If The Line Passes Through The Points (-8, R) And(2,-10) Is Perpendicular To The Line 2x - Y = 6.find

If The Line Passes Through The Points (-8, R) And (2, -10) Is Perpendicular To The Line 2x - Y = 6.find

Understanding how to determine the characteristics of a line passing through specific points and its relationship to other lines is fundamental in coordinate geometry. When given points with variables and the requirement to identify perpendicularly intersecting lines, it involves calculating slopes, equations, and verifying perpendicularity conditions. This comprehensive guide will walk you through the process of solving such problems step-by-step, ensuring you grasp the concepts clearly and can apply them to similar questions.

Understanding the Problem Statement

Before diving into calculations, it’s essential to interpret what the problem is asking:

- You are given two points: (-8, R) and (2, -10). Here, R is an unknown y-coordinate.
- You need to determine the value of R such that the line passing through these points is perpendicular to another line.
- The other line is given by the equation 2x - y = 6.
- Your goal is to find the value of R that makes the line through the points perpendicular to the given line.

This requires knowledge of:
- Calculating the slope of a line given two points.
- Finding the slope of the given line.
- Understanding the concept of perpendicular slopes.
- Using these to solve for R.

Key Concepts in Coordinate Geometry

1. Slope of a Line

The slope (m) of a line passing through two points (x_1, y_1) and (x_2, y_2) is calculated as:

$$ m = \frac{y_2 - y_1}{x_2 - x_1} $$

This measure indicates the steepness of the line and its direction.

2. Equation of a Line

The slope-intercept form is:

$$ y = mx + c $$

where:
- m is the slope,
- c is the y-intercept.

3. Perpendicular Lines

Two lines are perpendicular if the product of their slopes is -1:

$$ m_1 \times m_2 = -1 $$

This property is crucial for solving the problem, as it allows you to find the unknown slope (and R) by using the slopes’ relation.

Step-by-Step Solution Approach

Let's break down the problem into manageable steps:

Step 1: Find the slope of the given line 2x - y = 6

Rewrite the line in slope-intercept form:

$$ 2x - y = 6 $$
$$ -y = -2x + 6 $$
$$ y = 2x - 6 $$

Thus, the slope of the given line is:

$$ m_{line} = 2 $$

Step 2: Determine the slope of the line passing through the points (-8, R) and (2, -10)

Let’s denote this slope as m_{AB}:

$$m_{AB} = \frac{-10 - R}{2 - (-8)} = \frac{-10 - R}{10}$$

Step 3: Use the perpendicularity condition

Since the line passing through the points is perpendicular to the given line, their slopes satisfy:

$$m_{AB} \times m_{line} = -1$$

Substitute the known slope $m_{line} = 2$:

$$\left(\frac{-10 - R}{10} \right) \times 2 = -1$$

Simplify:

$$\frac{-10 - R}{10} \times 2 = -1$$
$$\frac{2(-10 - R)}{10} = -1$$
$$\frac{-20 - 2R}{10} = -1$$

Multiply both sides by 10 to clear the denominator:

$$-20 - 2R = -10$$

Step 4: Solve for R

Rearranged:

$$-2R = -10 + 20$$
$$-2R = 10$$
$$R = -\frac{10}{2}$$
$$R = -5$$

Final Answer and Explanation

The value of R that makes the line passing through (-8, R) and (2, -10) perpendicular to the line 2x - y = 6 is R = -5.

This result signifies that when the y-coordinate of the first point is -5, the line connecting it to the point (2, -10) is perpendicular to the given line.

Additional Insights and Related Concepts

1. Verifying the Result

It's good practice to verify the solution:

- Calculate the slope of the line passing through (-8, -5) and (2, -10):

$$ m_{AB} = \frac{-10 - (-5)}{2 - (-8)} = \frac{-10 + 5}{10} = \frac{-5}{10} = -\frac{1}{2} $$

- Confirm perpendicularity:

$$ m_{AB} \times m_{line} = -\frac{1}{2} \times 2 = -1 $$

which confirms the correctness.

