Let R Be The Region In The First Quadrant That Is Bounded By The Curves Y= =x X=0 And Y=2-x- Find The

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Introduction to the Region R and Its Significance in Calculus

Understanding the geometric region R bounded by specific curves and lines is fundamental in the study of multivariable calculus. This region often serves as the domain for double integrals, enabling the calculation of areas, volumes, and other physical quantities. When analyzing such a region, it is crucial to comprehend its boundaries, limits, and how these influence the integration process.

In this guide, we focus on the region R in the first quadrant bounded by the curves Y = x, the y-axis (X=0), and the line Y = 2 - x. Our goal is to thoroughly explore the characteristics of this region, interpret its geometric properties, and understand how to compute relevant integrals over R.

Defining the Region R: Boundaries and Description

Given Curves and Lines

The region R is bounded by the following:

- **Y = x**: a straight line passing through the origin with a slope of 1.
- **X = 0**: the y-axis, serving as the left boundary.
- **Y = 2 - x**: a straight line with a slope of -1, intercepting the y-axis at 2.

First Quadrant Constraints

Since R is in the first quadrant:

- $X \geq 0$
- $Y \geq 0$

This ensures that the region is confined to the positive x and y axes.

Visualizing the Curves

- The line $Y = x$ is a diagonal passing through the origin, rising equally along the x and y axes.
- The line $Y = 2 - x$ intercepts the y-axis at (0, 2) and the x-axis at (2, 0).
- The line $X=0$ is the y-axis itself.

Determining the Intersection Points

To accurately define R, find where the boundary lines intersect:

1. Set $Y = x$ and $Y = 2 - x$ to find their intersection:
    - $x = 2 - x \rightarrow 2x = 2 \rightarrow x = 1$
    - $Y = x \rightarrow Y = 1$

    So, the lines intersect at point (1, 1).

2. Check intersections with the axes:
    - $Y = 2 - x$ intersects the y-axis at (0, 2)
    - $Y = x$ intersects the x-axis at (0, 0)

Summary of Region R

Based on the above, R is the triangular area bounded by:

- The y-axis ($X=0$),
- The line $Y=2 - x$ from (0, 2) to (1, 1),
- The line $Y= x$ from (1, 1) to (0, 0).

Mathematical Representation of Region R

Choosing the Limits of Integration

To perform integrations over R, it’s essential to express the region in terms of variable limits.

Option 1: Integrate with respect to y first (dy dx):

- For a fixed x in [0,1]:

- The lower boundary of Y is $Y = x$ (from bottom line).

- The upper boundary of Y is $Y = 2 - x$ (from top line).

- Since the region is between these two lines:

- For x in [0,1], Y varies from $Y = x$ to $Y = 2 - x$.

Option 2: Integrate with respect to x first (dx dy):

- For a fixed y in [0,1]:

- The left boundary is $x=0$.

- The right boundary is $x = 2 - y$ (from rearranged line $Y=2 - x$ to $x=2 - y$).

- Alternatively, for y in [0,1], x varies from $x=0$ to $x= y$ (from $Y= x$).

Choosing the most straightforward approach:

Since the bounds are clearer when integrating with respect to y first, the limits become:

- x varies from 0 to 1.

- For each x, y varies from $y = x$ to $y= 2 - x$.

Setting Up Double Integrals Over Region R

Double Integral with respect to y then x

The integral of a function $f(x, y)$ over R:

$$
\iint_{R} f(x, y) \, dA = \int_{x=0}^{1} \int_{y=x}^{2 - x} f(x, y) \, dy \, dx
$$

This approach is often favored because the limits are straightforward and directly correspond to the geometric boundaries.

Alternative: Double Integral with respect to x then y

Alternatively, expressing the region as:

- y from 0 to 1,

- x from 0 to y (since along the line $Y = x$, $x=y$).

Then,

$$
\iint_{R} f(x, y) \, dA = \int_{y=0}^{1} \int_{x=0}^{y} f(x, y) \, dx \, dy
$$

Choosing the correct order depends on the function $f(x, y)$ being integrated and the simplicity of the limits.

Calculating Area of Region R

The area of R is an important measure, often the first step before calculating volume or center of mass.

Using the Double Integral

The area A is obtained by integrating the constant function 1 over R:

$$
A = \iint_{R} 1 \, dA
$$

Using the limits with respect to x first:

$$
A = \int_{x=0}^{1} \int_{y=x}^{2 - x} 1 \, dy \, dx
$$

Calculating the inner integral:

$$
\int_{y=x}^{2 - x} 1 \, dy = (2 - x) - x = 2 - 2x
$$

Now, integrating with respect to x:

$$
A = \int_{0}^{1} (2 - 2x) \, dx = \left[2x - x^2 \right]_0^1 = (2 \times 1 - 1^2) - 0 = (2 - 1) = 1
$$

Result: The area of R is 1 square unit.

