Let F(x) = 3x^2 - 8x + 4 And G(x) = X - 2. Perform The Function Operation And Then Find The Domain Of

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Introduction to Function Operations and Domain Analysis

Understanding how to perform operations on functions and determine their domains is fundamental in algebra and calculus. Functions serve as mathematical tools for modeling real-world phenomena, and knowing how to manipulate them is essential for solving complex problems. Given two functions, F(x) and G(x), we often need to combine these functions through various operations such as addition, subtraction, multiplication, or division. After performing these operations, it becomes crucial to analyze the domain of the resulting function because the domain defines the set of all possible input values (x-values) for which the function is defined.

This article explores the specific functions:

- $F(x) = 3x^2 - 8x + 4$

- $G(x) = x - 2$

We will perform different function operations involving these functions and meticulously analyze the domains of the resulting functions. This comprehensive guide aims to provide clarity on the process, mathematical reasoning, and practical applications.

Understanding the Given Functions

Description of F(x)

The function $F(x) = 3x^2 - 8x + 4$ is a quadratic function. Quadratic functions are polynomial functions of degree 2, characterized by their parabolic graphs. The general form is $ax^2 + bx + c$, where:

- $a = 3$

- $b = -8$

- $c = 4$

Quadratic functions are defined for all real numbers unless restricted by domain considerations, which typically do not apply unless specified.

Description of G(x)

The function $G(x) = x - 2$ is a linear function. It has a slope of 1 and a y-intercept of -2, making it a straightforward, continuous function defined for all real numbers.

Performing Function Operations

1. Addition of F(x) and G(x): $(F + G)(x)$

Definition:

$(F + G)(x) = F(x) + G(x)$

Calculation:

$$
(F + G)(x) = (3x^2 - 8x + 4) + (x - 2)
$$

Combine like terms:

$$
= 3x^2 - 8x + 4 + x - 2
$$

$$
= 3x^2 - 7x + 2
$$

Result:

$$
\boxed{(F + G)(x) = 3x^2 - 7x + 2}
$$

2. Subtraction of G(x) from F(x): $(F - G)(x)$

Definition:

$(F - G)(x) = F(x) - G(x)$

Calculation:

$$
(3x^2 - 8x + 4) - (x - 2) = 3x^2 - 8x + 4 - x + 2
$$

Combine like terms:

$$
= 3x^2 - 9x + 6
$$

Result:

$$
\boxed{(F - G)(x) = 3x^2 - 9x + 6}
$$

3. Multiplication of F(x) and G(x): $(F \times G)(x)$

Definition:

$$
(F \times G)(x) = F(x) \times G(x)
$$

Calculation:

$$
(3x^2 - 8x + 4) \times (x - 2)
$$

Apply distributive property (FOIL method):

$$
= (3x^2)(x - 2) + (-8x)(x - 2) + 4(x - 2)
$$

Calculations:

- $3x^2 \times x = 3x^3$
- $3x^2 \times -2 = -6x^2$
- $-8x \times x = -8x^2$
- $-8x \times -2 = +16x$
- $4 \times x = 4x$
- $4 \times -2 = -8$

Combine all:

$$
3x^3 - 6x^2 - 8x^2 + 16x + 4x - 8
$$

Simplify:

$$
3x^3 - (6x^2 + 8x^2) + (16x + 4x) - 8
$$

$$
= 3x^3 - 14x^2 + 20x - 8
$$

Result:

$$
\boxed{(F \times G)(x) = 3x^3 - 14x^2 + 20x - 8}
$$

4. Division of F(x) by G(x): $(F / G)(x)$

Definition:

$$
(F / G)(x) = \frac{F(x)}{G(x)} = \frac{3x^2 - 8x + 4}{x - 2}
$$

Note: Division is only defined where $G(x) \neq 0$.

Domain Analysis of the Resulting Functions

The domain of a function is the set of all real numbers x for which the function is defined.

Domain of Basic Functions $F(x)$ and $G(x)$

- F(x): Since $F(x)$ is a quadratic polynomial, it is defined for all real numbers.

