

In The Following Rc Circuit, Vs Is A Sinusoidal Wave With An Amplitude Of 2 V And Frequency Of 50 Hz.

In The Following Rc Circuit, Vs Is A Sinusoidal Wave With An Amplitude Of 2 V And Frequency Of 50 Hz.

Introduction to RC Circuits and Sinusoidal Excitations

An RC circuit, consisting of a resistor (R) and a capacitor (C) connected in series or parallel, is one of the fundamental building blocks in electrical engineering. These circuits are widely used in filters, timing applications, and signal processing due to their ability to modify and analyze AC signals. When an AC voltage source with sinusoidal variation is applied, the circuit's response—voltage across the capacitor, resistor, or the entire circuit—depends on the frequency of the input signal and the circuit's component values.

In this context, the input voltage, V_s , is a sinusoidal wave with an amplitude of 2 volts and a frequency of 50 Hz. Such a signal is typical in AC power systems and communication applications, making understanding its interaction with RC circuits vital for designing filters, phase shifters, and other electronic components.

Basic Concepts of Sinusoidal Voltages and Currents

Sinusoidal Waveform Characteristics

A sinusoidal voltage wave, $V_s(t)$, can be represented mathematically as:

$$V_s(t) = V_m \sin(2\pi f t + \phi)$$

Where:

- $V_m = 2\text{ V}$ is the amplitude,
- $f = 50\text{ Hz}$ is the frequency,
- ϕ is the phase angle (assumed zero unless otherwise specified),
- t is time in seconds.

This waveform oscillates between +2 V and -2 V, completing 50 cycles per second, a typical frequency for power systems in many countries.

Implications of Sinusoidal Excitation in RC Circuits

Applying a sinusoidal voltage source causes currents and voltages within the circuit to also vary sinusoidally, but with potential phase shifts and amplitude changes depending on the circuit's reactance and resistance.

Understanding the RC Circuit Components and Their Roles

Resistor (R)

- Resists the flow of current, dissipating energy as heat.
- Provides a voltage drop proportional to the current, per Ohm's Law $(V_R = IR)$.
- Does not introduce phase shift; the current and voltage across R are in phase.

Capacitor (C)

- Stores energy in an electric field.
- Opposes changes in voltage, leading to phase shifts between voltage and current.
- The capacitor's impedance (Z_C) decreases with increasing frequency:

$$|Z_C| = \frac{1}{\omega C}$$

where:

- j is the imaginary unit,
- ω is the angular frequency,
- C is the capacitance in farads.

Frequency Response of the RC Circuit

Impedance and Phase Shift

The total impedance (Z_{total}) of a series RC circuit is:

$$Z_{total} = R + \frac{1}{j\omega C}$$

The magnitude of impedance:

$$|Z| = \sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}$$

The phase angle (θ) (phase difference between voltage and current):

$$\theta = -\arctan\left(\frac{1}{2\pi f R C}\right)$$

This phase shift indicates that the current leads or lags the voltage depending on the frequency.

Cutoff Frequency

The cutoff frequency (f_c) is the frequency at which the circuit's impedance causes the output signal to be attenuated to 70.7% of its maximum amplitude:

$$f_c = \frac{1}{2\pi R C}$$

At frequencies below (f_c) , the circuit behaves more like a resistor; above (f_c) , the capacitor's reactance dominates, leading to a significant phase shift and attenuation.

Analyzing the Response to a 50 Hz Sinusoidal Voltage

Calculating the Reactance of the Capacitor

Given $(f = 50\text{ Hz})$, the capacitive reactance:

$$X_C = \frac{1}{2\pi f C}$$

Depending on the value of (C) , (X_C) can vary widely, affecting the circuit's behavior.

Phase Relationship and Impedance

- The total impedance's magnitude:

$$|Z| = \sqrt{R^2 + X_C^2}$$

- The phase angle:

$$\theta = -\arctan\left(\frac{X_C}{R}\right)$$

The negative sign indicates that the current leads the voltage by (θ) .

Current and Voltage Waveforms

- The current in the circuit:

$$I(t) = \frac{V_s(t)}{|Z|} \sin(2\pi f t + \phi + \theta)$$

- The voltage across the resistor:

$$V_R(t) = I(t) R$$

- The voltage across the capacitor:

$$V_C(t) = I(t) X_C$$

These relationships exhibit phase shifts and amplitude attenuation depending on the component values.

