# Let R Be The Region Bounded By The Following Curves. Find The Volume Of The Solid Generated When R Is

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Understanding how to determine the volume of a solid generated by a region bounded by curves is a fundamental concept in calculus. This topic is essential for students and professionals working in fields such as engineering, physics, and mathematics. In this comprehensive guide, we will explore the methods of finding the volume of solids generated when a region $R$ is bounded by various curves, with step-by-step explanations, examples, and tips to master this important calculus problem.

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## Understanding the Concept of Bounded Regions and Solid of Revolution

Before diving into the methods, it is important to understand what is meant by a region $R$ being bounded by curves and how this relates to generating a solid.

### What Is a Bounded Region?

A bounded region in the xy-plane is an area enclosed by specific curves. For example, consider the region enclosed between the curves $y = f(x)$ and $y = g(x)$, over an interval $[a, b]$. This region is finite in extent and forms the basis for constructing a three-dimensional solid when revolved about an axis.

### What Is a Solid of Revolution?

A solid of revolution is a three-dimensional object created by rotating a planar region $R$ around a fixed axis (usually the x-axis or y-axis). The volume of such a solid can be computed using methods like the disk method, washer method, or shell method, depending on the shape of the region and the axis of rotation.

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# Methods for Calculating the Volume of a Solid of Revolution

There are primarily three methods to find the volume of the solid generated by revolving a region $R$. The choice of method depends on the shape of the bounded region and the axis of rotation.

## 1. Disk Method

The disk method is suitable when the region is revolved around an axis that is perpendicular to the axis of the slices, and the cross-sections are disks or solid cylinders.

Steps to apply the disk method:

- Identify the axis of revolution.
- Express the radius of the disk as a function of $x$ or $y$.
- Set up the integral as the sum of the cross-sectional areas along the interval.

Formula:

- When revolving around the x-axis:

$$
V = \pi \int_{a}^{b} [f(x)]^2 \, dx
$$

- When revolving around the y-axis:

$$
V = \pi \int_{c}^{d} [g(y)]^2 \, dy
$$

Example:

Suppose $R$ is bounded by $y = x^2$ and $y = 4$, between $x = 0$ and $x = 2$, revolved around the x-axis.

The volume:

$$
V = \pi \int_{0}^{2} [\text{radius}]^2 dx = \pi \int_{0}^{2} (4 - x^2)^2 dx
$$

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## 2. Washer Method

The washer method extends the disk method for regions where the area is hollow (an inner and outer boundary), resulting in washers (disks with holes).

Steps:

- Identify the inner and outer radii as functions of $x$ or $y$.
- Set up an integral for the difference of the areas of the outer and inner disks.

Formula:

- When revolving around the x-axis:

$$
V = \pi \int_{a}^{b} \left( [R_{\text{outer}}(x)]^2 - [R_{\text{inner}}(x)]^2 \right) dx
$$

- When revolving around the y-axis:

$$
V = \pi \int_{c}^{d} \left( [R_{\text{outer}}(y)]^2 - [R_{\text{inner}}(y)]^2 \right) dy
$$

Example:

Find the volume when the region bounded by $y = \sqrt{x}$ and $y = x$ (from $x=0$ to $x=1$) is revolved around the x-axis.

The outer radius is $R_{\text{outer}} = \sqrt{x}$, and the inner radius is $R_{\text{inner}} = x$. Then,

$$
V = \pi \int_{0}^{1} \left( (\sqrt{x})^2 - x^2 \right) dx
$$

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## 3. Shell Method

The shell method is especially useful when the region is revolved around the y-axis or when slices are parallel to the axis of revolution.

Steps:

- Consider cylindrical shells formed by vertical or horizontal slices.
- Express the shell's radius and height.
- Integrate the lateral surface area to find the volume.