Applications and Further Calculations

Volume Under a Surface

Suppose we want to compute the volume under a surface z = f(x, y) over R. The double integral:

$$
V = \iint_{R} f(x, y) \, dA
$$

can be computed once the function f(x, y) is specified.

Centroid and Center of Mass

The centroid coordinates $(\bar{x}, \bar{y})$ are calculated as:

$$
\bar{x} = \frac{1}{A} \iint_{R} x \, dA, \quad \bar{y} = \frac{1}{A} \iint_{R} y \, dA
$$

which involve similar double integrals with functions x and y respectively.

Mass with Variable Density

If the region has a density function $\rho(x, y)$, then the mass M is:

$$
M = \iint_{R} \rho(x, y) \, dA
$$

and other properties like moments can be derived accordingly.

Visualization and Graphical Interpretation

Creating a visual of R helps in understanding the region:

- Draw the axes: x and y axes.
- Plot the line $Y = x$ (origin to (1, 1)).
- Plot the line $Y = 2 - x$ (intercepts at (0, 2) and (2, 0)).
- Shade the triangular region bounded by these lines and the y-axis.

This visualization clarifies the limits of integration and assists in setting up more complex integrals.

Summary and Key Takeaways

- The region R is a triangle in the first quadrant bounded by $Y = x$, $Y = 2 - x$, and the y-axis.
- Intersection points at (0, 0), (0, 2), and (1, 1) define the region.
- The area of R is 1 square unit.
- Integration limits are most straightforward when integrating with respect to y first: x from 0 to 1, y from x to 2 - x.
- Understanding the boundaries allows for accurate computation of integrals related to R, essential in various applications in physics, engineering, and mathematics

Frequently Asked Questions

What is the shape of the region R bounded by the curves y = x, x = 0, and y = 2 - x?

The region R is a triangle in the first quadrant bounded by the y-axis (x=0), the line y = x, and the line y = 2 - x, forming a right triangle with vertices at (0,0), (1,1), and (0,2).

How can we find the area of the region R bounded by the curves y = x, x = 0, and y = 2 - x?

The area can be found by integrating y = x over the interval from x=0 to x=1, or by integrating with respect to y. Using x as a variable: Area = $\int_0^1 (2 - x)\,dx$.

What is the significance of the line y = 2 - x in defining the region R?

The line y = 2 - x forms the hypotenuse of the triangular region, connecting the points (0,2) and (1,1), and together with the axes and y = x, it bounds the region in the first quadrant.

How do you set up the double integral to find the volume under the surface over the region R?

The double integral can be set up as $\int\int_R f(x,y)\,dA$, where R is bounded by x from 0 to 1 and y from x to 2 - x, or vice versa, depending on the order of integration.

What are the coordinates of the vertices of the region R, and how are they determined?

The vertices are at (0,0), (1,1), and (0,2). They are determined by finding intersections of the curves y = x with y = 2 - x, and the axes x=0 and y=0.

Additional Resources

Region Analysis in the First Quadrant: A Deep Dive into the Bounded Area Defined by \(y = x \), \(x=0 \), and \(y=2 - x \)

Introduction

Mathematics, especially calculus and analytical geometry, often involves the exploration of regions bounded by curves and lines. These regions are foundational for understanding concepts such as area, integration, and the behavior of functions. Today, we take an in-depth look at a specific region in the first quadrant, defined by the inequalities \(y = x \), \(x = 0 \), and \(y = 2 - x \). This analysis resembles a detailed product review or an expert feature article, aiming to elucidate every facet of this geometric construct.

Understanding the Region R

Before diving into calculations, it’s essential to visualize and understand the boundary definitions and their implications:

- First Quadrant Constraint: The region R lies where $x \geq 0$ and $y \geq 0$.
- Boundaries:
- Line 1: $y = x$ (a line passing through the origin with a slope of 1).
- Line 2: $y = 2 - x$ (a line crossing the y-axis at 2 and decreasing with a slope of -1).
- Line 3: $x = 0$ (the y-axis).

This setup results in a polygonal region enclosed by these lines, creating a bounded area in the first quadrant.

Visualizing the Region

To understand the area comprehensively, plotting the lines provides clarity:

- The line $y = x$ intersects the y-axis at $(0, 0)$ and extends diagonally upwards.
- The line $y = 2 - x$ intersects the y-axis at $(0, 2)$ and the x-axis at $(2, 0)$.
- The line $x=0$ is the y-axis itself.