Time-Domain and Frequency-Domain Analysis

Time-Domain Perspective

Using phasor analysis, the sinusoidal voltage and current can be represented as complex phasors, simplifying the calculations of amplitude and phase relationships.

Frequency-Domain Perspective

Applying Fourier analysis allows us to understand how the circuit responds to sinusoidal components at different frequencies—this is essential when analyzing signals with multiple frequency components or noise.

Practical Calculations with Given Parameters

Suppose the circuit has $R = 1 \text{ k}\Omega$ and an unknown capacitor C . We can analyze how the circuit responds to the 2 V, 50 Hz sinusoidal source.

Calculating the Capacitive Reactance

- For example, if $(C = 1\text{ }\mu\text{F})$:

$$X_C = \frac{1}{2\pi \times 50 \times 1 \times 10^{-6}} \approx 3.18\text{ k}\Omega$$

- Impedance magnitude:

$$|Z| = \sqrt{(1000)^2 + (3180)^2} \approx 3312\text{ }\Omega$$

- Phase angle:

$$\theta = -\arctan\left(\frac{3180}{1000}\right) \approx -72.5^\circ$$

This indicates the current leads the voltage by approximately 72.5 degrees.

Amplitude of the Circuit Current

$$I_{\max} = \frac{V_{\text{rms}}}{|Z|} = \frac{2\text{ V}}{3312\text{ }\Omega} \approx 0.603\text{ mA}$$

Voltage Across Components

- Voltage across resistor:

$$V_R = I_{\max} R \approx 0.603\text{ mA} \times 1000\text{ }\Omega = 0.603\text{ V}$$

- Voltage across capacitor:

$$V_C = I_{\max} X_C \approx 0.603\text{ mA} \times 3180\text{ }\Omega \approx 1.92\text{ V}$$

These voltages are phase-shifted relative to each other, with the capacitor voltage leading the resistor voltage due to the phase shift.

Implications for Circuit Design and Signal Processing

Filtering Applications

- RC circuits can be configured as low-pass or high-pass filters depending on the arrangement.
- For a low-pass filter, the output is taken across the capacitor, allowing signals below the cutoff frequency to pass while attenuating higher frequencies.

- For a high-pass filter, the output is across the resistor, permitting high-frequency signals and blocking low-frequency components.

Phase Shift and Signal Delay

- The phase shift introduced by RC circuits can be exploited in timing applications.
- Understanding the phase relationship helps in designing circuits for phase correction or delay.

Signal Attenuation

- At frequencies much higher than the cutoff, the circuit significantly attenuates the input voltage.
- The amplitude ratio (gain) can be calculated as:

$$\text{Gain} = \frac{V_{\text{out}}}{V_{\text{in}}} = \frac{1}{\sqrt{1 + (2\pi f R C)^2}}$$

- This helps in designing filters with desired attenuation characteristics.

Conclusion

The analysis of an RC circuit with a sinusoidal input of 2 V amplitude and 50 Hz frequency reveals critical insights into the behavior of such systems. The interplay between resistance and capacitance determines the circuit's impedance, phase shift, and frequency response. By understanding these relationships, engineers can design effective filters, phase shifters, and timing circuits tailored to specific applications. The key takeaway is that the frequency of the input signal heavily influences the circuit's response, particularly through the

Frequently Asked Questions

What is the maximum voltage of the sinusoidal source in the circuit?

The maximum voltage (amplitude) of the sinusoidal source V_s is 2 V.

What is the frequency of the sinusoidal voltage source in the RC circuit?

The frequency of the sinusoidal source is 50 Hz.

How does the frequency affect the behavior of the RC circuit?

The frequency determines the reactance of the capacitor, influencing the circuit's phase shift and the amplitude of the output voltage across the components.

What is the significance of the amplitude and frequency in analyzing the RC circuit's response?

The amplitude and frequency help in calculating the phasor voltages and currents, and in understanding how the circuit filters signals at different frequencies.

How can the impedance of the circuit be calculated given V_s 's parameters?

