Let C Be The Square With Vertices (0,0), (1,0), (1,1) And (0,1) (Oriented Counter Clockwise). Compute

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Understanding geometric figures and their properties is fundamental in mathematical analysis, especially in coordinate geometry. In this article, we delve into the problem of computing certain properties of a square defined by specific vertices, focusing on how to analyze and calculate key features such as area, perimeter, and possibly integrals over the region. Our goal is to provide a comprehensive guide, step-by-step, to solving such problems, emphasizing the importance of orientation, vertices, and the underlying mathematical principles.

Defining the Square and Its Orientation

Before diving into computations, it's crucial to understand the shape's configuration.

Vertices of the Square

The square \(C \) is given by the vertices:

- \((0,0) \)
- \((1,0) \)
- \((1,1) \)
- \((0,1) \)

These points form a unit square aligned with the coordinate axes.

Orientation: Counterclockwise

The vertices are ordered counterclockwise:

1. \((0,0) \)
2. \((1,0) \)
3. \((1,1) \)
4. \((0,1) \)

This orientation is important for certain calculations, such as computing the area via the shoelace formula or defining the direction of boundary integrals.

Computing the Area of the Square

One of the fundamental properties of a polygon like a square is its area. For a polygon with vertices listed in order, the shoelace formula provides an efficient way to compute the area.

Shoelace Formula for Polygon Area

Given vertices $(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$, the area A is:

$$
A = \frac{1}{2} \left| \sum_{i=1}^{n} (x_i y_{i+1} - y_i x_{i+1}) \right|
$$

where $(x_{n+1}, y_{n+1}) = (x_1, y_1)$.

Applying the Shoelace Formula to Our Square

Using the vertices:

Vertex	x_i	y_i
1	0	0
2	1	0
3	1	1
4	0	1

Calculate:

$$
\begin{aligned}
\text{Sum}_1 &= x_1 y_2 + x_2 y_3 + x_3 y_4 + x_4 y_1 \\
&= (0)(0) + (1)(1) + (1)(1) + (0)(0) = 0 + 1 + 1 + 0 = 2 \\
\text{Sum}_2 &= y_1 x_2 + y_2 x_3 + y_3 x_4 + y_4 x_1 \\
&= (0)(1) + (0)(1) + (1)(0) + (1)(0) = 0 + 0 + 0 + 0 = 0
\end{aligned}
$$

\]

Therefore,

\[
A = \frac{1}{2} | \text{Sum}_1 - \text{Sum}_2 | = \frac{1}{2} | 2 - 0 | = 1
\]

Result: The area of the square \(C \) is 1 square unit.

Calculating the Perimeter of the Square

Since the shape is a square with side length 1, the perimeter \(P \) is straightforward:

\[
P = 4 \times \text{side length} = 4 \times 1 = 4
\]

Alternatively, summing the lengths of each side:

- \(\text{Distance between } (0,0) \text{ and } (1,0) = 1 \)
- \((1,0) \text{ to } (1,1) = 1 \)
- \((1,1) \text{ to } (0,1) = 1 \)
- \((0,1) \text{ to } (0,0) = 1 \)

Total perimeter: 4 units.

Line Integrals Over the Square Boundary

In many applications, such as Green's theorem, calculating a line integral over the boundary of a region is essential. Given the counterclockwise orientation, the boundary integral's sign aligns with the positive orientation.

Applying Green's Theorem

Green's theorem relates a line integral around a simple closed curve \(C \) to a double integral over the region \(D \) it encloses:

$$
\oint_C (P \, dx + Q \, dy) = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx dy
$$

Suppose we want to compute the circulation or flux around the square.

Sample Computation: Line Integral of a Vector Field

Let's consider the vector field:

$$
\vec{F}(x, y) = (x, y)
$$

and compute the line integral over the boundary C:

$$
\oint_C \vec{F} \cdot d\vec{r}
$$

Given the orientation (counterclockwise), the integral becomes:

$$
\oint_C x\, dx + y\, dy
$$

Using parameterizations for each side:

1. Bottom side: from $(0,0)$ to $(1,0)$

- $x = t$, $y=0$, $t \in [0,1]$
- $dx = dt$, $dy=0$

Integral:

$$
\int_0^1 t\, dt = \frac{1}{2}
$$

2. Right side: from $(1,0)$ to $(1,1)$

- $x=1$, $y = t$, $t \in [0,1]$
- $dx=0$, $dy=dt$

Integral:

$$
\int_0^1 t\, dt = \frac{1}{2}
$$

3. Top side: from $(1,1)$ to $(0,1)$

- $x=1 - t$, $y=1$, $t \in [0,1]$
- $dx = -dt$, $dy=0$

Integral:

$$
\int_0^1 (1 - t)(-dt) = -\int_0^1 (1 - t) dt = -\left[t - \frac{t^2}{2} \right]_0^1 = -\left(1 - \frac{1}{2} \right) = -\frac{1}{2}
$$