Key intersection points:

1. $y = x$ and $y = 2 - x$:
Set $x = 2 - x$ to find their intersection:

$$
x = 2 - x \quad \Rightarrow \quad 2x = 2 \quad \Rightarrow \quad x=1
$$

Substitute back to find y:

$$
y = x = 1
$$

Intersection point: $(1, 1)$

2. $x=0$ and $y=2 - x$:
When $x=0$:

$$
y=2 - 0=2

\]

Point: $(0, 2)$

3. $x=0$ and $y=x$:
When $x=0$:

$$
y=0
$$

Point: $(0, 0)$

Describing the Region R

The region R is thus a triangle with vertices at:

- $(0, 0)$
- $(0, 2)$
- $(1, 1)$

The boundary lines are:

- Vertical boundary: $x=0$
- Top boundary: $y=2 - x$
- Diagonal boundary: $y=x$

This triangle lies entirely in the first quadrant, bounded by the axes and the two lines.

Calculating the Area of R

To compute the area, the most straightforward approach involves setting up an integral that captures the region's shape. There are two primary methods: integrating with respect to x or y. Given the region's shape, integrating with respect to x is often more intuitive.

Integration Approach: Setting up the Limits

Step 1: Identify the partition of the region along x:

- For $0 \leq x \leq 1$, the top boundary is $y=2 - x$ and the lower boundary is $y=x$.

Step 2: Determine which line forms the upper boundary and which forms the lower boundary in this interval:

- Between $x=0$ and $x=1$, the line $y=2-x$ is above $y=x$:

$$
2 - x \geq x \quad \Rightarrow \quad 2 \geq 2x \quad \Rightarrow \quad x \leq 1
$$

- For x in this interval, the region is bounded from below by $y=x$ and from above by $y=2 - x$.

Step 3: Write the integral for the area:

$$
A = \int_{x=0}^{1} \left[(2 - x) - x \right] dx
$$

which simplifies to:

$$
A = \int_{0}^{1} (2 - 2x) dx
$$

Performing the Integration

Calculating the integral:

$$
A = \int_{0}^{1} 2 dx - \int_{0}^{1} 2x dx
$$

$$
A = \left[2x \right]_0^1 - \left[x^2 \right]_0^1
$$

$$
A= (2 \times 1 - 2 \times 0) - (1^2 - 0) = 2 - 1 = 1
$$

Result: The area of the region R is 1 square unit.

Alternative Approach: Integration with Respect to y

While the previous method is straightforward, for completeness, we demonstrate the integration with respect to y:

- The region is bounded between $y=0$ and $y=2$, with the corresponding x-limits derived from the lines:

- For y between 0 and 1:

$$
y = x \quad \Rightarrow \quad x = y
$$

$$
y = 2 - x \quad \Rightarrow \quad x= 2 - y
$$

Since for $y \in [0,1]$, the left boundary is $x=y$ and the right boundary is $x=2 - y$.

- For y between 1 and 2:

The boundary is only $x=0$ (the y-axis) on the left, and $x=2 - y$ on the right.

However, because the shape is simple enough, the original x-integral approach remains the most efficient.

Significance of the Region and Its Applications

Understanding this bounded region is not merely an academic exercise; it has practical implications:

- Area Calculation: Fundamental in physics and engineering for calculating material properties, cross-sectional areas, or probability regions.
- Integration Practice: Serves as an excellent example for mastering double integrals and setting proper limits.
- Geometric Interpretation: Visualizing the region enhances spatial reasoning, essential in fields like computer graphics and CAD design.

Extending the Analysis: Centroid and Moments

Beyond the area, one might be interested in the centroid or moments of the region:

- Centroid Coordinates ($\bar{x}, \bar{y}$):

$$
\bar{x} = \frac{1}{A} \iint_R x \, dA, \quad \bar{y} = \frac{1}{A} \iint_R y \, dA
$$

- Calculating these involves setting up similar integrals over the region, which, given the simplicity, can be efficiently evaluated.

Conclusion

The region R, bounded in the first quadrant by $y = x$, $y=2 - x$, and $x=0$, forms a well-defined triangle with an area of 1 square unit. This analysis illustrates how geometric intuition, algebraic setup, and calculus techniques coalesce to provide a comprehensive understanding of the shape and size of the region.

Whether used as a stepping stone for more complex integral calculations or as a visualization exercise, this exploration exemplifies the elegance and utility of mathematical analysis in dissecting and understanding the spatial relationships within the first quadrant. As with any well-designed product, clarity of boundaries and precise calculations ensure robust and reliable insights—fundamental principles that underpin much of applied mathematics and engineering.

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