In The Geometric Sequence 6,12,24,48,dots, Which Term Is 768? The Second Term Of A Geometric Sequence

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Understanding geometric sequences is fundamental in algebra and mathematics as a whole. These sequences appear frequently in various real-life applications, such as population growth, financial calculations, and computer science algorithms. In this article, we will explore the geometric sequence starting with 6, 12, 24, 48, and continue to analyze which term equals 768. Additionally, we'll delve into the concept of the second term in a geometric sequence, the common ratio, and how to find specific terms within the sequence.

What Is a Geometric Sequence?

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous term by a constant called the common ratio, denoted as r. Mathematically, the sequence can be expressed as:

$$ a_n = a_1 \times r^{n-1} $$

where:
- a_n is the nth term,
- a_1 is the first term,
- r is the common ratio,
- n is the position of the term in the sequence.

Example: In the sequence 3, 6, 12, 24, 48, ..., each term is multiplied by 2 to obtain the next, so the common ratio $r = 2$.

Analyzing the Sequence 6, 12, 24, 48, ...

Let's examine the given sequence:

Sequence: 6, 12, 24, 48, ...

- The first term $a_1 = 6$
- The second term $a_2 = 12$
- The third term $a_3 = 24$
- The fourth term $a_4 = 48$

Step 1: Find the common ratio r

The common ratio is found by dividing any term by the previous term:

$$r = \frac{a_2}{a_1} = \frac{12}{6} = 2$$

Similarly,

$$r = \frac{a_3}{a_2} = \frac{24}{12} = 2$$

$$r = \frac{a_4}{a_3} = \frac{48}{24} = 2$$

Thus, the common ratio $r = 2$.

Step 2: Write the general term formula

Using the first term $a_1 = 6$ and $r = 2$, the nth term is:

$$a_n = 6 \times 2^{n-1}$$

Determining Which Term Is 768 in the Sequence

One of the most common questions when studying sequences is identifying the position of a specific term. In this case, we want to find the term number n such that $a_n = 768$.

Step 1: Set up the equation

$$6 \times 2^{n-1} = 768$$

Step 2: Solve for n

Divide both sides by 6:

$$2^{n-1} = \frac{768}{6}$$

$$2^{n-1} = 128$$

Recognize that 128 is a power of 2:

$$128 = 2^7$$

Thus,

$$2^{n-1} = 2^7$$

Since bases are equal, exponents are equal:

$$n - 1 = 7$$

$$n = 8$$

Result: The 8th term in the sequence is 768.

Summary of the Sequence and the Relevant Terms

Term Number (n)	Term (a_n)	Calculation
1	6	6×2^{0}
2	12	6×2^{1}
3	24	6×2^{2}
4	48	6×2^{3}
5	96	6×2^{4}
6	192	6×2^{5}
7	384	6×2^{6}
8	768	6×2^{7}

Therefore, the term 768 appears at position 8 in the sequence.

The Second Term of the Sequence

In the context of the sequence, the second term a_2 is 12. Since we have already established that:

$$a_2 = 6 \times 2^{2-1} = 6 \times 2^{1} = 6 \times 2 = 12$$

Understanding the importance of the second term:

- The second term helps verify the common ratio.
- It provides insight into the sequence's growth rate.
- Knowing the second term allows for quick reconstruction of the sequence.

In general, for any geometric sequence:

$$ a_2 = a_1 \times r $$

and in our specific sequence:

$$ a_2 = 6 \times 2 = 12 $$

which confirms that the common ratio $r = 2$.

Applications of Geometric Sequences

Geometric sequences are used in a variety of fields:

- Finance: Calculating compound interest over periods.
- Population Dynamics: Modeling population growth where each generation multiplies by a constant factor.
- Physics: Describing radioactive decay or exponential growth.
- Computer Science: Algorithm analysis, such as divide-and-conquer algorithms, often involve geometric progression.

Real-life Example: Suppose an investment of $1,000 compounds annually at a rate of 100%, the amount after n years:

$$ A_n = 1000 \times (1 + r)^n $$

which is a geometric sequence with $a_1 = 1000$ and $r = 1$ (or 100%).

