In A City, 1 Person In 5 Is Left Handed (a) Find The Probability That In A Random Sample Of 10 People

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Understanding probabilities in real-world scenarios can often seem abstract, but applying them to familiar contexts, such as the distribution of handedness in a city, offers a compelling way to grasp their significance. In this article, we explore the probability that, in a random sample of 10 individuals from a city where 1 in 5 people is left-handed, a specific configuration of left-handed individuals occurs. This investigation not only provides insight into probability theory but also demonstrates practical applications of binomial distributions in everyday life.

Understanding the Problem Context

Before delving into calculations, it's essential to clarify the problem's parameters and assumptions:

Key Details

- Proportion of left-handed people in the city: 1/5 (or 0.20)
- Sample size (number of people sampled): 10
- Objective: Find the probability of a specific number of left-handed individuals within the sample, such as exactly 2, 3, or any other count

Assumptions

- The selection of individuals is random and independent.
- The probability of being left-handed remains constant at 0.20 for each person.
- The population is large enough that sampling without replacement

approximates sampling with replacement for probability calculations.

Modeling the Situation: Binomial Distribution

Given the parameters, the problem naturally fits the binomial probability model.

What Is a Binomial Distribution?

The binomial distribution describes the number of successes (e.g., left-handed individuals) in a fixed number of independent Bernoulli trials (each trial being a person selected), where each trial has the same probability of success (p).

Mathematically, the probability of exactly k successes in n trials is given by:

$$
P(X = k) = \binom{n}{k} p^k (1-p)^{n-k}
$$

where:
- $\binom{n}{k}$ is the binomial coefficient ("n choose k")
- p is the probability of success on a single trial
- k is the number of successes

Applying the Binomial Model to Our Problem

In our context:
- $n = 10$ (sample size)
- $p = 0.20$ (probability of being left-handed)
- X = number of left-handed individuals in the sample

Calculating Specific Probabilities

Let's explore how to compute the probability that exactly k individuals in the sample are left-handed.

Example Calculations for Various Values of k

Suppose we want to find the probability that:

- Exactly 0 people are left-handed
- Exactly 1 person is left-handed
- Exactly 2 people are left-handed
- Exactly 3 people are left-handed

and so on, up to 10.

Probability of Exactly k Left-Handed People

$$
P(X = k) = \binom{10}{k} (0.20)^k (0.80)^{10 - k}
$$

Let's compute some of these probabilities:

For $k=0$:

$$
P(0) = \binom{10}{0} (0.20)^0 (0.80)^{10} = 1 \times 1 \times 0.1074 \approx 0.1074
$$

For $k=1$:

$$
P(1) = \binom{10}{1} (0.20)^1 (0.80)^9 = 10 \times 0.20 \times 0.1342 \approx 0.2684
$$

For $k=2$:

$$
P(2) = \binom{10}{2} (0.20)^2 (0.80)^8 = 45 \times 0.04 \times 0.1678 \approx 0.30199
$$

For $k=3$:

$$
P(3) = \binom{10}{3} (0.20)^3 (0.80)^7 = 120 \times 0.008 \times 0.2097 \approx 0.20133
$$

These calculations can be extended to any value of k, providing a complete picture of the distribution of left-handed individuals in the sample.

Interpreting the Probabilities

Understanding these probabilities helps in various decision-making scenarios and statistical analyses.

Examples of Practical Applications

- Estimating the likelihood of encountering a certain number of left-handed people in a survey.
- Planning for resource allocation in ergonomic design, considering the proportion of left-handed users.
- Analyzing the randomness and distribution of handedness in a population sample.

Key Observations

- The probability of finding exactly 0 left-handed individuals in the sample is approximately 10.7%.
- The most probable number of left-handed individuals in the sample can be determined by the mode of the binomial distribution, which often occurs near np. Here, $np = 10 \times 0.20 = 2$, indicating that 2 left-handed individuals is the most likely count.

Calculating Probabilities for Specific Scenarios

Suppose you are interested in the probability that at least 3 people are left-handed.

$$
P(X \geq 3) = 1 - P(X \leq 2) = 1 - [P(0) + P(1) + P(2)]
$$

From previous calculations:

$$
P(0) \approx 0.1074
$$
$$
P(1) \approx 0.2684
$$
$$
P(2) \approx 0.3020
$$

Thus,

$$
P(X \geq 3) = 1 - (0.1074 + 0.2684 + 0.3020) = 1 - 0.6778 \approx 0.3222
$$

This indicates about a 32.2% chance that at least 3 people in the sample are left-handed.

Using Statistical Tools for Calculations

Manually calculating binomial probabilities for larger samples or numerous scenarios can be cumbersome. Fortunately, statistical software and calculators can simplify these computations.

