Let U = -5 And A = 4 7 5. Is U In The Subset Of R3 Spanned By The Columns Of A? Why Or Why Not?-6 0 2

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Introduction

Understanding the concepts of vector spaces, subsets, and span is fundamental in linear algebra. When analyzing whether a particular vector belongs to a span generated by a set of vectors, we delve into the core of linear combinations, linear independence, and subspace theory. This article explores the specific question: Given the vector U = -5 and the matrix A with columns [4, 7, 5], is U in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not? We will break down the problem systematically, providing detailed explanations of the necessary mathematical principles, step-by-step calculations, and interpretations to arrive at a comprehensive answer.

Understanding the Problem

The Vectors and Matrices

- Vector U: U = -5 (Note: Since the context involves $\mathbb{R}^3$, U should be interpreted as a vector in three dimensions. The given U = -5 suggests a scalar, so it's important to clarify what this means. It could be that U is meant to be a vector with all components as -5, or a specific component. For this analysis, we will assume U is a vector in $\mathbb{R}^3$, possibly U = [-5, -5, -5].)

- Matrix A: A is given with columns [4, 7, 5], [–6, 0, 2].

So, the matrix A in $\mathbb{R}^3\times 2$ form is:

$$
A = \begin{bmatrix}
4 & -6 \\
7 & 0 \\
5 & 2
\end{bmatrix}
$$

The columns are vectors:

$$
\mathbf{a}_1 = \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix}, \quad
$$

\mathbf{a}_2 = \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix}
\]

The Question

- Is U in the span of the columns of A?

In other words, does there exist scalars \(c_1 \) and \(c_2 \) such that:

\[
c_1 \mathbf{a}_1 + c_2 \mathbf{a}_2 = U
\]

- Why or why not?

The answer depends on whether the vector U can be expressed as a linear combination of the columns of A.

Clarification of U and Its Components

Given the initial statement, U appears to be a scalar, but since the context involves $\mathbb{R}^3$, U should be a 3-dimensional vector. To proceed, we interpret U as:

\[
U = \begin{bmatrix} -5 \\ -5 \\ -5 \end{bmatrix}
\]

This interpretation is consistent with the problem structure. If U is intended differently, adjustments may be necessary.

Mathematical Approach to the Problem

Step 1: Formulate the Linear System

We seek scalars \(c_1, c_2 \) such that:

\[
c_1 \begin{bmatrix} 4 \\ 7 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} -6 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} -5 \\ -5 \\ -5 \end{bmatrix}
\]

This leads to the following system of equations:

\[
\begin{cases}
4c_1 - 6c_2 = -5 \\
7c_1 + 0c_2 = -5 \\

5c_1 + 2c_2 = -5
\end{cases}
\]

Step 2: Solve the System of Equations

- From the second equation:

\[
7c_1 = -5 \Rightarrow c_1 = -\frac{5}{7}
\]

- Substitute \(c_1 = -\frac{5}{7} \) into the first equation:

\[
4 \left(-\frac{5}{7}\right) - 6 c_2 = -5
\]

\[
-\frac{20}{7} - 6 c_2 = -5
\]

Multiply through by 7 to clear denominators:

\[
-20 - 42 c_2 = -35
\]

\[
-42 c_2 = -35 + 20 = -15
\]

\[
c_2 = \frac{-15}{-42} = \frac{15}{42} = \frac{5}{14}
\]

- Now, verify these values satisfy the third equation:

\[
5 c_1 + 2 c_2 = -5
\]

\[
5 \left(-\frac{5}{7}\right) + 2 \left(\frac{5}{14}\right) = -5
\]

Calculate each term:

\[
-\frac{25}{7} + \frac{10}{14}
\]

Note that $\frac{10}{14} = \frac{5}{7}$:

$$
-\frac{25}{7} + \frac{5}{7} = -\frac{20}{7}
$$

Compare to -5:

$$
-\frac{20}{7} \neq -5
$$

Express -5 with denominator 7:

$$
-5 = -\frac{35}{7}
$$

Since $-\frac{20}{7} \neq -\frac{35}{7}$, the third equation is not satisfied by the values of c_1 and c_2 found from the first two equations.

Step 3: Conclusion from the System

Because the third equation is not satisfied, the system is inconsistent, indicating that no solution exists for the scalars c_1 and c_2. Therefore:

> U is not in the span of the columns of A.

Why Is U Not in the Span?

Linear Dependence and Span

- The columns of A form a subspace in $\mathbb{R}^3$. The span of these columns is the set of all linear combinations of $\mathbf{a}_1$ and $\mathbf{a}_2$.

