Let D Be The Region Bounded By The Two Paraboloids Z = 2x + 2y - 4 And Z = 5-x-y Where X 20 And Y 20.

Let D Be The Region Bounded By The Two Paraboloids Z = 2x + 2y - 4 And Z = 5 - x - y Where X ≥ 20 And Y ≥ 20.

Introduction

In multivariable calculus, understanding regions bounded by surfaces is fundamental for evaluating integrals, analyzing volumes, and solving various physical problems. One intriguing case involves the region D bounded by two paraboloids: $Z = 2x + 2y - 4$ and $Z = 5 - x - y$, with the additional constraints $x \geq 20$ and $y \geq 20$. This article offers a comprehensive exploration of this region, detailing how to analyze, visualize, and compute properties associated with it.

Understanding the Surfaces and Region

The Surfaces Involved

- Paraboloid 1: $Z = 2x + 2y - 4$

- Paraboloid 2: $Z = 5 - x - y$

Both are plane-like paraboloids opening in different directions, intersecting along a curve in space. The region D is the volume between these two surfaces, constrained to the domain where $x \geq 20$ and $y \geq 20$.

Visualizing the Surfaces

Graphical Representation

Visualizing these surfaces helps understand the region D:

- Surface 1: $Z = 2x + 2y - 4$ is a plane with a positive slope in both x and y directions.

- Surface 2: $Z = 5 - x - y$ is a plane with negative slope, decreasing as x or y increases.

Their intersection forms a curve in the xy-plane, which can be found by setting the two equations equal:

$$
2x + 2y - 4 = 5 - x - y
$$

Finding the Intersection Curve

Step 1: Set the Equations Equal

$$
2x + 2y - 4 = 5 - x - y
$$

Step 2: Simplify the Equation

$$
2x + 2y - 4 = 5 - x - y
$$

Bring all terms to one side:

$$
2x + 2y - 4 - 5 + x + y = 0
$$

Combine like terms:

$$
(2x + x) + (2y + y) - 9 = 0
$$

$$
3x + 3y = 9
$$

Step 3: Express the Intersection Line

Divide through by 3:

$$
x + y = 3
$$

This line in the xy-plane represents the intersection of the two paraboloids.

Domain Constraints and Region D

Given the constraints $x \geq 20$ and $y \geq 20$, and the intersection line $x + y = 3$, it is clear that:

- The point (x, y) of intersection occurs at values less than 20 for both x and y.

- Since $x, y \geq 20$, the region of interest lies outside the intersection curve, specifically in the region where $x \geq 20$, $y \geq 20$.

- The paraboloids intersect at a point far from the region $x, y \geq 20$, meaning the two surfaces do not intersect within the constrained domain $x, y \geq 20$.

Analyzing the Region D

Geometric Intuition

- Since the paraboloids do not intersect within the domain $x, y \geq 20$, the volume D is the space between the two surfaces in the specified quadrant, extending from some lower bounds (here 20) to infinity or until other boundary conditions.

- For each fixed (x, y) with $x, y \geq 20$, the vertical "slice" of the region D is between the lower surface $Z = 2x + 2y - 4$ and the upper surface $Z = 5 - x - y$, if the upper surface is indeed above the lower surface.

Checking Which Surface Is On Top

To determine which surface lies above the other in the domain $x, y \geq 20$:

$$
Z_{1} = 2x + 2y - 4
$$
$$
Z_{2} = 5 - x - y
$$

Calculate the difference:

$$
Z_{1} - Z_{2} = (2x + 2y - 4) - (5 - x - y) = 2x + 2y - 4 - 5 + x + y = 3x + 3y - 9
$$

$$

Factor:

$$
Z_{1} - Z_{2} = 3(x + y - 3)
$$

- For $x, y \geq 20$, $x + y \geq 40$, so:

$$
Z_{1} - Z_{2} = 3(\text{value} \geq 40 - 3) = 3(37) = 111 > 0
$$

Thus, $Z_{1} > Z_{2}$ for all $x, y \geq 20$. The paraboloid $Z = 2x + 2y - 4$ is always above $Z = 5 - x - y$ in the domain of interest.