2. Practical Applications

Understanding these concepts is vital in various fields such as:
- Engineering design
- Computer graphics
- Physics for analyzing trajectories
- Data analysis for trend lines

3. Common Mistakes to Avoid

- Forgetting to convert equations into slope-intercept form
- Incorrectly calculating the slope
- Mixing up the order of points when computing the slope
- Not applying the perpendicularity condition correctly

Tips for Solving Similar Problems

- Always write the line equations in slope-intercept form for clarity.
- Carefully handle the signs during calculations.
- Use the property $m_1 \times m_2 = -1$ for perpendicular lines.
- Double-check your algebraic manipulations.

Conclusion

In summary, when you need to find the value of R such that a line passing through points (-8, R) and (2, -10) is perpendicular to a given line like 2x - y = 6, the process involves:

- Finding the slope of the given line.
- Establishing the slope of the passing line in terms of R.
- Applying the perpendicularity condition to set up an equation.
- Solving for R.

This problem demonstrates the power of coordinate geometry principles in solving real-world and academic problems efficiently. Mastering these steps enhances your ability to approach similar questions confidently and accurately.

Keywords for SEO Optimization:
- Coordinate Geometry
- Perpendicular Lines
- Slope Calculation
- Find R in Line Equation
- Line Through Two Points
- Equation of a Line
- Geometry Problems
- Math Problem Solving
- Algebra and Geometry
- Slope-Intercept Form

Frequently Asked Questions

How do you determine the value of R for a line passing through points (-8, R) and (2, -10) that is perpendicular to the line 2x - y = 6?

First, find the slope of the given line 2x - y = 6, convert it to slope-intercept form y = 2x - 6, so its slope is 2. The perpendicular line's slope is the negative reciprocal, which is -1/2. Then, use the two points to find R by substituting into the point-slope form.

What is the slope of the line passing through points (-8, R) and (2, -10)?

The slope is (−10 - R) / (2 + 8) = (−10 - R) / 10.

What is the slope of the line 2x - y = 6?

Rearranged to y = 2x - 6, so its slope is 2.

How do you find R such that the line through (-8, R) and (2, -10) is perpendicular to 2x - y = 6?

Set the slope of the line through the points equal to the negative reciprocal of 2, which is -1/2. So, (−10 - R)/10 = -1/2, then solve for R.

What is the value of R when the two lines are perpendicular?

Solve (−10 - R)/10 = -1/2: Cross-multiplied, gives −10 - R = -5, so R = -5 + 10 = 5.

What is the equation of the line passing through (-8, 5) and (2, -10)?

First, find the slope: (−10 - 5)/ (2 + 8) = (-15)/10 = -3/2. Using point-slope form: y - 5 = -3/2(x + 8).

Is the line passing through (-8, 5) and (2, -10) perpendicular to 2x - y = 6?

Yes, because its slope is -3/2, which is the negative reciprocal of 2, confirming perpendicularity.

How can you verify that two lines are perpendicular?

By confirming that the product of their slopes is -1.

What is the final answer for R in this problem?

R equals 5, making the line passing through (-8, 5) and (2, -10) perpendicular to the line 2x - y = 6.

Additional Resources

If The Line Passes Through The Points (-8, R) And (2, -10) Is Perpendicular To The Line 2x - y = 6 Find – this intriguing problem combines the fundamental concepts of coordinate geometry, specifically the determination of the equation of a line passing through given points and its relationship with another line. It challenges the learner to utilize the slopes, intercepts, and perpendicularity conditions to derive the unknowns, making it a comprehensive exercise in algebraic manipulation and geometric understanding.

Understanding the Problem and Its Context

This problem involves multiple layers of geometric reasoning. First, we are given two points: (-8, R) and (2, -10). The coordinate R is unknown, which means we need to express the problem in terms of R and then find its value based on the perpendicularity condition.

Secondly, the problem states that the line passing through these two points is perpendicular to the line described by the equation 2x - y = 6.

The key steps involve:

- Calculating the slope of the line passing through the given points, which depends on R.
- Determining the slope of the given line 2x - y = 6.
- Applying the perpendicularity condition, which states that the slopes of two perpendicular lines multiply to -1.
- Solving for R, and then possibly writing the equation of the required line.

This problem is an excellent illustration of how algebraic techniques intertwine with geometric properties, especially the concept of perpendicular slopes in the coordinate plane.

