify common misconceptions.

Fundamental Principles: The Triangle Inequality Theorem

What Is the Triangle Inequality Theorem?

The triangle inequality theorem is a key principle in geometry that states:

> The sum of the lengths of any two sides of a triangle must be greater than the length of the remaining side.

This theorem ensures that the three sides can indeed form a valid triangle. It applies to all triangles, whether they are acute, right, or obtuse.

Mathematically, for sides a, b, and c:

$$
a + b > c \\
a + c > b \\
b + c > a
$$

Applying the Theorem to Known Sides

Suppose two sides of a triangle are known: $a = 10\,cm$ and $b = 17\,cm$. Let the third side be c.

To find the range of possible values for c, we apply the triangle inequality to each pair:

1. $a + b > c \Rightarrow 10 + 17 > c \Rightarrow 27 > c$
2. $a + c > b \Rightarrow 10 + c > 17 \Rightarrow c > 7$
3. $b + c > a \Rightarrow 17 + c > 10 \Rightarrow c > -7$

Since c must be positive (lengths are positive real numbers), the third inequality simplifies to $c > 0$, which is always true for side lengths.

The crucial inequalities are:

- $c > 7$
- $c < 27$

Therefore, the third side c must be greater than 7 cm but less than 27 cm.

Deriving the Range of Possible Lengths

Explicit Range for the Third Side

Based on the inequalities, the possible lengths for the third side c are:

$$
\boxed{
\text{Range of } c: \quad 7\,cm < c < 27\,cm
}
$$

This is an open interval because the lengths cannot be exactly 7 or 27 cm (if they were, the sum of two sides would equal the third, resulting in a degenerate triangle).

Summary:

- The smallest possible length of the third side: just greater than 7 cm.
- The largest possible length of the third side: just less than 27 cm.

Visualizing the Range

Visual representations can be invaluable for understanding the constraints:

- Number Line Representation:

```
0 --------- 7 --------- 27 --------- ∞
| |
Not possible Possible
```

The third side length must lie strictly between 7 and 27 cm.

- Triangle Formation:

When c approaches 7 cm from above, the triangle becomes very "thin," with the two sides nearly forming a straight line. Conversely, as c approaches 27 cm from below, the triangle becomes more "stretched," with the longest side approaching the sum of the other two.

Special Cases and Limitations

Degenerate Triangles

- If the third side equals exactly 7 cm or 27 cm, the triangle becomes degenerate:

- At $c = 7\,cm$: The sum of the two smaller sides equals the third, resulting in a straight line.

- At $c = 27\,cm$: The sum of the two sides equals the third, forming a straight line again.

- Implication: For a valid, non-degenerate triangle, the side length must be strictly greater than 7 cm and less than 27 cm.

Impact of Triangle Type

- Acute, Right, or Obtuse? The possible types of the triangle depend on the length of the third side within this range.

- Right Triangle Condition: When the sides satisfy the Pythagorean theorem $c^2 = a^2 + b^2$:

$$
c^2 = 10^2 + 17^2 = 100 + 289 = 389
$$

$$
c = \sqrt{389} \approx 19.72\,cm
$$

- Since $7 < 19.72 < 27$, a right triangle with sides 10 cm, 17 cm, and approximately 19.72 cm is possible.

- Acute or Obtuse?
- If $c < 19.72\,cm$, the triangle is acute.
- If $c > 19.72\,cm$, the triangle is obtuse.

Mathematical and Geometric Applications

Practical Examples

- Construction and Engineering: When designing structures requiring specific side lengths,

understanding these constraints prevents invalid configurations.
- Navigation: Triangulation methods often rely on such principles; knowing the range of possible distances aids in accurate positioning.
- Computer Graphics: Algorithms for rendering shapes often calculate possible side lengths to generate valid triangles.

Extended Problems and Variations

- Given a fixed side and a range for the third side, one might explore how the shape and area of the triangle change.
- For isosceles or equilateral triangles: The constraints differ, leading to different ranges and geometric properties.

Additional Considerations and Common Misconceptions

Misconception: The Third Side Must Be Exactly 17 cm

It's a common mistake to assume the third side must be a certain length rather than within a range. Remember, unless more constraints are specified, the third side can vary continuously within the valid interval.

Misconception: Triangle Inequality Is Not Strict

The inequality must be strict: the sum of two sides must be greater than the third, not equal. Equal sums lead to degenerate triangles which are not considered proper triangles in most contexts.

Real-World Constraints

In physical applications, factors such as material limitations, angles, and environmental conditions might further restrict possible side lengths, but mathematically, the range derived remains valid.

Summary and Key Takeaways

- Given two sides of a triangle measuring 10 cm and 17 cm, the third side \(c \) must satisfy:

$$
7\,cm < c < 27\,cm
$$

- This range ensures the triangle inequality holds, allowing the sides to form a valid triangle.

- The range is derived directly from the triangle inequality theorem, emphasizing the importance of these fundamental principles.

- Special cases, such as $c = 7\,cm$ or $c = 27\,cm$, lead to degenerate triangles, which are generally not considered valid for most purposes.

- The actual shape of the triangle (acute, right, obtuse) depends on the specific length of c, with the Pythagorean relation providing a special case at approximately 19.72 cm.

Final Note: Understanding the permissible side lengths of a triangle given partial information is essential not only in pure mathematics but also across various scientific and engineering fields. Mastery of the triangle inequality theorem provides a foundation for exploring more complex geometric and algebraic problems, facilitating accurate modeling, design, and analysis.

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