- G(x): As a linear polynomial, also defined for all real numbers.

Domain of Combined Functions

1. Domain of $(F + G)(x)$

Since both $F(x)$ and $G(x)$ are defined for all real numbers, their sum $(F + G)(x)$ is also defined for all real numbers.

Domain:

$$
\boxed{\mathbb{R}}
$$

2. Domain of $(F - G)(x)$

Similarly, the difference of two polynomials is defined everywhere.

Domain:

$$
\boxed{\mathbb{R}}
$$

3. Domain of $(F \times G)(x)$

The product of two polynomials is defined for all real numbers.

Domain:

$$
\boxed{\mathbb{R}}
$$

4. Domain of $(F / G)(x)$

Division introduces restrictions. Specifically, the denominator cannot be zero:

$$
G(x) = x - 2 \neq 0 \Rightarrow x \neq 2
$$

Domain:

$$
\boxed{\mathbb{R} \setminus \{2\}}
$$

Visualizing the Domains and Function Behavior

Understanding the domains visually can be very helpful, especially for students and practitioners dealing with more complex functions.

Graphical Interpretation

- The quadratic functions $F(x)$ and their combinations are continuous and smooth for all real numbers.

- The linear function $G(x) = x - 2$ is a straight line, continuous everywhere.

- The division $(F / G)(x)$ exhibits a vertical asymptote at $x = 2$, where the denominator zeroes out, and the function is undefined.

Practical Applications of Function Operations and Domain Analysis

Function operations are not purely theoretical; they have practical implications in various fields such as physics, engineering, economics, and computer science.

In Physics

- Combining functions models systems where multiple effects occur simultaneously, such as combining velocity and acceleration functions.

In Economics

- Summing cost and revenue functions to derive profit functions.

In Engineering

- Multiplying transfer functions in control systems to analyze overall system behavior.

In Computer Science

- Composing functions for data transformations and ensuring the input constraints are respected.

Summary of Key Points

- Function Addition and Subtraction: The resulting function is defined wherever both original functions are defined. Since $F(x)$ and $G(x)$ are polynomials, their sums and differences are defined over all real numbers.

- Function Multiplication: Similar to addition and subtraction, polynomials are defined everywhere; thus, their product is also defined over the entire real line.

- Function Division: The domain excludes points where the denominator equals zero. For $(F / G)(x)$, the only restriction is $x \neq 2$.

- Domain Notation: The domain of combined functions is expressed either as $\mathbb{R}$ or $\mathbb{R} \setminus \{x_0\}$ where x_0 is a point of discontinuity.

Final Remarks

Mastering function operations and domain analysis is an essential skill in mathematics that lays the groundwork for more advanced topics, including calculus, differential equations, and mathematical modeling. Always pay close attention to restrictions introduced by division or other operations that could lead to undefined points. Visualizing functions and understanding their behavior at domain boundaries can greatly enhance comprehension and problem-solving efficiency.

By practicing these

Frequently Asked Questions

What is the sum of the functions F(x) and G(x) for the given functions?

The sum, $(F + G)(x)$, is $(3x^2 - 8x + 4) + (x - 2) = 3x^2 - 7x + 2$.

How do you perform the difference operation between F(x) and G(x)?

The difference, $(F - G)(x)$, is $(3x^2 - 8x + 4) - (x - 2) = 3x^2 - 9x + 6$.

What is the product of F(x) and G(x)?

The product, $(F G)(x)$, is $(3x^2 - 8x + 4) (x - 2)$. Expanding, it becomes $3x^3 - 6x^2 - 8x^2 + 16x + 4x - 8$, which simplifies to $3x^3 - 14x^2 + 20x - 8$.

How do you perform the division of F(x) by G(x)?

The division, $(F / G)(x)$, is $(3x^2 - 8x + 4)$ divided by $(x - 2)$, with the domain restrictions where $G(x) \neq 0$, i.e., $x \neq 2$.

What is the domain of the combined functions after performing the division operation?

Since $G(x) = x - 2$ in the division operation, the domain excludes $x = 2$. Therefore, the domain is all real numbers except $x \neq 2$.