Computing the Volume V of Region D

Given the above, the volume V of D can be expressed as a double integral over the xy-domain:

$$
V = \iint_{R} \left[Z_{\text{top}}(x, y) - Z_{\text{bottom}}(x, y) \right] dx dy
$$

where:

- $Z_{\text{top}}(x, y) = 2x + 2y - 4$

- $Z_{\text{bottom}}(x, y) = 5 - x - y$

and the region R in the xy-plane is:

$$
x \geq 20,\quad y \geq 20
$$

Since the surfaces do not intersect within this domain, and the region extends infinitely, the integral becomes:

$$
V = \int_{x=20}^{\infty} \int_{y=20}^{\infty} \left[(2x + 2y - 4) - (5 - x - y) \right] dy dx
$$

Simplifying the Integrand

Calculate the integrand:

$$
(2x + 2y - 4) - (5 - x - y) = 2x + 2y - 4 - 5 + x + y = (2x + x) + (2y + y) - 9 = 3x + 3y - 9
$$

Thus,

$$
V = \int_{20}^{\infty} \int_{20}^{\infty} (3x + 3y - 9) dy dx
$$

Computing the Double Integral

Step 1: Inner integral with respect to y:

$$
I_{y} = \int_{y=20}^{\infty} (3x + 3y - 9) dy
$$

For fixed x, integrate:

$$
I_{y} = \int_{20}^{\infty} (3x - 9 + 3y) dy
$$

Note that $3x - 9$ is constant with respect to y:

$$
I_{y} = (3x - 9) \int_{20}^{\infty} dy + 3 \int_{20}^{\infty} y dy
$$

Evaluate:

$$
I_{y} = (3x - 9) [y]_{20}^{\infty} + 3 \left[\frac{y^{2}}{2} \right]_{20}^{\infty}
$$

- The term $(3x - 9) [y]_{20}^{\infty}$ diverges unless $3x - 9 = 0$ and the integral converges.

- Since $3x - 9 \neq 0$ for $x \geq 20$, the integral diverges to infinity.

Therefore, the volume V is unbounded (infinite) over the given domain.

Implications and Interpretations

- The infinite result indicates that the region D extends infinitely in the y and x directions, and the volume between the surfaces is unbounded.

- For practical purposes, to compute a finite volume, boundary constraints (

Frequently Asked Questions

What is the region D bounded by the paraboloids Z = 2x + 2y - 4 and Z = 5 - x - y?

Region D is the set of points (x, y) where the two paraboloids intersect, meaning their Z-values are equal, and within the constraints $x \leq 20$ and $y \leq 20$.

How do you find the intersection curve of the paraboloids Z = 2x + 2y - 4 and Z = 5 - x - y?

Set the two equations equal: $2x + 2y - 4 = 5 - x - y$. Simplify to find the intersection line: $3x + 3y = 9$, which simplifies to $x + y = 3$.

What are the bounds for x and y in the region D?

The bounds are $x \leq 20$, $y \leq 20$, and from the intersection line $x + y = 3$, typically considering the region where both paraboloids are above or below certain levels, depending on the context.

How can the volume of the region D be computed?

The volume can be found by setting up a double integral over the region D, integrating the difference between the upper and lower paraboloids: Volume = $\iint_D (Z_{upper} - Z_{lower})\, dx\, dy$.

What is the significance of the constraints x ≤ 20 and y ≤ 20 in defining the region D?

These constraints limit the region D to a finite portion of the paraboloids, ensuring the region is bounded and the volume integral converges.

Can the region D be described in terms of its

projection onto the xy-plane?

Yes, the projection of D onto the xy-plane is bounded by the intersection curve x + y = 3 and the limits x ≤ 20, y ≤ 20, forming a finite region where the paraboloids enclose space.