Popular Tools Include:

- Graphing calculators (e.g., TI-83/84)
- Spreadsheet programs (Excel, Google Sheets)
- Statistical software (R, SPSS, SAS)

Example: Calculating Binomial Probabilities in Excel

In Excel, the function `BINOM.DIST(k, n, p, FALSE)` computes the probability $P(X=k)$.

For example, to find $P(3)$:

```
=BINOM.DIST(3, 10, 0.2, FALSE)
```

Conclusion

The probability that in a random sample of 10 people from a city where 1 in 5 is left-handed can be precisely calculated using the binomial distribution. By understanding the parameters and applying the binomial formula, one can determine the likelihood of various configurations of left-handed individuals.

This analysis demonstrates how probability theory provides valuable insights into real-world distributions, aiding in research, planning, and decision-making. Whether it's predicting the number of left-handed students in a class or designing products for ergonomic comfort, understanding these probabilities is both practical and insightful.

In summary:
- The binomial distribution is the ideal model for this scenario.
- Exact probabilities can be computed for any number k of left-handed individuals.
- The most probable count is around 2, aligning with the expected value $np=2$.
- Using statistical tools can streamline these calculations, making probability analysis accessible and efficient.

By mastering these concepts, you can confidently analyze similar problems involving proportions, sample sizes, and probabilities in various fields ranging from social sciences to engineering.

Keywords: probability, binomial distribution, left-handed people, sample size, statistical analysis, probability calculation, binomial formula, success probability

Frequently Asked Questions

What is the probability that a randomly selected person from the city is left-handed?

The probability that a randomly selected person is left-handed is 1/5 or 0.2.

How can we calculate the probability that exactly 3

out of 10 people are left-handed?

Use the binomial probability formula: P = C(10,3) (1/5)^3 (4/5)^7, where C(10,3) is the combination of 10 items taken 3 at a time.

What is the probability that none of the 10 people are left-handed?

The probability that none are left-handed is (4/5)^10, since each person has a 4/5 chance of being right-handed.

How do you find the probability that at least 1 person out of 10 is left-handed?

Calculate 1 minus the probability that none are left-handed: 1 - (4/5)^10.

What is the expected number of left-handed people in a sample of 10?

Expected number = total sample size probability of being left-handed = 10 1/5 = 2.

How does the binomial distribution help in this problem?

It models the probability of a fixed number of left-handed individuals in the sample, based on the probability of each person being left-handed and the total number sampled.

What is the probability that exactly 5 people out of 10 are left-handed?

Use the binomial formula: P = C(10,5) (1/5)^5 (4/5)^5, which calculates the probability of exactly 5 left-handed people.

If someone wanted to find the probability that more than 2 people are left-handed in the sample, how would they do it?

Calculate 1 minus the probability that 0, 1, or 2 are left-handed: 1 - [P(0) + P(1) + P(2)], where each P(k) is computed using the binomial formula.

Additional Resources

In A City, 1 Person In 5 Is Left Handed (a) Find The Probability That In A Random Sample Of 10 People

In a bustling city where diversity and individuality thrive, understanding the distribution of traits such as handedness offers fascinating insights into population characteristics. Among these traits, left-handedness has long piqued the curiosity of psychologists, statisticians, and the general public alike. Recent studies suggest that approximately one in five residents in this city is left-handed, prompting a compelling question: what is the probability that, in a random sample of ten people, a specific number of them are left-handed? This article delves into the mathematical foundations of this probability problem, exploring how to model such situations with binomial probability, and providing a detailed, step-by-step approach to understand and compute these probabilities.

Understanding the Scenario: The 'One in Five' Statistic

Before diving into calculations, it's essential to understand the context and assumptions:

- The probability that any randomly selected individual is left-handed is \(p = \frac{1}{5} = 0.2 \).
- Conversely, the probability that an individual is right-handed (or not left-handed) is \(1 - p = 0.8 \).
- We consider a random sample of 10 people, each selected independently and with the same probability distribution.

This setting naturally lends itself to a binomial probability model because:

- There are a fixed number of trials (here, 10 people).
- Each trial (person) has two possible outcomes: left-handed or not.
- The probability of 'success' (being left-handed) remains constant across trials.
- The trials are independent; one person's handedness doesn't influence another's.

Given these conditions, the number of left-handed individuals in the sample follows a binomial distribution.

The Binomial Distribution: The Mathematical Backbone

Definition and Formula

The binomial distribution is a discrete probability distribution that models the number of successes in a fixed number of independent Bernoulli trials. Its probability mass function (PMF) is given by:

$$
P(X = k) = \binom{n}{k} p^k (1 - p)^{n - k}
$$

where:
- n = total number of trials (here, 10),
- k = number of successes (number of left-handed individuals),
- p = probability of success in a single trial (here, 0.2),
- $\binom{n}{k}$ = binomial coefficient, representing the number of ways to choose k successes from n trials.