If you perform the composition F(G(x)), what is the resulting function?

$F(G(x)) = F(x - 2) = 3(x - 2)^2 - 8(x - 2) + 4$, which simplifies to $3(x^2 - 4x + 4) - 8x + 16 + 4 = 3x^2 - 12x + 12 - 8x + 16 + 4 = 3x^2 - 20x + 32$.

What is the domain of the composite function F(G(x))?

Since $G(x) = x - 2$ is defined for all real numbers, and $F(x)$ is a polynomial, the domain of $F(G(x))$ is all real numbers.

Can the functions F(x) and G(x) be combined through any other operations, and what would their domains be?

Yes, they can be added, subtracted, multiplied, divided (excluding $x = 2$ for division), or composed. The domains depend on the operation: for division, $x \neq 2$; for composition, the domain is all real numbers, since $G(x)$ and F are polynomials.

What is the overall domain of the functions after performing all operations, considering restrictions?

The domain excludes $x = 2$ due to the division operation. For all other operations, the domain remains all real numbers except $x = 2$, ensuring the functions are well-defined.

Additional Resources

Let $F(x) = 3x^2 - 8x + 4$ And $G(x) = x - 2$. Perform The Function Operation And Then Find The Domain Of

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Introduction

In the realm of mathematical analysis, functions serve as fundamental tools for modeling relationships between variables. Understanding the composition and combination of functions is crucial, especially when considering their domains, which define the set of permissible inputs for the functions to produce valid outputs. This article embarks on an in-depth exploration of two quadratic and linear functions: $F(x) = 3x^2 - 8x + 4$ and $G(x) = x - 2$. Our primary objectives are to perform various function operations—such as addition, subtraction, multiplication, division, and composition—and to rigorously analyze the resulting domains. Such an investigation not only enhances conceptual clarity but also provides insights into the behavior of combined functions, which is essential for applications across mathematics, physics, engineering, and data science.

Defining the Functions

Before delving into operations, it is essential to clearly understand the given functions:

- F(x): A quadratic function represented as $F(x) = 3x^2 - 8x + 4$. Its quadratic nature suggests a parabola opening upwards (since the coefficient of x^2 is positive).

- G(x): A linear function given by $G(x) = x - 2$. Straightforward in behavior, with a slope of 1 and a y-intercept at -2.

Fundamental Function Operations

The core of our analysis involves performing the following operations on $F(x)$ and $G(x)$:

1. Addition: $(F + G)(x) = F(x) + G(x)$
2. Subtraction: $(F - G)(x) = F(x) - G(x)$
3. Multiplication: $(F \times G)(x) = F(x) \times G(x)$
4. Division: $\left(\frac{F}{G}\right)(x) = \frac{F(x)}{G(x)}$, where $G(x) \neq 0$
5. Composition: $(F \circ G)(x) = F(G(x))$ and $(G \circ F)(x) = G(F(x))$

Each operation has unique considerations regarding the domain, especially division and composition, which can impose restrictions based on the functions' outputs or inputs.

Performing the Operations

1. Addition: $(F + G)(x)$

Calculating directly:

$$
(F + G)(x) = (3x^2 - 8x + 4) + (x - 2) = 3x^2 - 8x + 4 + x - 2
$$

Simplify:

$$
(F + G)(x) = 3x^2 - 7x + 2
$$

Domain: Since the sum of two polynomials is always a polynomial, the domain of $(F + G)(x)$ is all real numbers, i.e., $\boxed{\mathbb{R}}$.

---

2. Subtraction: $(F - G)(x)$

Calculating:

$$
(F - G)(x) = (3x^2 - 8x + 4) - (x - 2) = 3x^2 - 8x + 4 - x + 2
$$

Simplify:

$$

(F - G)(x) = 3x^2 - 9x + 6
\]

Domain: Again, a polynomial difference yields a polynomial, so the domain is all real numbers.