The impedance can be calculated using $Z = \sqrt{R^2 + (X_c)^2}$, where $X_c = 1/(2\pi fC)$, considering the resistor R and capacitor C in the circuit.

What is the phase relationship between the voltage across the resistor and capacitor in the circuit?

In an RC circuit with sinusoidal excitation, the voltage across the resistor is in phase with the current, while the voltage across the capacitor lags the current by 90 degrees, resulting in a phase shift between voltages depending on the frequency and component values.

Additional Resources

RC Circuit Analysis with Sinusoidal Input: An Expert Overview

Introduction: The Significance of Sinusoidal Inputs in RC Circuits

In the realm of electrical engineering, understanding the behavior of RC (Resistor-Capacitor) circuits under various input signals is fundamental. These circuits serve as building blocks in filters, timers, and signal processing systems. When a sinusoidal voltage source, V_s , is applied, the circuit's response provides insight into its filtering characteristics, phase relationships, and frequency-dependent behavior.

This article delves into a specific scenario: an RC circuit powered by a sinusoidal voltage source, V_s , with an amplitude of 2 V and a frequency of 50 Hz. We will explore the circuit's theoretical foundations, analyze its response, and discuss practical implications. Whether you're a student, educator, or practicing engineer, this comprehensive review aims to deepen your understanding of RC circuit dynamics under sinusoidal excitation.

Fundamental Concepts of RC Circuits with Sinusoidal Excitation

What is an RC Circuit?

An RC circuit typically comprises a resistor (R) and a capacitor (C) connected in series or parallel. For the most common analysis, we consider the series RC circuit, where the input voltage, $V_s(t)$, is applied across the series combination, and the output is often taken across the capacitor or resistor.

Sinusoidal Voltage Sources

A sinusoidal source, $V_s(t)$, can be mathematically expressed as:

$$V_s(t) = V_m \sin(2\pi f t)$$

where:

- V_m = Amplitude of the sinusoidal wave (here, 2 V)
- f = Frequency (here, 50 Hz)
- t = Time variable

The sinusoidal input is characterized by its frequency and amplitude, both crucial in defining the circuit's response.

Analyzing the Given Parameters

Input Voltage Specifications

- Amplitude (V_m): 2 V
- Frequency (f): 50 Hz

The choice of 50 Hz is significant because it aligns with standard power line frequencies in many parts of the world, making the analysis relevant for real-world applications such as audio processing and power systems.

Theoretical Framework: Circuit Response to Sinusoidal Input

Impedance of the RC Circuit

The key to understanding the circuit's behavior lies in the concept of impedance (Z), which combines resistance and reactance:

$$Z = R + jX_C$$

where:

- R = Resistance (ohms, Ω)
- X_C = Capacitive reactance = $\frac{1}{2\pi f C}$

- j = Imaginary unit

The capacitive reactance decreases with increasing frequency, affecting how the circuit filters signals.

Transfer Function and Frequency Response

The transfer function $H(j\omega)$, representing the ratio of output to input in the frequency domain, is given by:

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1 + j\omega RC}$$

where:

$$\omega = 2\pi f$$

This function describes how the circuit attenuates and shifts the phase of the input signal across frequencies.

Step-by-Step Analysis of the Circuit Response

1. Calculating Reactance at 50 Hz

Given:

$$X_C = \frac{1}{2\pi f C}$$

Assuming a capacitance value (say, $C = 1 \mu\text{F}$ for illustration), then:

$$X_C = \frac{1}{2\pi \times 50 \times 1 \times 10^{-6}} \approx 3.18 \text{ k}\Omega$$

This high reactance indicates that at 50 Hz, the capacitor presents a significant impedance, influencing the circuit's filtering behavior.

2. Impedance and Phase Shift

The total impedance:

$$Z = R + jX_C$$

The magnitude of the impedance:

$$|Z| = \sqrt{R^2 + X_C^2}$$

And the phase angle:

$$\phi = -\arctan\left(\frac{X_C}{R}\right)$$

This phase angle indicates how much the output signal lags behind the input.