- Since the system has no solution, U cannot be written as a linear combination of the columns of A.

Geometric Interpretation

- The columns $\mathbf{a}_1$ and $\mathbf{a}_2$ span a subspace of at most dimension 2 (since there are only two vectors).

- The vector U, with all components -5, is not in this subspace because it lies outside the plane spanned by the two vectors.

Additional Considerations

Matrix Rank and Column Space

- The rank of matrix A determines the dimension of the column space.

- Computing the rank:

$$
A = \begin{bmatrix}
4 & -6 \\
7 & 0 \\
5 & 2
\end{bmatrix}
$$

- The two columns are not scalar multiples, so they are linearly independent, and the rank of A is 2.

- The column space is a 2-dimensional subspace of $\mathbb{R}^3$.

Implication for the Vector U

- Since U is not in the column space, the linear system has no solution, confirming that U is outside the subspace spanned by the columns.

Summary and Final Verdict

Aspect	Explanation	Result
Vector U	Interpreted as $[-5, -5, -5]$	Given in $\mathbb{R}^3$
Columns of A	$\mathbf{a}_1 = [4,7,5]$, $\mathbf{a}_2 = [-6,0,2]$	Generate a 2D subspace
System solution	No consistent solution found	U not in span
Geometric interpretation	U not in the plane spanned by columns	Confirmed

Therefore, U is not in the subset of $\mathbb{R}^3$ spanned by the columns of A because it cannot be expressed as a linear combination of those columns.

Conclusion

In linear algebra, determining whether a vector belongs to a subspace generated by a set of vectors involves solving a system of linear equations. If the system is inconsistent, as in this case, the vector does not lie within the span. Our detailed analysis shows that the vector U = [-5, -5, -5] cannot be expressed as a linear combination of the columns of matrix A, which span a 2-dimensional subspace of $\mathbb{R}^3$. Consequently, U is outside the subspace spanned by A's columns.

Understanding these principles is crucial for various applications, such as solving systems of linear equations, analyzing vector spaces, and performing projections in higher dimensions. This comprehensive approach demonstrates how to methodically analyze similar problems to determine vector inclusion in subspaces.

Additional Resources

- Linear Algebra Textbooks: For foundational concepts on vector spaces, span, and linear independence.
- Online Calculators: Tools for solving systems of equations.
- Educational Videos: Visual explanations of span and subspace concepts.

Frequently Asked Questions

Given U = (-5, -6, 0, 2) and A = [4, 7, 5], is the vector U in the subset of R^3 spanned by the columns of A?

No, U is not in the span of the columns of A because the dimensions do not match; A's columns are in R^3, but U has four components, so U cannot be expressed as a linear combination of the columns of A.

What does it mean for a vector U to be in the span of the columns of matrix A?

It means that there exist scalars such that when multiplied by the columns of A and summed, they produce the vector U.

Can a vector with four components be in the span of a set of vectors in R^3?

No, because the span of vectors in R^3 can only produce vectors with three components. A four-component vector cannot be expressed as a linear combination of vectors in R^3.

Is matrix A given as a 3x1 or 1x3 matrix, and how does that affect whether U can be in its span?

If A is a 3x1 matrix, its span is a line in R^3. U must then be a scalar multiple of that column to be in the span. If A is 1x3, it represents a row vector, which doesn't define a span in R^3. Clarification of A's shape is necessary to determine span.

How can we determine if a vector U is in the span of the columns of A?

Set up a linear system where the columns of A are multiplied by scalars, and see if the resulting vector equals U. If the system has a solution, U is in the span; otherwise, it is not.

Given U = (-5, -6, 0, 2), what is the dimensionality issue with the set A = [4, 7, 5] in relation to U?

The vector U has four components, but the set A's columns are in R^3. Since they are in different dimensions, U cannot be expressed as a linear combination of A's columns, indicating U is not in the span.

What is the importance of matching dimensions when checking if a vector is in the span of a set of vectors?

Matching dimensions is crucial because a vector can only be expressed as a linear combination of vectors in the same space. Mismatched dimensions mean the vector cannot be represented as such, and thus, is not in the span.

Additional Resources

Let U = -5 And A = 4 7 5. Is U In The Subset Of R3 Spanned By The Columns Of A? Why Or Why Not?

Understanding whether a vector is contained within a particular subspace is a fundamental question in linear algebra, especially when dealing with vector spaces such as R^3. In this context, we are asked to determine whether a given vector U, which is -5, and another matrix A composed of specific column vectors, spans a subspace of R^3 that contains U. This involves examining the relationship between the vector U and the subspace generated by the columns of A. To thoroughly analyze this, we need to clarify the definitions, interpret the data correctly, and perform the necessary computations.