4. Left side: from $(0,1)$ to $(0,0)$

- $x=0$, $y=1 - t$, $t \in [0,1]$
- $dx=0$, $dy = -dt$

Integral:

$$
\int_0^1 (0)(0) + (1 - t)(-dt) = -\int_0^1 (1 - t) dt = -\left[t - \frac{t^2}{2} \right]_0^1 = -\left(1 - \frac{1}{2} \right) = -\frac{1}{2}
$$

Adding all parts:

$$
\frac{1}{2} + \frac{1}{2} - \frac{1}{2} - \frac{1}{2} = 0
$$

Conclusion: The line integral over the boundary C for the vector field $\vec{F} = (x, y)$ is 0, which aligns with the fact that the curl of $\vec{F}$ is zero everywhere.

Computing Double Integrals Over the Square Region

Double integrals are instrumental when calculating areas, mass distributions, or other quantities over a region.

Example: Computing the Double Integral of a Function

Suppose we want to compute:

$$
\iint_{C} f(x, y) \, dx\, dy
$$

where $f(x, y) = x + y$.

Since C is the unit square:

$$
0 \leq x \leq 1, \quad 0
$$

Frequently Asked Questions

What are the vertices of the square C in the given problem?

The vertices of square C are (0,0), (1,0), (1,1), and (0,1).

In what order are the vertices of square C given?

The vertices are oriented counterclockwise: (0,0), (1,0), (1,1), and (0,1).

What are common geometric computations to perform on the square C?

Common computations include calculating the area, perimeter, centroid, and equations of its sides.

How do you find the area of the square C with vertices at (0,0), (1,0), (1,1), and (0,1)?

Since the side length is 1, the area is side × side = 1 × 1 = 1.

What is the significance of the vertices being oriented counterclockwise?

Counterclockwise orientation is important for correctly applying certain algorithms, such as calculating polygon area using the shoelace formula.

How can the shoelace formula be used to verify the area of square C?

By plugging the vertices into the shoelace formula, the computed area will confirm it is 1, consistent with the shape's dimensions.

What is the next step if the problem asks to compute a property like the centroid or perimeter of C?

For the centroid, average the x and y coordinates of the vertices; for the perimeter, sum the lengths of all sides.

Additional Resources

Let C Be The Square With Vertices (0,0), (1,0), (1,1), And (0,1) (Oriented Counter Clockwise). Compute

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Introduction

In the realm of mathematical analysis, geometry, and vector calculus, the study of simple geometric shapes such as squares forms the foundation for understanding complex concepts like line integrals, Green's theorem, and surface area computations. The problem statement here involves a square C with explicitly given vertices and an orientation, prompting us to explore the geometric properties, parameterizations, and integrals associated with this shape. This article aims to dissect the problem comprehensively, providing detailed explanations, step-by-step computations, and contextual insights, all within a structured and accessible narrative.

Defining the Square C

Geometric Description and Coordinates

The square C is specified with vertices:

- $(0,0)$
- $(1,0)$

- $(1,1)$
- $(0,1)$

These four points form a unit square aligned with the Cartesian axes. The problem notes that the orientation is counterclockwise, which is significant when applying certain theorems such as Green's theorem or calculating line integrals, as the direction of traversal influences the sign of the computed quantities.

Significance of Orientation

The orientation determines the direction in which the boundary C is traversed. In counterclockwise orientation, the boundary proceeds from $(0,0)$ to $(1,0)$, then to $(1,1)$, then to $(0,1)$, and finally back to $(0,0)$. This orientation is standard for applying Green's theorem, which relates line integrals around a simple closed curve to double integrals over the region it encloses.

Mathematical Context and Objectives

The core of this analysis revolves around computing specific quantities associated with the square C. Depending on the context, this could involve:

- Evaluating line integrals of vector fields around C.
- Applying Green's theorem to convert line integrals into double integrals.
- Calculating flux or circulation.
- Computing areas or other geometric properties.

While the problem statement hints at "compute" without specifying which integral or quantity, the typical scenario involves evaluating a line integral or applying Green's theorem to derive related quantities.

Parameterization of the Square C

Why Parameterize?

Parameterization simplifies the process of calculating line integrals or applying theorems. It involves expressing each boundary segment as a vector function of a parameter t, allowing us to integrate over the parameter domain.