3. Output Voltage and Attenuation

The output is often considered across the capacitor:

$$V_{\text{out}}(t) = |H(j\omega)| \times V_{\text{s}}(t)$$

where:

$$|H(j\omega)| = \frac{X_C}{\sqrt{R^2 + X_C^2}}$$

For the example with $C = 1 \mu\text{F}$:

$$|H(j\omega)| \approx \frac{3180}{\sqrt{R^2 + 3180^2}}$$

This ratio demonstrates how frequency and component values filter the input signal.

Practical Implications: Filter Characteristics

Low-Pass Filter Behavior

An RC circuit with the output taken across the capacitor functions as a low-pass filter, allowing signals with frequencies below a certain cutoff to pass while attenuating higher frequencies.

The cutoff frequency f_c :

$$f_c = \frac{1}{2\pi RC}$$

Using the earlier example ($C = 1 \mu\text{F}$):

$$f_c \approx \frac{1}{2\pi R \times 10^{-6}}$$

If R is 10 kΩ:

$$f_c \approx \frac{1}{2\pi \times 10^4 \times 10^{-6}} \approx 15.9, \text{ Hz}$$

Since 50 Hz > 15.9 Hz, the circuit would significantly attenuate the 50 Hz signal, illustrating its filter characteristic.

Visualizing the Response: Time Domain and Frequency Domain

Time-Domain Response

The output signal across the capacitor will be a sinusoid of the same frequency but with reduced amplitude and a phase lag. The precise amplitude and phase depend on R, C, and frequency.

Frequency-Domain Response

Plotting the magnitude and phase of $(H(j\omega))$ across a range of frequencies reveals the circuit's filtering profile:

- Magnitude Response: Shows how much of the input amplitude is preserved at each frequency.
- Phase Response: Demonstrates how the phase of the output shifts relative to the input.

This analysis is essential for designing circuits tailored to specific frequency ranges, such as audio filters or power line noise suppression.

Practical Examples and Applications

1. Power Line Noise Filtering

In many electronic devices, RC filters are employed to suppress 50 Hz or 60 Hz power line noise, ensuring cleaner signals in sensitive circuits.

2. Signal Conditioning

In sensor systems, RC circuits can shape or filter incoming signals, emphasizing desired frequency components while reducing noise.

3. Audio Processing

Low-pass RC filters smooth high-frequency components, providing a basis for tone control and audio equalization.

Key Takeaways and Design Considerations

- Component Values: Choose R and C based on desired cutoff frequency and attenuation characteristics.
- Frequency Response: Understand how the circuit filters signals at different frequencies, especially when the input is sinusoidal.
- Phase Shift: Recognize that sinusoidal signals passing through RC filters undergo phase shifts, impacting the timing in signal processing applications.
- Impedance Effects: High reactance at certain frequencies can significantly influence circuit behavior, necessitating precise calculations.

Conclusion: Mastering RC Circuit Response to Sinusoidal Inputs

The application of a sinusoidal voltage source with a 2 V amplitude at 50 Hz to an RC circuit exemplifies fundamental principles of electrical filtering and signal analysis. By evaluating impedance, transfer function, and frequency response, engineers can design circuits optimized for specific tasks, from noise filtering to signal shaping.

Understanding these responses not only enhances circuit design precision but also broadens the capacity to innovate in fields such as communications, instrumentation, and power systems. As technology advances, mastering the sinusoidal behavior in RC circuits remains a cornerstone of electrical engineering expertise.

Final Remarks

Whether for academic pursuits or practical engineering, the detailed analysis of RC circuits under sinusoidal excitation underscores their versatility and importance. Embracing the interplay between frequency, impedance, and phase response empowers professionals to craft sophisticated, reliable electronic systems that perform seamlessly across diverse applications.

In The Following Rc Circuit Vs Is A Sinusoidal Wave With An Amplitude Of 2 V And Frequency Of 50 Hz

In The Following Rc Circuit Vs Is A Sinusoidal Wave With An Amplitude Of 2 V And Frequency Of 50 Hz

Related Articles

- [For The Given Equation, Find The Values Of A, B, And C, Determine The Direction In Which The Parabola](#)
- [JUSTIFY REASONING You Are Evaluating A Conditional Statement In Which The Hypothesis Is](#)

[True, But The](#)

- [How Does Disney Practice Their Stakeholder Management. Explainhow The Company Manages To Reach Out To](#)

[Back to Home](#)