

In The Hydrogen Atom, What Is The Electric Potential Energy Of The Electron When It Is Found In The N

In The Hydrogen Atom, What Is The Electric Potential Energy Of The Electron When It Is Found In The N – this question delves into one of the fundamental aspects of atomic physics and quantum mechanics. Understanding the electric potential energy of an electron in a hydrogen atom when it is in a specific energy level, characterized by the principal quantum number (n) , provides insight into atomic structure, energy quantization, and the forces at play at the atomic scale. This article explores the concept thoroughly, explaining the underlying physics, the mathematical formulation, and its significance in the broader context of atomic theory.

Overview of the Hydrogen Atom and Electron Energy Levels

The hydrogen atom is the simplest atomic system, consisting of a single proton in the nucleus and a single electron orbiting around it. Its simplicity makes it a fundamental model for understanding atomic physics, quantum mechanics, and electromagnetic interactions. The electron in a hydrogen atom does not orbit the nucleus in a classical sense but exists in quantized energy states described by quantum numbers.

Principal Quantum Number (n)

The principal quantum number (n) (where $(n = 1, 2, 3, \dots)$) determines the energy level or shell in which the electron resides. It indicates the size and energy of the orbital; higher (n) values correspond to larger, higher-energy orbitals.

Energy Quantization

Energy levels in the hydrogen atom are discrete, meaning the electron can only occupy specific energy states. These energy levels are derived from solving the Schrödinger equation for the Coulomb potential between the negatively charged electron and the positively charged proton.

Electric Potential Energy in the Hydrogen Atom

The electric potential energy (U) of the electron in the hydrogen atom is a measure of the electrostatic interaction between the negatively charged

electron and the positively charged proton. Since both charges are point charges, their interaction can be described by Coulomb's law.

Coulomb Potential Energy

The Coulomb potential energy between two point charges (q_1) and (q_2) separated by a distance (r) is given by:

$$U(r) = \frac{k q_1 q_2}{r}$$

where:

- (k) is Coulomb's constant, $(8.9875 \times 10^9 \text{ Nm}^2/\text{C}^2)$,
- (q_1) and (q_2) are the charges,
- (r) is the separation distance.

In the hydrogen atom:

- $(q_1 = -e)$ (electron charge),
- $(q_2 = +e)$ (proton charge),
- (r) is the radius of the electron's orbital.

Hence, the potential energy becomes:

$$U(r) = -\frac{k e^2}{r}$$

The negative sign indicates that the potential energy is lower (more negative) when the electron is bound to the nucleus, reflecting a bound state due to attractive Coulomb force.

Expressing Potential Energy in Terms of Quantum Number (n)

Since the electron's orbital radius depends on the quantum number (n) , the potential energy can be expressed as a function of (n) . To do this, we first need to understand the relationship between (r) and (n) .

Bohr Radius and Electron Orbit

The Bohr model provides a simplified picture, where the radius of the (n) -th orbit is:

$$r_n = n^2 a_0$$

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where:

- a_0 is the Bohr radius, approximately $0.529 \times 10^{-10} \text{ m}$.

The Bohr radius is derived from fundamental physical constants:

$$a_0 = \frac{4\pi \epsilon_0 \hbar^2}{m_e e^2}$$

with:

- ϵ_0 : vacuum permittivity,
- $\hbar$: reduced Planck's constant,
- m_e : electron mass,
- e : elementary charge.

Potential Energy in the n -th Level

Replacing r with r_n :

$$U_n = -\frac{k e^2}{r_n} = -\frac{k e^2}{n^2 a_0}$$

Substituting $k = \frac{1}{4\pi \epsilon_0}$, the expression simplifies to:

$$U_n = -\frac{e^2}{4\pi \epsilon_0 n^2 a_0}$$

Given that the electrostatic potential energy at the n -th level is proportional to $-1/n^2$, the potential energy becomes less negative (less bound) as n increases.

Final Expression

The electric potential energy of the electron when it is in the n -th energy level of the hydrogen atom is:

$$\boxed{U_n = -\frac{13.6 \text{ eV}}{n^2}}$$

This numerical value corresponds to the ionization energy at the ground state ($n=1$), where:

- For $n=1$, $U_1 = -13.6 \text{ eV}$,
- For higher n , the potential energy magnitude decreases, approaching zero as $n \rightarrow \infty$.

Relationship Between Potential Energy and Total Energy

In the hydrogen atom, the total energy E_n of the electron in the n -th level is related to the potential energy U_n by:

$$E_n = \frac{U_n}{2}$$

This relationship arises from the virial theorem for Coulomb potentials, which states that for bound systems:

$$\langle T \rangle = -\frac{1}{2} \langle U \rangle$$

where:

- $\langle T \rangle$ is the average kinetic energy,
- $\langle U \rangle$ is the average potential energy.

Thus, the total energy is negative and equal to half the potential energy:

$$E_n = -\frac{13.6 \text{ eV}}{n^2} = -\frac{6.8 \text{ eV}}{n^2}$$

This energy indicates the energy required to remove the electron from the atom (ionization energy).

