Let F Be The Function Given By F(x)=3xsinx. What Is The Average Value Of F On The Closed Interval 1x7

Let F Be The Function Given By F(x)=3xsinx. What Is The Average Value Of F On The Closed Interval 1x7

Understanding how to compute the average value of a function over a specific interval is a fundamental concept in calculus, with applications ranging from physics to economics. In this article, we will explore the process of finding the average value of the function $F(x) = 3x \sin x$ over the closed interval $[1,7]$. We will break down each step clearly, discuss the mathematical principles involved, and provide insights into the importance of such calculations.

Introduction to the Average Value of a Function

Before delving into the specific function $F(x) = 3x \sin x$, it is essential to understand what the average value of a function over an interval entails.

Definition of the Average Value

The average value f_{avg} of a continuous function $f(x)$ over a closed interval $[a, b]$ is given by the formula:

$$
f_{avg} = \frac{1}{b - a} \int_a^b f(x) \, dx
$$

This formula essentially computes the mean height of the function on the interval, providing a single value that represents the overall behavior of the function within that range.

Significance of Calculating the Average Value

- Helps in understanding the overall trend of the function.
- Useful in applications such as physics (average velocity), economics (average cost), and engineering.
- Forms a foundation for more advanced calculus concepts like mean value theorems.

Analyzing the Function $F(x) = 3x \sin x$

Let's analyze the function at hand and understand its properties before computing the average value.

Function Characteristics

- Type: The function is a product of a linear term $3x$ and a trigonometric function $\sin x$.
- Domain: The function is continuous for all real numbers, especially on the interval $[1, 7]$.
- Behavior: As x increases, the amplitude of $3x \sin x$ fluctuates due to the sine component, while the linear term $3x$ scales the oscillations.

Importance of the Interval $[1,7]$

- The interval is finite and contains multiple periods of the sine function.
- The function exhibits oscillatory behavior, making the calculation of the average value interesting and insightful.

Calculating the Average Value of $F(x) = 3x \sin x$ on $[1,7]$

The core task involves evaluating the integral:

$$
F_{avg} = \frac{1}{7 - 1} \int_1^7 3x \sin x \, dx
$$

which simplifies to:

$$
F_{avg} = \frac{1}{6} \int_1^7 3x \sin x \, dx
$$

Let's proceed step-by-step.

Step 1: Factor out constants

$$
F_{avg} = \frac{1}{6} \times 3 \int_1^7 x \sin x \, dx = \frac{1}{2} \int_1^7 x \sin x \, dx
$$

Now, the problem reduces to evaluating:

$$
I = \int_1^7 x \sin x \, dx
$$

Step 2: Use Integration by Parts

To compute I, we apply the integration by parts formula:

$$
\int u \, dv = uv - \int v \, du
$$

Let:

- $u = x$ so that $du = dx$
- $dv = \sin x \, dx$ so that $v = -\cos x$

Applying the formula:

$$
I = -x \cos x + \int \cos x \, dx
$$

The integral of $\cos x$ is $\sin x$, so:

$$
I = -x \cos x + \sin x + C
$$

Since we are computing a definite integral from 1 to 7:

$$
I = \left[-x \cos x + \sin x \right]_1^7
$$

Step 3: Evaluate the definite integral

Calculate at $x=7$:

$$
-7 \cos 7 + \sin 7
$$

Calculate at $x=1$:

$$
-1 \cos 1 + \sin 1
$$

Thus:

$$
I = \left(-7 \cos 7 + \sin 7 \right) - \left(-1 \cos 1 + \sin 1 \right)
$$

Simplify:

$$
I = -7 \cos 7 + \sin 7 + \cos 1 - \sin 1
$$

Step 4: Final computation of the average value

Recall that:

$$
F_{avg} = \frac{1}{2} I = \frac{1}{2} \left(-7 \cos 7 + \sin 7 + \cos 1 - \sin 1 \right)
$$

Expressed explicitly:

$$
F_{avg} = \frac{1}{2} \left(-7 \cos 7 + \sin 7 + \cos 1 - \sin 1 \right)
$$

This is the exact average value of $F(x) = 3x \sin x$ over the interval $[1,7]$.

Numerical Approximation of the Average Value

While the exact value involves trigonometric functions at specific points, for practical purposes, numerical approximation is often useful.

Using calculator or computational tools:

- $\cos 7 \approx 0.7539$
- $\sin 7 \approx 0.6569$
- $\cos 1 \approx 0.5403$
- $\sin 1 \approx 0.8415$

Plugging these into the formula:

$$
F_{avg} \approx \frac{1}{2} \left(-7 \times 0.7539 + 0.6569 + 0.5403 - 0.8415 \right)
$$

Calculate step-by-step:

- $-7 \times 0.7539 \approx -5.2773$
- Sum inside parentheses:

$$
-5.2773 + 0.6569 + 0.5403 - 0.8415 = (-5.2773 - 0.8415) + (0.6569 + 0.5403) = -6.1188 + 1.1972 = -4.9216
$$

Finally, multiply by $\frac{1}{2}$:

$$
F_{avg} \approx \frac{1}{2} \times -4.9216 = -2.4608
$$

Therefore, the approximate average value of $F(x) = 3x \sin x$ over $[1,7]$ is about -2.46.