How to Approach Similar Problems

When faced with a sequence and asked to find a specific term or term position:

1. Identify the sequence type: Is it arithmetic or geometric?
2. Find the first term (a_1) and common ratio (r).
3. Write the general term formula: $a_n = a_1 \times r^{n-1}$.
4. Solve for n if the term's value is given: Set a_n equal to the given value and solve for n.

Example: If the sequence was 3, 6, 12, 24, ..., and you wanted to find which term equals 192:

$$3 \times 2^{n-1} = 192$$

Divide both sides by 3:

$$2^{n-1} = 64$$

Since $64 = 2^6$:

$$n - 1 = 6$$

$$n = 7$$

So, the 7th term is 192.

Conclusion

Understanding geometric sequences is essential for grasping exponential growth and decay phenomena. In the given sequence starting with 6, 12, 24, 48, the common ratio is 2, and the formula for the nth term is:

$$a_n = 6 \times 2^{n-1}$$

The term 768 appears at position 8. Recognizing the pattern allows for quick calculations and problem-solving in various mathematical and practical contexts.

Key takeaways:

- The sequence is geometric with $a_1 = 6$ and $r = 2$.
- The 8th term is 768.
- The second term is 12.
- The sequence reflects exponential growth, applicable in multiple fields.

By mastering the principles of geometric sequences, you can efficiently analyze and solve a wide range of mathematical problems involving exponential patterns.

Frequently Asked Questions

What is the common ratio in the geometric sequence 6, 12, 24, 48, ...?

The common ratio is 2, since each term is multiplied by 2 to get the next term.

How many terms are needed for the sequence 6, 12, 24, 48, ... to reach 768?

We can find the term number n where the sequence equals 768 using the formula: $6\ 2^{n-1} = 768$. Solving for n gives $n = 8$.

Which term in the sequence 6, 12, 24, 48, ... is 768?

The 8th term is 768.

How do you find the nth term in the sequence 6, 12, 24, 48, ...?

Use the formula: $a_n = a_1\ r^{n-1}$, where $a_1 = 6$ and $r = 2$. So, $a_n = 6\ 2^{n-1}$.

What is the second term of the geometric sequence 6, 12, 24, 48, ...?

The second term is 12.

Is 768 a term in the geometric sequence 6, 12, 24, 48, ...?

Yes, 768 is the 8th term of the sequence.

How can you verify that 768 is part of the sequence 6, 12, 24, 48, ...?

Calculate the term number n using $6\ 2^{n-1} = 768$. Solving yields $n=8$, confirming that 768 is the 8th term.

What is the first term and common ratio of the sequence 6, 12, 24, 48, ...?

The first term is 6 and the common ratio is 2.

If the second term of a geometric sequence is 12 and the first term is 6, what is the common ratio?

The common ratio r is 2, since $12 / 6 = 2$.

Can we find the term number for any given term in the sequence 6, 12, 24, 48, ...?

Yes, by rearranging the nth term formula: n = 1 + log_2 (a_n / 6).

Additional Resources

In The Geometric Sequence 6, 12, 24, 48, dots, Which Term Is 768? The Second Term Of A Geometric Sequence

Understanding geometric sequences is fundamental in mathematics, especially in algebra, calculus, and various applied sciences. The sequence 6, 12, 24, 48, ... exemplifies a classic geometric progression where each term is obtained by multiplying the previous term by a constant ratio. This article aims to explore the properties of this sequence, determine which term equals 768, and analyze the significance of the second term within this sequence. Whether you're a student preparing for exams or a math enthusiast seeking deeper insight, this comprehensive review will clarify the concepts and methods involved.

Overview of Geometric Sequences

Before delving into the specific sequence in question, it's essential to understand what defines a geometric sequence.

Definition and Properties

A geometric sequence is a list of numbers where each term after the first is found by multiplying the previous term by a fixed, non-zero constant called the common ratio (denoted as r). Formally, if the first term is a_1, then the sequence can be written as:

$a_1, a_1 \cdot r, a_1 \cdot r^2, a_1 \cdot r^3, ..., a_1 \cdot r^{n-1}$

Key features of geometric sequences include:

- Constant ratio: The ratio between successive terms remains the same throughout the sequence.
- Explicit formula: The n-th term can be calculated using $a_n = a_1 \cdot r$^(n-1).
- Applications: Growth models, compound interest calculations, population models, and more.