Step 1: Analyzing the Given Line 2x - y = 6

Converting to Slope-Intercept Form

To understand the slope of the given line, we first convert it into the slope-intercept form y = mx + c:

2x - y = 6
=> -y = -2x + 6
=> y = 2x - 6

Slope of the given line (m_1): 2

Features:

- The slope is positive, indicating the line ascends from left to right.

- The y-intercept is at (0, -6).

Pros:

- Easy to analyze the slope directly from the line's equation.
- Provides a straightforward basis for applying the perpendicularity condition.

Cons:

- Does not provide information about the slope of the perpendicular line directly; further calculations are needed.

Understanding the Slope's Role in Perpendicularity

The perpendicularity condition between two lines states:

If line 1 has slope m_1 and line 2 has slope m_2, then they are perpendicular if:

$m_1 \times m_2 = -1$

Given $m_1 = 2$, the slope of the line perpendicular to it (m_2) must satisfy:

$2 \times m_2 = -1$
$\Rightarrow m_2 = -1/2$

This is a critical step because it sets the target slope for the line passing through the points (-8, R) and (2, -10).

Step 2: Deriving the Slope of the Line Passing Through the Given Points

The line passes through points (-8, R) and (2, -10). The slope m of this line is:

$m = (y_2 - y_1) / (x_2 - x_1)$
$\Rightarrow m = (-10 - R) / (2 - (-8))$
$\Rightarrow m = (-10 - R) / (10)$

This simplifies to:

m = (-10 - R) / 10

Since the line must be perpendicular to the line with slope 2, its slope must be -1/2. Therefore:

(-10 - R) / 10 = -1/2

Step 3: Solving for R

Set the slope equal to -1/2:

(-10 - R) / 10 = -1/2

Cross-multiplied:

2(-10 - R) = -10

Simplify:

-20 - 2R = -10

Add 20 to both sides:

-2R = 10

Divide both sides by -2:

R = -5

Result: R = -5

This is the value of R for which the line passing through (-8, R) and (2, -10) is perpendicular to the line 2x - y = 6.

Step 4: Writing the Equation of the Required Line

Now that R is known, the points are:

- (-8, -5)
- (2, -10)

We already have the slope $m = -1/2$. Using point-slope form:

$$y - y_1 = m(x - x_1)$$

Choose point (-8, -5):

$$y - (-5) = -1/2 (x - (-8))$$

$$\Rightarrow y + 5 = -1/2 (x + 8)$$

Distribute:

$$y + 5 = -1/2 x - 4$$

Subtract 5 from both sides:

$$y = -1/2 x - 4 - 5$$

$$\Rightarrow y = -1/2 x - 9$$

Final Equation of the line:

$$y = -1/2 x - 9$$

Summary and Reflection

This problem elegantly combines algebra and geometry, requiring an understanding of slopes, perpendicular lines, and coordinate points. The key steps involved:

- Recognizing the importance of converting line equations into slope-intercept form.
- Applying the perpendicularity condition (product of slopes = -1).
- Solving for the unknown coordinate R via algebraic manipulation.
- Writing the explicit equation of the line once R was known.

Features of this problem:

- Reinforces the understanding of slopes and intercepts.
- Demonstrates how to handle unknown parameters in geometric contexts.
- Highlights the importance of the perpendicularity property in coordinate geometry.

Pros:

- Provides a comprehensive approach to solving complex geometric problems.
- Encourages algebraic reasoning alongside geometric intuition.

- Useful for exam preparations and strengthening foundational concepts.

Cons:

- Assumes familiarity with slope-intercept form and algebraic techniques.
- May be challenging if one forgets the perpendicularity condition or makes algebraic errors.

Conclusion

The problem of determining R and the equation of the line passing through (-8, R) and (2, -10), which is perpendicular to the line 2x - y = 6, encapsulates key principles of coordinate geometry. It underscores the importance of understanding slopes, intercepts, and the relationships between perpendicular lines. By breaking down the steps systematically—from analyzing the given line to deriving the unknown coordinate and formulating the final line equation—students can develop a robust understanding of geometric problem-solving techniques. This exercise not only enhances algebraic skills but also deepens the geometric insight necessary for tackling similar problems confidently in exams or real-world applications.

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