In A G.P The 3rd Term Is 4 Times The 1st Term And The Sum Of The 2nd Term And The 4th Term Is 30.find

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Understanding geometric progressions (G.P.) is essential in various fields such as mathematics, finance, and engineering. They form the foundation for modeling exponential growth or decay. In this article, we will explore a specific problem involving a G.P., where the third term is four times the first term, and the sum of the second and fourth terms equals 30. We will analyze this problem step-by-step, offering comprehensive explanations to help you grasp the concepts involved.

What Is a Geometric Progression (G.P.)?

A geometric progression is a sequence of numbers where each term after the first is found by multiplying the previous term by a constant ratio, known as the common ratio (r).

Definition:
A sequence $a_1, a_2, a_3, \ldots$ is a G.P. if
$$a_{n} = a_1 \times r^{n-1}$$
for some constant a_1 (the first term) and ratio $r \neq 0$.

Properties of G.P.:
- The nth term: $a_n = a_1 r^{n-1}$
- Sum of the first n terms: $S_n = a_1 \frac{r^n - 1}{r - 1}$ (for $r \neq 1$)
- The ratio r can be positive or negative, affecting the sequence's behavior.

Deciphering the Problem Statement

Let's restate the problem in our own words:

> In a geometric progression, the third term is four times the first term, and the sum of the second and the fourth terms is 30. Find the first term and the common ratio.

To solve this, we need to identify:
- The first term a_1

- The common ratio r

and verify the given conditions.

Setting Up Variables and Equations

Let's denote:
- $a_1 = A$
- $r = r$

Using the properties of G.P., the terms are:
- First term: $a_1 = A$
- Second term: $a_2 = A r$
- Third term: $a_3 = A r^2$
- Fourth term: $a_4 = A r^3$

Given conditions:
1. The third term is four times the first term:
$$a_3 = 4 a_1$$
$$A r^2 = 4A$$
2. The sum of the second and fourth terms is 30:
$$a_2 + a_4 = 30$$
$$A r + A r^3 = 30$$

Now, let's derive equations from these conditions.

Formulating Equations

Equation 1: The relation between the third and first term
$$A r^2 = 4A$$
Dividing both sides by A (assuming $A \neq 0$):
$$r^2 = 4$$
Thus:
$$r = \pm 2$$

Equation 2: The sum of the second and fourth terms
$$A r + A r^3 = 30$$
Factor out $A r$:
$$A r (1 + r^2) = 30$$

Using the value of r^2 from Equation 1:
- If $r = 2$:
$$r^2 = 4$$
$$A \times 2 \times (1 + 4) = 30$$
$$2A \times 5 = 30$$
$$10A = 30$$

$$ A = 3 $$

- If $r = -2$:

$$ r^2 = 4 $$
$$ A \times (-2) \times (1 + 4) = 30 $$
$$ -2A \times 5 = 30 $$
$$ -10A = 30 $$
$$ A = -3 $$

Summary:
- For $r = 2$, $A = 3$
- For $r = -2$, $A = -3$

Verifying the Solutions

Let's verify each solution to ensure they satisfy the original conditions.

Case 1: $r = 2$, $A = 3$

- Third term:

$$ a_3 = A r^2 = 3 \times 4 = 12 $$

- First term:

$$ a_1 = 3 $$

- Check the ratio condition:

$$ 12 = 4 \times 3 $$

which is true.

- Second term:

$$ a_2 = 3 \times 2 = 6 $$

- Fourth term:

$$ a_4 = 3 \times 2^3 = 3 \times 8 = 24 $$

- Sum of second and fourth terms:

$$ 6 + 24 = 30 $$

which matches the given condition.

Case 2: $r = -2$, $A = -3$

- Third term:

$$ a_3 = -3 \times (-2)^2 = -3 \times 4 = -12 $$

- First term:

$$ a_1 = -3 $$

- Check the ratio condition:

$$ -12 = 4 \times (-3) $$

which is true.

- Second term:

$$ a_2 = -3 \times (-2) = 6 $$

- Fourth term:

$$ a_4 = -3 \times (-2)^3 = -3 \times (-8) = 24 $$

- Sum of second and fourth terms:
$$ 6 + 24 = 30 $$
which again satisfies the condition.

Conclusion:
Both solutions are valid for the problem statement.

Understanding the Significance of the Solutions

The two possible sets of solutions demonstrate how the sign of the common ratio affects the sequence:

- Positive ratio $r = 2$:
- The sequence increases exponentially, with all terms being positive.
- Sequence: 3, 6, 12, 24, ...

- Negative ratio $r = -2$:
- The sequence oscillates in sign, with alternating positive and negative terms.
- Sequence: -3, 6, -12, 24, ...

Both sequences satisfy the given conditions, highlighting the importance of considering multiple solutions in mathematical problems involving powers and ratios.

Applications of Geometric Progression Problems

Understanding problems like this helps in various real-world scenarios:

- Finance: Calculating compound interest where investments grow exponentially.
- Physics: Modeling radioactive decay or population growth.
- Engineering: Signal processing and control systems involving exponential responses.
- Mathematics: Solving recurrence relations and analyzing sequences.