Boundary Segments

Given the vertices, the square's boundary comprises four line segments:

1. Segment 1: from $(0,0)$ to $(1,0)$
2. Segment 2: from $(1,0)$ to $(1,1)$
3. Segment 3: from $(1,1)$ to $(0,1)$
4. Segment 4: from $(0,1)$ back to $(0,0)$

Each segment can be parameterized separately:

- Segment 1: $\mathbf{r}_1(t) = (t, 0)$, with $t \in [0,1]$
- Segment 2: $\mathbf{r}_2(t) = (1, t)$, with $t \in [0,1]$
- Segment 3: $\mathbf{r}_3(t) = (1 - t, 1)$, with $t \in [0,1]$
- Segment 4: $\mathbf{r}_4(t) = (0, 1 - t)$, with $t \in [0,1]$

This explicit parameterization facilitates straightforward computation of line integrals along each segment.

Computing Line Integrals: Step-by-Step Approach

Suppose we wish to evaluate a line integral of a vector field $\mathbf{F}(x, y) = P(x, y) \mathbf{i} + Q(x, y) \mathbf{j}$ along the boundary C. The general form is:

$$
\oint_C \mathbf{F} \cdot d\mathbf{r} = \sum_{k=1}^4 \int_{t=0}^1 \mathbf{F}(\mathbf{r}_k(t)) \cdot \mathbf{r}_k'(t) dt
$$

where $\mathbf{r}_k(t)$ parameterizes the k^{th} segment, and $\mathbf{r}_k'(t)$ is its derivative.

Note: Since the specific $\mathbf{F}$ isn't provided, we'll illustrate the process with an example, then discuss how to generalize.

Applying Green's Theorem for Simplification

Green's theorem relates the line integral around a simple, positively oriented, closed curve C to a double integral over the region D it encloses:

$$
\oint_C P\, dx + Q\, dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dx\, dy
$$

This theorem provides a powerful shortcut for evaluating certain integrals, especially when the vector field components P and Q are well-behaved functions.

Analytical Procedure for a Specific Example

Suppose, for illustrative purposes, we are asked to compute the circulation of the vector field $\mathbf{F}(x,y) = (-y, x)$ around the square C. This is a classic case, often related to rotational fields.

Step 1: Identify P and Q

- $P(x, y) = -y$
- $Q(x, y) = x$

Step 2: Compute the derivatives

- $\frac{\partial Q}{\partial x} = 1$
- $\frac{\partial P}{\partial y} = -1$

Thus,

$$
\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} = 1 - (-1) = 2
$$

Step 3: Set up the double integral over the region D

Since D is the unit square with vertices at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$, the double integral becomes:

$$
\iint_D 2 \, dx\, dy = 2 \times \text{Area}(D) = 2 \times 1 = 2
$$

Result: The circulation of $\mathbf{F}$ around C is 2.

This example underscores how Green's theorem simplifies such calculations dramatically.

Generalization and Other Quantities

Depending on the original problem's intent, we might need to:

- Compute flux: involving P and Q in different arrangements.
- Evaluate specific line integrals: such as $\int_C y\, dx + x\, dy$.
- Determine area: by choosing appropriate integrands.

The key takeaway is that the shape's simplicity allows for multiple analytical approaches, including direct parameterization or applying fundamental theorems.

Significance in Broader Context

The square C serves as an archetype in multivariable calculus, illustrating core principles:

- Line Integrals: foundational in understanding work done by fields or circulation.
- Green’s Theorem: transforming boundary integrals into area integrals simplifies calculations.
- Orientation: crucial for correct application of theorems and sign conventions.
- Parameterization: essential for explicit integral evaluations.

These concepts extend beyond the square to more complex shapes, making this analysis a vital educational stepping stone.

Practical Applications and Implications

Understanding how to compute integrals over shapes like C has implications across physics, engineering, and computer graphics:

- Electromagnetism: calculating magnetic flux or electric circulation.
- Fluid Dynamics: determining flow patterns around obstacles.
- Robotics and Path Planning: evaluating paths and work along boundaries.
- Computer-Aided Design (CAD): surface and boundary computations.

Mastering the computation over C provides a solid foundation for tackling real-world problems involving fields and flows.

Conclusion

The problem involving the square C with vertices at $(0,0)$, $(1,0)$, $(1,1)$, and $(0,1)$, oriented counterclockwise, encapsulates fundamental principles of vector calculus and geometry. Whether through direct parameterization or applying powerful theorems like Green's, the analysis illustrates the elegance and utility of mathematical tools in simplifying and solving complex integral problems.

By dissecting the shape, understanding the importance of orientation, and leveraging the symmetry of the unit square, we gain insights not only into the specific computations but also into the broader mathematical structures that underpin much of science and engineering. The techniques discussed serve as essential tools for students and professionals alike, fostering a deeper appreciation for the interplay between geometry, calculus,

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