Formula:

- When revolving around the y-axis:

$$
V = 2\pi \int_{a}^{b} x \cdot f(x) \, dx
$$

- When revolving around the x-axis:

$$
V = 2\pi \int_{c}^{d} y \cdot g(y) \, dy
$$

Example:

Revolve the region between $y = x^3$ and $y=0$ from $x=0$ to $x=1$ around the y-axis:

$$
V = 2\pi \int_{0}^{1} x \cdot x^3 dx = 2\pi \int_{0}^{1} x^4 dx
$$

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# Step-by-Step Guide to Solving Volume Problems

To effectively solve problems involving the volume of solids generated by bounded regions, follow this structured approach:

## Step 1: Understand the Region $R$

- Sketch the curves involved.
- Find the points of intersection.
- Determine the bounds of the region.

## Step 2: Decide the Axis of Revolution

- Is the region revolved around the x-axis, y-axis, or another line?
- The axis influences the choice of method.

## Step 3: Choose the Appropriate Method

- Use the disk method if the slices are perpendicular to the axis and no holes are present.
- Use the washer method if the region has holes or is hollow.
- Use the shell method if slices are parallel to the axis or the region is more easily described in terms of $x$ or $y$.

## Step 4: Set Up the Integral

- Express the radii or shell dimensions as functions of the variable of integration.
- Determine the limits of integration based on the intersection points.

## Step 5: Compute the Integral

- Carefully evaluate the integral.
- Simplify and check for errors.

## Step 6: Interpret the Result

- Ensure the volume makes sense physically and mathematically.
- Consider units if applicable.

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# Practical Examples and Applications

Let's explore some practical problems to illustrate the concepts.

## Example 1: Volume of a Solid Revolved Around the x-Axis

Problem:

Find the volume of the solid generated when the region bounded by $y = x^2$ and $y = 4$ between $x=0$ and $x=2$ is revolved around the x-axis.

Solution:

- Since the region is between $y = x^2$ and $y=4$, and the axis is the x-axis, the outer radius is constant at $R=4$, and the inner radius varies as $R = x^2$.

- Using the washer method:

$$
V = \pi \int_{0}^{2} \left( R_{\text{outer}}^2 - R_{\text{inner}}^2 \right) dx = \pi \int_{0}^{2} (4)^2 - (x^2)^2 dx = \pi \int_{0}^{2} 16 - x^4 dx
$$

- Evaluate:

$$
V = \pi \left[ 16x - \frac{x^5}{5} \right]_0^2 = \pi \left( 16 \times 2 - \frac{32}{5} \right) = \pi \left( 32 - \frac{32}{5} \right) = \pi \left( \frac{160}{5} - \frac{32}{5} \right) = \pi \frac{128}{5} = \frac{128\pi}{5}
$$

Result:

The volume is $\frac{128\pi}{5}$ cubic units.

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## Example 2: Volume Using the Shell Method

Problem:

Find the volume generated when the region between $y = x^2$ and $y=0$ for $x \in [0,1]$ is revolved around the y-axis.

Solution:

- Since the axis of revolution is the y-axis, and the region is bounded by $y = x^2$, the shell method

# Frequently Asked Questions

## Let R be the region bounded by the curves $y = x^2$ and $y = 4$. Find the volume of the solid generated when R is revolved about the x-axis.

First, find the points of intersection: $x^2 = 4 \Rightarrow x = \pm 2$. Using the disk method, the volume $V = \pi \int_{-2}^{2} (4 - x^2)^2 dx$. Evaluating this integral gives $V = (16\pi/3)$ units$^3$.

## Let R be the region bounded by $y = \sqrt{x}$ and $y = x/4$. Find the volume of the solid generated when R is revolved

## about the y-axis.

Find the intersection points by solving $\sqrt{x} = x/4$, which yields $x = 0$ and $x = 16$. Using the shell method, the volume $V = 2\pi \int_{0}^{16} x (\sqrt{x} - x/4)\, dx$. Computing this integral results in $V = (256\pi/15)$ units³.

## Let R be the region bounded by y = x^3 and y = x. Find the volume of the solid generated when R is revolved about the y-axis.

Find intersection points: $x^3 = x \Rightarrow x = 0, 1$. Using the shell method, volume $V = 2\pi \int_{0}^{1} x (x - x^3)\, dx = 2\pi \int_{0}^{1} (x^2 - x^4)\, dx = 2\pi [x^3/3 - x^5/5]_0^1 = 2\pi (1/3 - 1/5) = 2\pi (2/15) = (4\pi/15)$ units³.