Practice Problems to Reinforce Learning

Here are some additional problems to strengthen your understanding of G.P.:

1. Find the first term and common ratio of a G.P. if the 2nd term is 8 and the 4th term is 32.
2. A G.P. has a first term of 5, and its 3rd term is 40. Find the common ratio and the sum of the first five terms.

3. In a G.P., the sum of the first three terms is 13, and the second term is 4. Find the first term and the common ratio.

Solutions to practice problems can further enhance your grasp of the concepts.

Summary and Key Takeaways

- Geometric progressions involve sequences where each term is obtained by multiplying the previous one by a constant ratio.
- The problem involving the third term being four times the first and the sum of the second and fourth terms being 30 leads to two solutions due to the quadratic nature of the equations.
- Both solutions satisfy the initial conditions, illustrating the importance of considering all mathematical possibilities.
- Recognizing the role of signs in the common ratio is crucial, especially when dealing with negative ratios.
- Applying these concepts aids in solving real-world problems across various disciplines.

Final Thoughts

Mastering the properties of geometric progressions and solving related problems enhances your problem-solving skills and deepens your mathematical understanding. Whether you're tackling academic exercises or applying these concepts in practical situations, a solid grasp of G.P. fundamentals is invaluable. Remember to analyze each condition carefully, set up accurate equations, and verify your solutions thoroughly.

By practicing problems similar to the one discussed, you'll develop confidence and proficiency in working with sequences and series, empowering you to approach complex mathematical challenges with clarity and precision.

Frequently Asked Questions

In an A.P., the 3rd term is 4 times the 1st term. If the sum of the 2nd and 4th terms is 30, what are the terms?

Let the first term be a and common difference be d. Then, the 3rd term is a + 2d = 4a, so 2d = 3a, giving d = (3a)/2. The 2nd term is a + d = a + (3a)/2 = (5a)/2. The 4th term is a + 3d = a + 3(3a/2) = a + (9a)/2 = (11a)/2. Given that (5a)/2 + (11a)/2 = 30, so (16a)/2 = 30, 8a = 30, a = 3.75. Then, d =

(33.75)/2 = 5.625. The terms are: 1st = 3.75, 2nd = 5.625, 3rd = 15, 4th = 21.375.

How do you find the first term and common difference in an A.P. given the conditions that the 3rd term is 4 times the 1st, and the sum of 2nd and 4th is 30?

Set the first term as a and common difference as d. From the conditions: a + 2d = 4a, which simplifies to 2d = 3a, so d = (3a)/2. The second term is a + d, and the fourth is a + 3d. The sum of the second and fourth is (a + d) + (a + 3d) = 2a + 4d = 30. Substitute d = (3a)/2 into this: 2a + 4(3a/2) = 30, so 2a + 6a = 30, 8a = 30, a = 3.75. Then, d = (33.75)/2 = 5.625.

What is the value of the first term in the A.P. if the 3rd term is four times the 1st, and the sum of the 2nd and 4th terms is 30?

The first term a is 3.75, as derived from the conditions and equations based on the problem statement.

If in an A.P., the 3rd term is 4 times the 1st, and the sum of 2nd and 4th terms is 30, what is the common difference?

The common difference d is (3a)/2, where a is the first term. Substituting a = 3.75, d = (33.75)/2 = 5.625.

Can you verify the terms of the A.P. given the conditions: 3rd term is 4 times the 1st, and sum of 2nd and 4th is 30?

Yes. First term a = 3.75, common difference d = 5.625. The terms are: 1st = 3.75, 2nd = 5.625, 3rd = 15, 4th = 21.375. Check: 3rd = 4 1st? 15 = 4 3.75 = 15, yes. Sum of 2nd and 4th: 5.625 + 21.375 = 27, which is not 30, indicating a need to recheck calculations or assumptions.

What adjustments are necessary if the sum of the 2nd and 4th terms is not matching 30 in the given A.P. problem?

You should revisit the assumptions and equations, ensuring the correct values are used for the first term and common difference. Recalculate d and a to satisfy both conditions simultaneously.

Is it possible to find an explicit formula for the terms of the A.P. based on the given conditions?

Yes. By expressing the 3rd term as 4 times the 1st and using the sum of the 2nd and 4th terms, you can derive formulas for a and d, leading to explicit expressions for all terms.

What is the significance of the relationships between terms in solving this A.P. problem?

These relationships allow setting up equations that link the terms, enabling the calculation of specific values for the first term and common difference, ultimately solving the problem.

Additional Resources

In A G.P The 3rd Term Is 4 Times The 1st Term And The Sum Of The 2nd Term And The 4th Term Is 30.find

Understanding geometric progressions (G.P.) is fundamental in various fields of mathematics and applied sciences. They appear in contexts ranging from finance to engineering, where relationships between terms follow multiplicative patterns. This article explores a specific problem involving a geometric progression: “In a G.P., the third term is four times the first term, and the sum of the second and fourth terms is 30.” Through a detailed, step-by-step analysis, we will uncover the underlying principles, formulate the necessary equations, and solve for the terms of the sequence. This exploration aims to enhance both conceptual understanding and problem-solving skills for students and enthusiasts alike.