Importance in Mathematics and Science

Geometric sequences model exponential growth or decay phenomena. They are fundamental in understanding systems where quantities increase or decrease multiplicatively, such as bacteria populations, radioactive decay, or financial investments.

Analyzing the Sequence: 6, 12, 24, 48, ...

The sequence provided is:

6, 12, 24, 48, ...

Let's analyze its characteristics.

Identifying the Common Ratio

To find the common ratio, divide any term by the previous one:

- 12 / 6 = 2
- 24 / 12 = 2
- 48 / 24 = 2

Since all these are equal, the sequence has a common ratio of r = 2.

First Term and General Term

- The first term, a_1, is 6.
- The n-th term, a_n, can be expressed as:

$a_n = 6 \cdot 2^{(n-1)}$

This formula allows us to find any term directly without listing all previous terms.

Which Term Is 768? Determining the Position of 768 in the Sequence

A common problem involving sequences is identifying the position of a specific term. Here, we want to find n such that:

$a_n = 768$

Using the explicit formula:

$768 = 6 \cdot 2^{(n-1)}$

Dividing both sides by 6:

$768 / 6 = 2^{(n-1)}$

$128 = 2^{(n-1)}$

Now, since 128 is a power of 2:

$128 = 2^7$

Therefore:

$2^{(n-1)} = 2^7$

This implies:

$n - 1 = 7$

$n = 8$

Conclusion: The 8th term of the sequence is 768.

Verification:

Calculate a_8:

$a_8 = 6 \cdot 2^{(8-1)} = 6 \cdot 2^7 = 6 \cdot 128 = 768$

This confirms our calculation.

The Second Term of the Sequence

Understanding the second term is straightforward once the sequence's initial parameters are known.

Calculating the Second Term

Given $a_1 = 6$ and $r = 2$:

$a_2 = a_1 \cdot r = 6 \cdot 2 = 12$

Significance of the Second Term:

- It indicates how quickly the sequence grows.
- In applications like compound interest, it models the amount after the first period.
- It helps in verifying the common ratio.

Features:

- The second term is often used to confirm the sequence's ratio.
- It provides insight into the sequence's progression early on.

Summary of Key Findings

Feature	Details
First term (a_1)	6
Common ratio (r)	2
General term formula	$a_n = 6 \cdot 2^{(n-1)}$
Term equal to 768	8th term ($n = 8$)
Second term	12

Practical Applications and Examples

Understanding this sequence's properties can be applied in various real-world scenarios.

Financial Growth

Suppose an investment starts with $6 and doubles each year. After 8 years, the investment will grow to:

$a_8 = 6 \cdot 2^7 = \$768$

This example illustrates exponential growth and how geometric sequences model compound interest.

Population Dynamics

If a population of bacteria starts at 6 individuals and doubles every hour, after 8 hours, the population will be 768.

Physics and Engineering

Radioactive decay or signal amplification often follows geometric progression principles.

Pros and Cons of Using Geometric Sequences

Pros:

- Simple to compute and understand.
- Widely applicable in modeling exponential processes.
- Easily predictable with explicit formulas.
- Useful in financial calculations, biology, physics, and computer science.

Cons:

- Assumes constant ratio, which may not reflect real-world complexities.
- Sensitive to initial parameters; small errors can lead to significant deviations.
- Not suitable for sequences where growth is not multiplicative or varies over time.

Conclusion

The sequence 6, 12, 24, 48, ... exemplifies the fundamental properties of geometric progressions, with a clear common ratio of 2 and a well-defined explicit formula. By analyzing the sequence, we determined that the term 768 appears as the 8th term. Additionally, the second term, which is 12, plays a crucial role in understanding the sequence's growth rate and serves as a reference point for calculations.

Understanding such sequences enhances mathematical literacy and provides tools to model various phenomena accurately. Whether in finance, biology, physics, or computer science, recognizing geometric sequences and their properties empowers learners and professionals alike to analyze systems characterized by exponential change.

In summary:

- The sequence's common ratio is 2.
- The 8th term of the sequence is 768.
- The second term is 12.
- The explicit formula facilitates quick calculations.

Through this detailed exploration, learners can appreciate the elegance and utility of geometric sequences and apply these concepts in practical scenarios with confidence.

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