## Let R be the region bounded by y = sin x and y = 0, between x = 0 and x = π. Find the volume of the solid generated when R is revolved about the x-axis.

Using the disk method, $V = \pi \int_{0}^{\pi} (\sin x)^2\, dx$. Since $\int_{0}^{\pi} \sin^2 x\, dx = \pi/2$, the volume $V = \pi (\pi/2) = (\pi^2)/2$ units³.

## Let R be the region bounded by the curves y = x + 2 and y = 3x, between x = 0 and x = 1. Find the volume when R is revolved about the y-axis.

Express x in terms of y: from $y = 3x$, $x = y/3$; from $y = x + 2$, $x = y - 2$. The bounds in y are $y = 2$ (at $x=0$) and $y = 3$ (at $x=1$). Using the shell method, $V = 2\pi \int_{2}^{3} y (y/3 - (y - 2))\, dy$. Simplify and evaluate the integral to find the volume.

# Additional Resources

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Introduction

In the realm of calculus and three-dimensional geometry, the process of determining the volume of a solid generated by a region bounded by curves is both fundamental and fascinating. This problem not only tests our understanding of integration techniques but also showcases the elegance of mathematical methods to solve real-world shape problems. When given a region \( R \) bounded by specific curves, the key challenge lies in translating the geometric constraints into suitable mathematical expressions and then applying integration methods—such as the disk, washer, or shell methods—to find the volume of the resulting solid.

This article aims to provide a comprehensive, step-by-step guide to solving such problems. We will explore the concepts behind the methods, walk through typical examples, and clarify common pitfalls. Whether you're a student seeking clarity or a curious enthusiast eager to deepen your understanding, this discussion will serve as a valuable resource for approaching volume calculations involving bounded regions.

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## Understanding the Region $R$

Before diving into the calculation of volume, it is crucial to understand the nature of the region $R$. Usually, in problems like this, $R$ is specified by certain curves—be they lines, circles, parabolas, or other functions.

Key steps:

- Identify the curves: Determine the equations defining the boundary of $R$. For example, $y = f_1(x)$, $y = f_2(x)$, or $x = g_1(y)$, $x = g_2(y)$.
- Determine the intersection points: Find where the curves intersect to establish the limits of integration.
- Visualize the region: Sketching the region helps in selecting the appropriate method and understanding the shape of the solid.

Example: Suppose the region $R$ is bounded by the curves $y = x^2$ and $y = 4$, between $x = 0$ and $x = 2$. The region is a parabola segment capped at $y=4$, which is straightforward to analyze.

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## Methods for Finding the Volume

Once the region $R$ is understood, the next step is choosing an appropriate method to generate the volume of the solid when the region is revolved around a specified axis.

### 1. Disk Method

Applicability: When the solid is generated by revolving $R$ around an axis (commonly the x-axis or y-axis), and the cross-sections perpendicular to the axis are disks.

Concept: The volume is obtained by summing up the disk-shaped slices' areas along the axis of revolution.

Formula:

- When revolving around the x-axis:

$$
V = \pi \int_{a}^{b} [f(x)]^2\, dx
$$

- When revolving around the y-axis:

$$
V = \pi \int_{c}^{d} [g(y)]^2\, dy
$$

Example: If $R$ is bounded between $x=a$ and $x=b$, and the curve is $y = f(x)$, then revolving around the x-axis creates disks with radius $y = f(x)$.

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2. Washer Method

Applicability: When the region is rotated about an axis, but the cross-section is a ring or washer with a hole, rather than a simple disk.

Concept: The volume is obtained by subtracting the volume of the inner radius from the outer radius at each slice.

Formula:

- For revolution around the x-axis:

$$
V = \pi \int_{a}^{b} \left( [R_{\text{outer}}(x)]^2 - [R_{\text{inner}}(x)]^2 \right) dx
$$

- Similarly for the y-axis.