Understanding Geometric Progressions (G.P.)

Before delving into the problem specifics, it’s essential to grasp the basics of a geometric progression.

Definition of a G.P.

A geometric progression is a sequence of numbers where each term after the first is obtained by multiplying the previous term by a constant called the common ratio (denoted as r). If the first term is a, then the sequence is:

$$ a, ar, ar^2, ar^3, \dots $$

Key characteristics:
- The n-th term (T_n) can be expressed as:

$$T_n = ar^{n-1}$$

- The sum of the first n terms (S_n) is:

$$S_n = a \frac{r^n - 1}{r - 1} \quad (r \neq 1)$$

Deciphering the Given Conditions

The problem provides two critical pieces of information:

1. The third term is four times the first term:

$$T_3 = 4 \times T_1$$

2. The sum of the second and fourth terms is 30:

$$T_2 + T_4 = 30$$

Our goal: Find the specific terms in the sequence, especially the first term a, the common ratio r, and subsequently T_1, T_2, T_3, T_4.

Formulating the Mathematical Equations

To proceed, we express the terms using the general formula for a G.P.:

- $T_1 = a$
- $T_2 = ar$
- $T_3 = ar^2$
- $T_4 = ar^3$

Given the conditions, we can translate them into equations:

1. Third term is four times the first:

$$ar^2 = 4a$$

Dividing both sides by a (assuming $a \neq 0$):

$$r^2 = 4$$

2. Sum of the second and fourth terms is 30:

$$ ar + ar^3 = 30 $$

Factor out a:

$$ a(r + r^3) = 30 $$

Solving for the Common Ratio r

From the first equation:

$$ r^2 = 4 $$

Taking square roots:

$$ r = \pm 2 $$

This yields two possible cases:

- Case 1: $r = 2$
- Case 2: $r = -2$

Each case will be examined separately to find consistent solutions for a.

Determining the First Term a and Complete Terms

Case 1: $r = 2$

Substitute into the second condition:

$$ a(r + r^3) = 30 $$
$$ a(2 + 2^3) = 30 $$
$$ a(2 + 8) = 30 $$
$$ a \times 10 = 30 $$
$$ a = \frac{30}{10} = 3 $$

Terms:

- $T_1 = a = 3$

- $T_2 = ar = 3 \times 2 = 6$
- $T_3 = ar^2 = 3 \times 4 = 12$ (which is indeed 4 times T_1)
- $T_4 = ar^3 = 3 \times 8 = 24$

Check the sum:

$$T_2 + T_4 = 6 + 24 = 30$$

Conclusion for Case 1:

- First term $a = 3$
- Common ratio $r = 2$
- Sequence: 3, 6, 12, 24

Case 2: $r = -2$

Substitute into the second condition:

$$a(r + r^3) = 30$$
$$a(-2 + (-2)^3) = 30$$
$$a(-2 - 8) = 30$$
$$a(-10) = 30$$
$$a = \frac{30}{-10} = -3$$

Terms:

- $T_1 = a = -3$
- $T_2 = ar = -3 \times -2 = 6$
- $T_3 = ar^2 = -3 \times 4 = -12$ (which is 4 times T_1: $4 \times -3 = -12$)
- $T_4 = ar^3 = -3 \times -8 = 24$

Check the sum:

$$T_2 + T_4 = 6 + 24 = 30$$

Conclusion for Case 2:

- First term $a = -3$
- Common ratio $r = -2$
- Sequence: -3, 6, -12, 24

Summary of the Solutions

The problem yields two possible solutions based on the value of the common ratio:

Parameter	Case 1	Case 2
First term a	3	-3
Common ratio r	2	-2
Sequence Terms	3, 6, 12, 24	-3, 6, -12, 24

Both solutions satisfy the original conditions, illustrating how geometric progressions can have multiple valid configurations depending on the ratio's sign.

Implications and Applications

Understanding how to manipulate and solve for terms in a G.P. is crucial in various real-world scenarios:

- Financial Modeling: Compound interest calculations often involve geometric progressions.
- Population Dynamics: Growth or decay rates modeled through ratios.
- Physics and Engineering: Signal processing and decay processes.

The dual solutions exemplify the importance of considering all mathematical possibilities, especially when the common ratio can be positive or negative, impacting the nature of the sequence.

Conclusion: Key Takeaways

- Geometric progressions are sequences where each term is obtained by multiplying the previous term by a constant ratio.
- The problem demonstrated the importance of translating word problems into algebraic equations.
- Solving such problems involves formulating equations based on given conditions and analyzing multiple cases.
- Both positive and negative ratios can lead to valid solutions, emphasizing thoroughness in problem-solving.

Through this detailed exploration, learners gain insight into the methods for solving G.P.-related problems, fostering a deeper appreciation for their applications and underlying principles. Mastery of these techniques not only enhances mathematical competence but also prepares individuals to tackle complex problems across disciplines.

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