Summary of Key Points

- The average value of a continuous function over an interval is calculated using the integral average formula.
- For $F(x) = 3x \sin x$, the integral was tackled using integration by parts.
- The exact average value involves evaluating the expression:

$$
\frac{1}{2} \left(-7 \cos 7 + \sin 7 + \cos 1 - \sin 1 \right)
$$

- Numerical approximations provide practical insights, especially when dealing with transcendental functions.

Applications and Further Insights

Understanding how to compute the average value of functions like $3x \sin x$ has numerous real-world applications:

- Physics: Calculating average displacement or velocity over a period.
- Engineering: Evaluating the mean load or stress over a component.
- Economics: Determining average cost or revenue over a period.

Additionally, mastering integration techniques such as integration by parts is crucial for tackling more complex functions in calculus.

Conclusion

Calculating the average value of the function $F(x) = 3x \sin x$ over the interval $[1,7]$ combines essential calculus concepts, including definite integrals and integration by parts. The precise value, expressed in terms of cosine and sine functions, provides a deeper understanding of the function's behavior over the interval. Moreover, numerical estimates facilitate practical applications and further analysis. Mastery of these techniques enhances one’s ability to analyze various functions and solve real-world problems involving averages and integral calculus.

Remember: Calculus is a powerful tool that helps quantify and interpret the behavior of functions, and understanding how to compute their average values is fundamental to this understanding.

Frequently Asked Questions

What is the function F(x) given in the problem?

The function is F(x) = 3x sin x.

What is the interval over which we need to find the average value of F?

The interval is from x = 1 to x = 7.

How is the average value of a function over an interval [a, b] calculated?

The average value is given by (1 / (b - a)) times the integral of the function from a to b.

What is the formula for the average value of F on [1,7]?

Average value = (1 / (7 - 1)) ∫ from 1 to 7 of F(x) dx, which simplifies to (1/6) $\int_1^7$ 3x sin x dx.

How do you evaluate the integral ∫ 3x sin x dx?

Use integration by parts, letting u = 3x and dv = sin x dx, then differentiate and integrate accordingly.

What is the result of the integral ∫ 3x sin x dx?

The integral evaluates to -3x cos x + 3 sin x + C.

How do you compute the definite integral $\int_1^7$ 3x sin x dx?

Calculate the antiderivative -3x cos x + 3 sin x at x = 7 and x = 1, then subtract the results.

What is the value of the antiderivative at x = 7?

At x = 7, the antiderivative is -3 7 cos 7 + 3 sin 7.

What is the value of the antiderivative at x = 1?

At x = 1, the antiderivative is -3 1 cos 1 + 3 sin 1.

What is the final step to find the average value of F on [1,7]?

Subtract the antiderivative values at 7 and 1, then multiply by 1/6 to find the average value.

Additional Resources

Understanding the Average Value of a Function: An In-Depth Analysis of F(x) = 3x sin x on [1, 7]

Introduction

In the realm of calculus, the concept of an average value of a function over a specific interval is fundamental for understanding the overall "behavior" or "mean" output of the function within that range. It provides a way to summarize the function's values into a single representative number, which is particularly useful in applications like physics, engineering, economics, and data analysis.

This review delves into the detailed process of finding the average value of the function $F(x) = 3x \sin x$ over the closed interval $[1, 7]$. We will explore the mathematical foundation, the step-by-step calculation process, and contextual insights that deepen understanding of this important concept.

Mathematical Foundations of Average Value of a Function

Definition

For a continuous function $f(x)$ on a closed interval $[a, b]$, the average value f_{avg} is given by:

$$
f_{avg} = \frac{1}{b - a} \int_{a}^{b} f(x) \, dx
$$

This integral computes the total "accumulated" value of the function over the interval, and dividing by the interval length normalizes it to an average.

Significance

- Physical Interpretation: If $f(x)$ represents a rate (like velocity), then f_{avg} over $[a, b]$ is the average rate during that interval.
- Mathematical Insight: The average value can be viewed as the value of a constant function that has the same total integral as $f(x)$ over $[a, b]$.