---

3. Multiplication: \((F \times G)(x)\)

Calculating:

\[
(F \times G)(x) = (3x^2 - 8x + 4)(x - 2)
\]

Expanding:

\[
= 3x^2(x - 2) - 8x(x - 2) + 4(x - 2)
\]

\[
= 3x^3 - 6x^2 - 8x^2 + 16x + 4x - 8
\]

Combine like terms:

\[
= 3x^3 - 14x^2 + 20x - 8
\]

Domain: The product of polynomials is a polynomial, hence all real numbers.

---

4. Division: \(\left(\frac{F}{G}\right)(x)\)

Calculating:

\[
\left(\frac{F}{G}\right)(x) = \frac{3x^2 - 8x + 4}{x - 2}
\]

Domain considerations: The denominator cannot be zero:

$$
x - 2 \neq 0 \implies x \neq 2
$$

Thus, the domain is:

$$
\boxed{\mathbb{R} \setminus \{2\}}
$$

---

5. Composition: $F(G(x))$ and $G(F(x))$

a) $F(G(x))$

Substitute $G(x) = x - 2$ into F:

$$
F(G(x)) = 3(x - 2)^2 - 8(x - 2) + 4
$$

Expand:

$$
= 3(x^2 - 4x + 4) - 8x + 16 + 4
$$

Distribute:

$$
= 3x^2 - 12x + 12 - 8x + 16 + 4
$$

Combine like terms:

$$
= 3x^2 - 20x + 32
$$

Domain: Since $G(x)$ is linear with domain $\mathbb{R}$, and F is polynomial, the composition

$F(G(x))$ is defined for all real numbers:

$$
\boxed{\mathbb{R}}
$$

b) $G(F(x))$

Substitute $F(x) = 3x^2 - 8x + 4$ into G:

$$
G(F(x)) = (3x^2 - 8x + 4) - 2 = 3x^2 - 8x + 2
$$

Domain: Since $F(x)$ is polynomial and G is linear, the composition is defined for all real numbers.

Domain Analysis and Key Observations

The domains of the combined functions hinge on the initial functions and the operations performed:

- Addition, Subtraction, Multiplication, and Composition: These operations typically have the domain as all real numbers because polynomials and linear functions are defined everywhere.

- Division: The only restriction arises from the denominator; here, $G(x) = x - 2$ must not be zero, leading to a domain of $\mathbb{R} \setminus \{2\}$.

Implications: The restriction at $x=2$ impacts the division operation but not the others. Recognizing and explicitly stating these domain restrictions is essential for accurate function analysis.

Broader Implications and Applications

Understanding the behavior of combined functions and their domains is vital in various fields:

- Mathematical Modeling: Ensuring input restrictions are accounted for prevents incorrect assumptions in models involving multiple functions.

- Calculus and Analysis: Domain considerations are critical when differentiating and integrating composite functions.

- Engineering: System stability and response often depend on the domains of transfer functions or signal transformations.

- Data Science: Function compositions appear in feature engineering and transformations; domain restrictions can affect data preprocessing pipelines.

Conclusion

This comprehensive analysis confirms that, for the given functions $F(x) = 3x^2 - 8x + 4$ and $G(x) = x - 2$, most operations yield functions with domains spanning all real numbers, owing to their polynomial and linear nature. The division operation, however, introduces a domain restriction at $x=2$, excluding this point from the domain of $\frac{F}{G}$. Such investigations exemplify the importance of meticulous domain analysis in function operations—a cornerstone in advanced mathematical problem-solving, modeling, and theoretical research.

Understanding these nuances not only enhances mathematical literacy but also ensures accurate application across scientific disciplines. As functions become more complex, this foundational knowledge facilitates the development of robust analytical frameworks and prevents potential pitfalls arising from overlooked domain restrictions.

In summary:

Operation	Resulting Function	Domain
Addition	$3x^2 - 7x + 2$	$\mathbb{R}$
Subtraction	$3x^2 - 9x + 6$	$\mathbb{R}$
Multiplication	$3x^3 - 14x^2 + 20x - 8$	$\mathbb{R}$
Division	$\frac{3x^2 - 8x + 4}{x - 2}$	$\mathbb{R} \setminus \{2\}$
Composition $F(G(x))$	\(3	

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