What is the shape of the intersection curve between the two paraboloids?

The intersection curve lies along the line x + y = 3, which is a straight line in the xy-plane, representing the set of points where the paraboloids meet.

How does the bounded region D relate to the visualization of the paraboloids?

Region D represents the volume enclosed between the two paraboloids, limited by the specified constraints, and can be visualized as the area between the two surfaces above the xy-plane within the bounds.

Additional Resources

Let D Be The Region Bounded By The Two Paraboloids Z = 2x + 2y - 4 And Z = 5 - x - y Where X ≥ 0 And Y ≥ 0.

Introduction

In the realm of multivariable calculus, understanding the geometric regions bounded by surfaces is fundamental. These regions not only serve as the foundation for calculating volumes and integrals but also provide insight into the spatial relationships between complex surfaces. Today, we explore a particular region in three-dimensional space defined by two paraboloids: $Z = 2x + 2y - 4$ and $Z = 5 - x - y$, constrained to the first quadrant where $x \geq 0$ and $y \geq 0$. This exploration offers a window into the interplay between linear and quadratic surfaces, demonstrating how calculus enables us to analyze and visualize such regions with precision.

Defining the Region D: An Overview

The region D is specified by two surfaces:

- Paraboloid 1: $Z = 2x + 2y - 4$
- Paraboloid 2: $Z = 5 - x - y$

Both are defined over the domain where $x \geq 0$ and $y \geq 0$. To comprehend the shape and extent of D, we need to analyze where these surfaces intersect, their relative positions, and the bounds imposed by the constraints.

Key steps in understanding D:

1. Identify the intersection curve between the two paraboloids.
2. Determine the region in the xy-plane over which the volume is bounded.
3. Set the limits for integration based on the intersection and the constraints.

Geometric Interpretation of the Surfaces

The First Paraboloid: $Z = 2x + 2y - 4$

This surface is a plane inclined in the z-direction, with a linear relation involving x and y. It intersects the xy-plane (where $Z=0$) along the line:

$$
0 = 2x + 2y - 4 \Rightarrow 2x + 2y = 4 \Rightarrow x + y = 2
$$

This line in the xy-plane forms the boundary where the paraboloid touches the plane $Z=0$.

The Second Paraboloid: $Z = 5 - x - y$

This surface is also a plane with a negative slope, intersecting the xy-plane along:

$$
0 = 5 - x - y \Rightarrow x + y = 5
$$

This boundary is farther out in the xy-plane compared to the first paraboloid's intersection line.

Finding the Intersection of the Surfaces

To find the curve where the two paraboloids meet, set their Z expressions equal:

$$
2x + 2y - 4 = 5 - x - y
$$

Rearranged:

$$
2x + 2y - 4 = 5 - x - y
$$

$$
2x + 2y + x + y = 5 + 4
$$

$$
3x + 3y = 9
$$

$$
x + y = 3
$$

The intersection line in the xy-plane is thus:

$$
x + y = 3
$$

Given the constraints $x \geq 0$, $y \geq 0$, the intersection segment runs from $(0, 3)$ to $(3, 0)$.

Visualizing the Region D

The region D is bounded below by the surface $Z = 2x + 2y - 4$ and above by $Z = 5 - x - y$, with the intersection line $x + y = 3$ acting as the boundary in the xy-plane.

In the xy-plane:

- The feasible region is the triangle with vertices at:
- $(0, 0)$
- $(0, 3)$
- $(3, 0)$

This triangle is bounded by the axes and the line $x + y = 3$, all within the first quadrant.

Setting Up the Double Integral

To compute the volume of D, we integrate the difference between the upper surface and the lower surface over the region R:

$$
\text{Volume} = \iint_{R} [(5 - x - y) - (2x + 2y - 4)] \, dA
$$

Simplify the integrand:

$$
(5 - x - y) - (2x + 2y - 4) = 5 - x - y - 2x - 2y + 4 = (5 + 4) - (x + 2x) - (y + 2y) = 9 - 3x - 3y
$$

Factor out 3:

$$
= 3(3 - x - y)
$$

The limits for x and y:

- For a fixed x in $[0, 3]$, y varies from 0 up to $3 - x$.