Application to $F(x) = 3x \sin x$

Since the function $F(x)$ is continuous on $[1, 7]$, the formula applies directly.

$$
F_{avg} = \frac{1}{7 - 1} \int_{1}^{7} 3x \sin x \, dx = \frac{1}{6} \int_{1}^{7} 3x \sin x \, dx
$$

Our task reduces to evaluating the integral:

$$
I = \int_{1}^{7} 3x \sin x \, dx
$$

Computing the Integral $\int 3x \sin x \, dx$

Methodology

This integral involves the product of a polynomial x and a transcendental function $\sin x$. The most appropriate technique here is integration by parts, which states:

$$
\int u \, dv = uv - \int v \, du
$$

Given the structure of the integrand $3x \sin x$, we can factor out the constant 3:

$$
I = 3 \int x \sin x \, dx
$$

Our focus is on evaluating:

$$
J = \int x \sin x \, dx
$$

Step-by-Step Integration by Parts

Step 1: Choose u and dv:

- $u = x$ (since differentiation simplifies it)
- $dv = \sin x \, dx$

Step 2: Compute \(du \) and \(v \):

- \(du = dx \)
- \(v = -\cos x \) (since \(\int \sin x \, dx = - \cos x \))

Step 3: Apply integration by parts:

\[
J = uv - \int v \, du = -x \cos x + \int \cos x \, dx
\]

Step 4: Integrate \(\cos x\):

\[
\int \cos x \, dx = \sin x
\]

Step 5: Write the result:

\[
J = -x \cos x + \sin x + C
\]

Step 6: Recall that \(I = 3J\), so

\[
I = 3 \left(-x \cos x + \sin x \right) + C
\]

Since we're performing a definite integral from 1 to 7, the constant \(C \) cancels out, and the definite integral becomes:

\[
\int_{1}^{7} 3x \sin x \, dx = 3 \left[-x \cos x + \sin x \right]_{x=1}^{x=7}
\]

Evaluating the Definite Integral

Now, substitute the bounds:

\[
I = 3 \left([-7 \cos 7 + \sin 7] - [-1 \cos 1 + \sin 1] \right)
\]

Expressed explicitly:

\[

$$
I = 3 \left(-7 \cos 7 + \sin 7 + \cos 1 - \sin 1 \right)
$$

Calculating Numerical Values

To proceed, approximate the trigonometric functions using a calculator or computational tool:

- $\cos 7 \approx 0.7539$
- $\sin 7 \approx 0.6569$
- $\cos 1 \approx 0.5403$
- $\sin 1 \approx 0.8415$

Substitute these into the expression:

$$
I = 3 \left(-7 \times 0.7539 + 0.6569 + 0.5403 - 0.8415 \right)
$$

Compute step by step:

1. $-7 \times 0.7539 \approx -5.2773$
2. Sum inside the parentheses:

$$
-5.2773 + 0.6569 + 0.5403 - 0.8415 = (-5.2773 + 0.6569) + (0.5403 - 0.8415)
$$
$$
= -4.6204 - 0.3012 = -4.9216
$$

Finally, multiply by 3:

$$
I \approx 3 \times -4.9216 = -14.7648
$$

Calculating the Average Value

Recall:

\[

$$
F_{avg} = \frac{1}{6} \times I \approx \frac{1}{6} \times -14.7648 \approx -2.4608
$$

Therefore, the average value of $F(x) = 3x \sin x$ on the interval $[1, 7]$ is approximately -2.4608.

Interpretation and Insights

Graphical Perspective

- The function $F(x) = 3x \sin x$ oscillates due to the sine component, but the linear term x amplifies the amplitude as x increases.
- Over the interval $[1, 7]$, the function exhibits multiple oscillations, with increasing amplitude.
- The negative average value indicates that, over the interval, the function's negative contributions outweigh the positive ones, pulling the mean below zero.

Mathematical Significance

- The average value being approximately -2.4608 suggests that if you were to "flatten" the function into a constant across $[1,7]$, that constant would be roughly -2.46.
- This has implications in applications like physics, where such an average could represent net displacement or average force when oscillations are involved.

Behavioral Observations

- The rapid oscillations of $\sin x$ combined with the linear growth trend of x create a complex pattern.
- The integral's negative value indicates the dominant contribution of the negative lobes of the sine wave when scaled by $3x$.

Extensions and Related Concepts

Average Value of Other Functions

- The process demonstrated here can be extended to various functions, including polynomials, exponentials, trigonometric functions, or combinations thereof.
- For functions with more complex behavior, numerical methods or software tools may be necessary to approximate integrals.

Alternative Techniques

- When integrals are difficult, techniques such as substitution, partial fractions, or numerical integration (Simpson's rule, trapezoidal rule) can be employed.
- For functions involving special functions or non-elementary integrals, approximation methods or computer algebra systems become invaluable.

Physical and Practical Applications

- Calculating average values is crucial in fields like physics (average velocity, average acceleration), engineering (average stress or strain), and economics (average rate of return).
- Understanding the integral's behavior over an interval helps in designing systems, predicting outcomes, or analyzing data.

Conclusion

The detailed exploration of the average value of $F(x) = 3x \sin x$ over $[1, 7]$ reveals the elegance and utility of calculus techniques

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