Example: If the region is between two functions $y = f_1(x)$ and $y = f_2(x)$, revolving around the x-axis, each cross-section is a washer with outer radius $f_2(x)$ and inner radius $f_1(x)$.

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3. Shell Method

Applicability: When revolving a region around an axis parallel to the coordinate axis, especially advantageous when the region is easier to describe in terms of one variable.

Concept: Instead of slicing perpendicular to the axis, the shell method involves slicing parallel to the axis of revolution, forming cylindrical shells.

Formula:

- For revolution around the y-axis:

$$
V = 2\pi \int_{a}^{b} x \cdot f(x) \, dx
$$

- For revolution around the x-axis:

$$
V = 2\pi \int_{c}^{d} y \cdot g(y) \, dy
$$

Advantages: Often simplifies calculations when the region is bounded by functions of $x$ and $y$, and the axis of rotation is parallel to one of the axes.

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Step-by-Step Example: Calculating Volume for a Specific Region

Let's consider a concrete example to illustrate these concepts:

Problem: Find the volume of the solid generated when the region $R$, bounded by $y = x^2$ and $y = 4$, from $x=0$ to $x=2$, is revolved around the x-axis.

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Step 1: Visualize the Region

- The curves $y = x^2$ and $y=4$ intersect at $x=2$, since $2^2=4$.
- The region $R$ is between $x=0$ and $x=2$, bounded below by $y=x^2$, above by $y=4$.

Sketch: A parabola opening upward from the origin, capped at $y=4$, between $x=0$ and $x=2$.

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Step 2: Choose the Method

Since the region will be revolved around the x-axis, and the slices perpendicular to the x-axis are disks with varying radii, the disk method is appropriate.

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Step 3: Determine the Radii

- The outer radius of each disk is the distance from the x-axis to the top of the region, which is $y=4$.
- The inner radius is the distance from the x-axis to the lower curve $y = x^2$.

Note: Because the region is bounded between $y=x^2$ and $y=4$, revolving around the x-axis generates a solid with a hollow core where the inner radius corresponds to $y=x^2$.

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Step 4: Set Up the Integral

The volume of the solid of revolution (a washer at each $x$) is:

$$
V = \pi \int_{0}^{2} \left[ (\text{outer radius})^2 - (\text{inner radius})^2 \right] dx
$$

- Outer radius: $R_{\text{outer}} = 4$
- Inner radius: $R_{\text{inner}} = x^2$

Therefore:

$$
V = \pi \int_{0}^{2} \left( 4^2 - (x^2)^2 \right) dx = \pi \int_{0}^{2} (16 - x^4) dx
$$

---

Step 5: Compute the Integral

Calculate:

$$
V = \pi \left[ 16x - \frac{x^5}{5} \right]_{0}^{2}
$$

Plugging in the limits:

$$
V = \pi \left( 16 \times 2 - \frac{(2)^5}{5} \right) - \pi (0 - 0) = \pi \left( 32 - \frac{32}{5} \right)
$$

Simplify:

$$
V = \pi \left( \frac{160}{5} - \frac{32}{5} \right) = \pi \times \frac{128}{5} = \frac{128\pi}{5}
$$

Final Answer:

$$
\boxed{
V = \frac{128\pi}{5}
}
$$

This result quantifies the volume of the solid generated by revolving the specified region around the x-axis.

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Variations and Additional Considerations

While the above example uses the disk method for a straightforward region, real-world problems often involve more complex regions or axes of rotation. Here are some important considerations:

- Revolving around different axes: When rotating around the y-axis or an arbitrary line, the shell method may be more efficient.
- Multiple regions: For regions bounded by multiple curves or disjoint areas, split the integral accordingly.
- Symmetry: Use symmetry to simplify calculations when applicable.
- Changing the order of integration: Sometimes, integrating with respect to \( y \) first (or \( x \) first) simplifies the process.

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Common Pitfalls and Tips

- Incorrect limits: Always find the intersection points carefully to set the correct bounds.
- Choosing the wrong method: Visualize the problem to decide whether disks, washers, or shells are most suitable.
- Misidentifying radii: Ensure radii are measured from the axis of rotation and correspond to the correct curves.
- Ignoring

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