Significance of Electric Potential Energy in Atomic Physics

Understanding the electric potential energy in the hydrogen atom is crucial for multiple reasons:

- It explains the stability of the atom: the negative potential energy

signifies a bound state.

- It helps in calculating spectral lines: transitions between energy levels involve changes in potential and total energy.
- It underpins the quantization of energy: the $(1/n^2)$ dependence leads to discrete emission and absorption spectra.
- It provides foundational insight into more complex atomic systems, where Coulomb interactions dominate.

Summary

In conclusion, the electric potential energy of an electron in the hydrogen atom when found in the (n) -th energy level is given by:

$$U_n = -\frac{13.6 \text{ eV}}{n^2}$$

This negative value reflects the bound state of the electron within the atom, with the magnitude decreasing as the electron moves to higher energy levels. The relationship between potential energy, total energy, and quantum number (n) forms the basis for understanding atomic spectra, stability, and the fundamental principles of quantum mechanics.

Key Takeaways:

- The potential energy becomes less negative as (n) increases.
- It is directly proportional to $(-1/n^2)$.
- Ground state potential energy is approximately -13.6 eV.
- The potential energy is essential for understanding atomic structure and spectral phenomena.

By grasping these concepts, students and researchers can better understand atomic behavior and the quantum nature of matter at microscopic scales.

Frequently Asked Questions

What is the formula for the electric potential energy of an electron in the n th energy level of the hydrogen atom?

The electric potential energy (U) of an electron in the n th energy level of a hydrogen atom is given by $U = -13.6 \text{ eV} / n$, where 13.6 eV is the ionization energy of hydrogen.

How does the electric potential energy vary with the principal quantum number n in the hydrogen atom?

The electric potential energy becomes less negative (approaches zero) as n increases, meaning the electron is farther from the nucleus and has higher potential energy in higher energy levels.

Is the electric potential energy of the electron positive or negative in the hydrogen atom, and why?

It is negative because the electron is bound to the nucleus; negative potential energy indicates a bound system, with energy required to free the electron.

What is the electric potential energy of an electron in the ground state ($n=1$) of hydrogen?

In the ground state ($n=1$), the electric potential energy of the electron is approximately -13.6 eV.

How does the electric potential energy relate to the total energy of the electron in the hydrogen atom?

The electric potential energy is exactly twice the magnitude of the total energy but with opposite sign; total energy $E = U/2$, where U is the electric potential energy.

What happens to the electric potential energy of the electron as it transitions from a higher to a lower energy level in hydrogen?

The electric potential energy becomes more negative, releasing energy that can be emitted as photons during the transition.

Can the electric potential energy of the electron in the n th level be positive? Why or why not?

No, the electric potential energy is always negative for bound states in the hydrogen atom because the electron is attracted to the nucleus; positive potential energy would imply a free or unbound electron.

How is the electric potential energy related to the radius of the electron's orbit in the hydrogen atom?

Electric potential energy is inversely proportional to the radius of the orbit; as the radius increases with higher n , the potential energy becomes

less negative.

Why is understanding the electric potential energy important for explaining spectral lines in hydrogen?

Because changes in the electron's electric potential energy during transitions lead to photon emission or absorption, which produce the characteristic spectral lines.

Additional Resources

In The Hydrogen Atom, What Is The Electric Potential Energy Of The Electron When It Is Found In The N

Understanding the behavior of electrons within atoms is fundamental to quantum mechanics and atomic physics. Among the simplest yet most insightful systems is the hydrogen atom—a single proton nucleus orbited by one electron. This atom serves as the cornerstone for understanding more complex atomic structures and the principles governing subatomic particles. A key concept in this context is the electric potential energy of the electron when situated in a specific energy level characterized by the principal quantum number (n) . This article delves into the nature of electric potential energy in the hydrogen atom, elucidating how it is calculated, what it signifies physically, and its role in the broader framework of atomic physics.

The Hydrogen Atom: A Fundamental Model

Before exploring the electric potential energy, a brief overview of the hydrogen atom's structure is warranted. The hydrogen atom comprises a positively charged nucleus—essentially a single proton—and a negatively charged electron. The Coulomb force, which is the electrostatic attraction between these opposite charges, governs their interaction.

In classical physics, the electron is envisioned as orbiting the nucleus much like planets orbit the sun, but quantum mechanics refines this picture. Instead of fixed orbits, electrons are described by wavefunctions that provide probability distributions of their positions. These wavefunctions are characterized by quantum numbers, with the principal quantum number (n) playing a pivotal role in defining the energy levels and spatial extent of the electron's probable location.

Quantum Energy Levels in the Hydrogen Atom

The energy of an electron in the hydrogen atom is quantized, meaning it can only take specific discrete values. These energies depend primarily on the

principal quantum number (n) , which can take positive integer values: $(n = 1, 2, 3, \dots)$.