This formula allows us to compute the probability that exactly k people in our sample are left-handed.

Applying the Binomial Model to Our Problem

Suppose we are interested in finding:

- The probability that exactly k out of the 10 people are left-handed.
- The probability that at least k are left-handed.
- The probability that no more than k are left-handed.

The binomial formula provides the foundation for all these calculations.

Step-by-Step Calculation: Examples

Let's explore specific cases to illustrate the process.

Example 1: Probability that exactly 2 out of 10 are left-handed

Using the binomial formula:

$$
P(X = 2) = \binom{10}{2} (0.2)^2 (0.8)^{8}
$$

Calculating:

- $\binom{10}{2} = \frac{10!}{2! \times 8!} = 45$,
- $(0.2)^2 = 0.04$,
- $(0.8)^8 \approx 0.16777$.

Thus:

$$
P(X=2) = 45 \times 0.04 \times 0.16777 \approx 45 \times 0.0067108 \approx 0.30198
$$

\]

So, there's approximately a 30.2% chance that exactly 2 people in the sample are left-handed.

Example 2: Probability that at least 3 are left-handed

This involves summing probabilities from $k = 3$ up to 10:

$$
P(X \geq 3) = 1 - P(X \leq 2) = 1 - [P(X=0) + P(X=1) + P(X=2)]
$$

Calculating each:

- $P(X=0) = \binom{10}{0} (0.2)^0 (0.8)^{10} = 1 \times 1 \times 0.10737 \approx 0.10737$,
- $P(X=1) = \binom{10}{1} (0.2)^1 (0.8)^9 = 10 \times 0.2 \times 0.1342 \approx 10 \times 0.02684 \approx 0.2684$,
- $P(X=2)$ as calculated above: approximately 0.302.

Adding these:

$$
P(X \leq 2) \approx 0.10737 + 0.2684 + 0.302 = 0.67777
$$

Thus,

$$
P(X \geq 3) \approx 1 - 0.67777 = 0.32223
$$

There’s roughly a 32.2% chance that at least 3 people in the sample are left-handed.

Using Statistical Software and Calculators

While manual calculations are illustrative, they become cumbersome for larger n or multiple probability sums. Fortunately, modern statistical software and scientific calculators simplify this process:

- Excel: The function `BINOM.DIST(k, n, p, cumulative)` computes binomial probabilities.
- Python: Libraries like SciPy include `scipy.stats.binom` which can compute probabilities efficiently.
- R: The `dbinom(k, size, prob)` function provides the probability mass function, while `pbinom(k, size, prob)` computes cumulative probabilities.

For example, in Python:

```python
from scipy.stats import binom

Probability exactly 2 are left-handed
prob_2 = binom.pmf(2, 10, 0.2)

Probability at least 3 are left-handed
prob_at_least_3 = 1 - binom.cdf(2, 10, 0.2)
```

Broader Implications and Interpretations

Understanding these probabilities isn't just a mathematical exercise; it offers practical insights:

- Population Variability: While the average suggests 2 out of 10 are left-handed, actual samples may vary due to chance.
- Statistical Significance: If, for instance, a sample yields 5 left-handed individuals, we might question whether the 1/5 assumption holds or if there are underlying factors influencing handedness.
- Application in Surveys: Researchers designing surveys or experiments can use these models to estimate the likelihood of observing certain distributions, aiding in planning and interpretation.

Limitations and Assumptions

While the binomial model is robust, it relies on several assumptions:

- Independence: Each person's handedness is independent of others. Social or environmental factors could influence this in real populations.
- Constant Probability: The probability $p = 0.2$ is assumed uniform. In reality, demographic factors might cause variations.
- Sample Size: Small samples are more susceptible to random fluctuations, whereas larger samples provide more stable estimates.

Understanding these limitations ensures that probability calculations are interpreted within appropriate contexts.

Conclusion: Harnessing Probability to Decipher Population Traits

The question "In a city where 1 in 5 residents is left-handed, what is the probability that in a sample of 10 people a certain number are left-handed?" exemplifies how statistical models like the binomial distribution serve as powerful tools in understanding real-world phenomena. By applying these principles, we can predict, analyze, and interpret the likelihood of various outcomes, aiding researchers, policymakers, and the curious alike.

In essence, the mathematics behind binomial probabilities enables us to quantify uncertainty and variability in human traits, fostering a deeper appreciation of the diversity within our communities. Whether assessing handedness, health outcomes, or behavioral traits, these models form a cornerstone of modern statistical analysis, bridging the gap between raw data and meaningful insights.

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