Thus, the volume becomes:

$$
V = \int_{x=0}^{3} \int_{y=0}^{3 - x} 3(3 - x - y) \, dy \, dx
$$

Computing the Volume

Let's compute the inner integral:

$$
I(x) = \int_{0}^{3 - x} 3(3 - x - y) \, dy
$$

Expanding:

$$
I(x) = 3 \int_{0}^{3 - x} (3 - x - y) \, dy
$$

Integrate with respect to y:

$$
I(x) = 3 \left[(3 - x) y - \frac{1}{2} y^2 \right]_0^{3 - x}
$$

Evaluate at the upper limit:

$$
I(x) = 3 \left[(3 - x)(3 - x) - \frac{1}{2} (3 - x)^2 \right]
$$

Simplify:

$$
I(x) = 3 \left[(3 - x)^2 - \frac{1}{2}(3 - x)^2 \right] = 3 \left[\frac{1}{2} (3 - x)^2 \right] = \frac{3}{2} (3 - x)^2
$$

Now, integrate with respect to x:

$$
V = \int_0^{3} \frac{3}{2} (3 - x)^2 \, dx
$$

Factor out constants:

$$
V = \frac{3}{2} \int_0^{3} (3 - x)^2 \, dx
$$

Substitute $u = 3 - x$, $du = -dx$. When $x = 0$, $u = 3$; when $x = 3$, $u = 0$:

$$
V = \frac{3}{2} \int_{u=3}^{0} u^2 \, (-du) = \frac{3}{2} \int_0^{3} u^2 \, du
$$

Compute:

$$
V = \frac{3}{2} \left[\frac{u^3}{3} \right]_0^{3} = \frac{3}{2} \left(\frac{3^3}{3} - 0 \right) = \frac{3}{2} \times \frac{27}{3} = \frac{3}{2} \times 9 = \frac{27}{2} = 13.5
$$

Final Result and Interpretation

The volume V of the region D bounded by the two paraboloids in the specified domain is 13.5 cubic units.

This calculation exemplifies how calculus enables the precise measurement of complex three-dimensional regions, transforming visual intuition into exact numerical results. The key steps involved identifying the intersection curve, establishing the bounds in the xy-plane, setting up the integral of the difference between the surfaces, and performing integration systematically.

Practical Applications of Such Calculations

Understanding the volume enclosed between surfaces like paraboloids has practical significance across various fields:

- Engineering: Designing mechanical parts where material boundaries are defined by curved surfaces.
- Physics: Calculating mass or charge distributions in regions bounded by specific surface equations.
- Environmental Science: Modeling terrains or basins with curved boundaries to assess volume-related properties like water capacity.
- Mathematics Education: Enhancing spatial reasoning and integral calculus skills through real-world inspired problems.

Extending the Analysis: Surface Area and Beyond

While volume calculation is fundamental, further analysis might include:

- Computing the surface area of the region \(D \).
- Determining centroid or moments of inertia for the bounded

Let D Be The Region Bounded By The Two Paraboloids Z 2x 2y 4 And Z 5 X Y Where X 20 And Y 20

Let D Be The Region Bounded By The Two Paraboloids Z 2x 2y 4 And Z 5 X Y Where X 20 And Y 20

Related Articles

- QIA. Consider The Market For Paper Products In Kelowna. This Market Is Perfectly Competitive, And The
- Read The Poem.The Wild Swans At Cooleby William Butler YeatsThe Trees Are In Their Autumn Beauty, The
- Rewrite The Function In Simplest Form, Including Theappropriate Domain.f (x) = 3x2 + X - 2_________3x

Back to Home