The quantized energy levels (E_n) are given by:

$$E_n = - \frac{13.6 \text{ eV}}{n^2}$$

where 13.6 eV (electron volts) is the ionization energy of hydrogen—the energy required to completely remove the electron from the atom in its ground state.

These energy levels are negative, indicating that the electron is bound to the nucleus; energy must be supplied to free it. The more significant the principal quantum number (n) , the higher (less negative) the energy, approaching zero as the electron becomes free from the atom.

Electric Potential Energy: Concept and Significance

Electric potential energy in the hydrogen atom context refers to the electrostatic potential energy stored due to the Coulomb attraction between the electron and the nucleus. Unlike kinetic energy, which relates to the electron's motion, potential energy encapsulates the position-dependent energy stored in the electrostatic field.

Physically, the potential energy indicates how much work must be done to bring the electron from a reference point at infinity (where the Coulomb potential is zero) to its position within the atom at a specific energy level (n) .

In atomic physics, the electric potential energy (U) of the electron when in a specific quantum state is directly related to the total energy (E_n) but with an essential sign difference.

Deriving the Electric Potential Energy in the Hydrogen Atom

The relationship between the total energy (E_n) , the kinetic energy (K) , and the potential energy (U) in the hydrogen atom stems from the virial theorem. For Coulomb potentials, the virial theorem states that:

$$\boxed{K = - \frac{1}{2} U}$$

and the total energy is:

$$E_n = K + U$$

Using the virial theorem, the total energy can be expressed as:

$$E_n = K + U = -\frac{1}{2} U + U = \frac{1}{2} U$$

Rearranging, the potential energy:

$$U = 2 E_n$$

Since the total energy (E_n) for the hydrogen atom is known:

$$E_n = -\frac{13.6 \text{ eV}}{n^2}$$

the electric potential energy becomes:

$$U_n = 2 \times \left(-\frac{13.6 \text{ eV}}{n^2}\right) = -\frac{27.2 \text{ eV}}{n^2}$$

This formula indicates that the potential energy of the electron in the (n^{th}) energy level is negative and its magnitude decreases as (n) increases.

Physical Interpretation of the Electric Potential Energy

The negative sign of (U_n) signifies that the electron is in a bound state within the atom. It would require an input of energy equal to $(27.2 \text{ eV} / n^2)$ to free the electron from the atom, bringing it from the energy level (n) to an infinite separation where potential energy is zero.

As (n) increases, the electron's average distance from the nucleus grows, and the system's potential energy approaches zero from below. For example:

- At $(n=1)$ (ground state):

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$$U_1 = -27.2 \text{ eV}$$

- At $(n=2)$:

$$U_2 = -\frac{27.2 \text{ eV}}{4} = -6.8 \text{ eV}$$

- At high (n) (say, $(n=10)$):

$$U_{10} = -\frac{27.2 \text{ eV}}{100} = -0.272 \text{ eV}$$

This progression illustrates how the electron's electrostatic potential energy becomes less negative as it moves to higher energy levels, reflecting a looser binding to the nucleus.

Significance in Spectroscopy and Atomic Transitions

The electric potential energy's quantized nature underpins the phenomena of spectral lines and atomic emission or absorption. When an electron transitions between energy levels (n_i) and (n_f) , the change in potential energy corresponds to the energy of the emitted or absorbed photon:

$$\Delta U = U_{n_f} - U_{n_i}$$

which, due to the relation $(U_n = -\frac{13.6 \text{ eV}}{n^2})$, simplifies calculations of spectral lines.

For example, the Lyman series involves transitions down to $(n=1)$, with the energy difference—and thus the photon wavelength—directly tied to the change in potential energy.

Broader Context and Applications

Understanding the electric potential energy in the hydrogen atom is not just academic; it forms the basis for various applications:

- **Quantum Mechanics Foundations:** The hydrogen atom model is a cornerstone for quantum theory development, helping refine our understanding of atomic structure.
- **Spectroscopy:** Precise knowledge of energy levels and potential energies

aids in interpreting spectral lines, crucial in fields like astrophysics and plasma physics.

- Chemical Bonding: Insights from atomic potential energies extend to molecular bonding and the behavior of more complex atoms and molecules.

- Quantum Computing and Nanotechnology: Manipulating electron energy states relies on fundamental principles derived from hydrogenic models.

Conclusion

The electric potential energy of an electron in the hydrogen atom at a principal quantum number (n) embodies the electrostatic binding that holds the electron in its quantized state. It is given by:

$$U_n = - \frac{27.2 \text{ eV}}{n^2}$$

This negative value signifies the bound nature of the electron, and its magnitude diminishes as the electron ascends to higher energy levels. The profound relationship between total energy and potential energy, established through the virial theorem, not only clarifies the electron's energetic landscape but also underpins many observable phenomena, from spectral lines to quantum transitions.

By understanding the electric potential energy in the hydrogen atom, scientists continue to unravel the intricacies of atomic and subatomic systems, laying the groundwork for advancements in technology, spectroscopy, and our fundamental comprehension of matter.

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