Understanding the Components of the Problem

What is the Vector U?

The vector U is given as -5. At first glance, this appears to be a scalar; however, since the context involves R^3 and columns of a matrix, U should be a vector in three-dimensional space. It’s possible that the notation "-5" refers to a vector with all components equal to -5, or perhaps a specific component. Clarifying the notation is essential.

Possible interpretations:
- U = (-5, -5, -5): a vector with all components equal to -5.
- U = (-5, 0, 0): if only the first component is specified.
- U = -5: a scalar, which cannot directly be compared to vectors in R^3.

Given the context and typical problem notation, it is most plausible that U is the vector U = (-5, -5, -5), representing a vector in R^3.

Pros:
- Consistent with the setting of R^3 and the problem statement.
- Allows straightforward comparison with the span of columns of A.

Cons:
- The original notation is ambiguous; clarification would be better.

Understanding the Matrix A

The matrix A is given as:
$$ A = \begin{bmatrix} 4 & 7 & 5 \\ -6 & 0 & 2 \end{bmatrix} $$

However, a matrix in R^3 typically has three rows to match the dimension. Here, only two rows are provided, which suggests an inconsistency.

Possible interpretations:
- The matrix A is a 2x3 matrix, meaning its columns are vectors in R^2, not R^3.
- Alternatively, perhaps the matrix is intended to be a 3x3 matrix with missing components.

Given the explicit mention of R^3, the most logical assumption is that the columns of A are vectors in R^3, and the data provided is incomplete or misformatted.

Assumption for clarity:
Let’s assume the columns of A are:
$$ A = \begin{bmatrix} 4 & 7 & 5 \\ -6 & 0 & 2 \\ ? & ? & ? \end{bmatrix} $$

But since only two rows are given, perhaps the columns are:
- First column: (4, -6)
- Second column: (7, 0)
- Third column: (5, 2)

In this case, the columns are vectors in R^2, not R^3, which conflicts with the initial problem statement.

Conclusion:
Given the ambiguity, the most reasonable interpretation is that the matrix A is a 3x3 matrix with columns:
$$ A = \begin{bmatrix} 4 & 7 & 5 \\ -6 & 0 & 2 \\ ? & ? & ? \end{bmatrix} $$

But since the data is incomplete, perhaps the intended matrix is:
$$ A = \begin{bmatrix} 4 & 7 & 5 \\ -6 & 0 & 2 \end{bmatrix} $$

which would be a 2x3 matrix, representing vectors in R^2, not R^3.

Key point:

The problem likely aims to test the span of the columns in R^3, so we should clarify that the columns of A are:
- $c_1 = (4, -6, ?)$
- $c_2 = (7, 0, ?)$
- $c_3 = (5, 2, ?)$

But because the data is incomplete, we will proceed under the assumption that the columns of A are in R^3, and the missing third row is zero or not provided, and that the columns are:
$$ c_1 = (4, -6, 0), \quad c_2 = (7, 0, 0), \quad c_3 = (5, 2, 0) $$

This assumption allows us to perform the necessary analysis.

Determining Whether U Is in the Span of Columns of A

What Is the Span of the Columns of A?

The span of the columns of A is the set of all linear combinations:
$$ \text{Span}\{c_1, c_2, c_3\} = \left\{ \alpha c_1 + \beta c_2 + \gamma c_3 \mid \alpha, \beta, \gamma \in \mathbb{R} \right\} $$

To check whether U is in this span, we need to see if there exist scalars α, β, γ such that:
$$ U = \alpha c_1 + \beta c_2 + \gamma c_3 $$

Assuming U = (-5, -5, -5), and the columns are as above, the system becomes:
$$ \begin{cases}
4 \alpha + 7 \beta + 5 \gamma = -5 \\
-6 \alpha + 0 \beta + 2 \gamma = -5 \\
0 \alpha + 0 \beta + 0 \gamma = -5
\end{cases} $$

The third equation simplifies to $0 = -5$, which is impossible. Therefore, U cannot be expressed as a linear combination of these columns.

Conclusion:
U is not in the span of the columns of A, given the current data.

Why Is U Not in the Span of Columns of A?

Because the third equation derived from the assumptions yields an inconsistency (0 = -5), this indicates that U cannot be generated by the columns of A. This is a straightforward linear algebra conclusion based on the infeasibility of the system of equations.

Features:
- The inconsistency directly shows U is outside the subspace spanned by the columns.
- The problem reduces to solving a system of linear equations, a fundamental approach in linear algebra.

Pros:
- Clear and decisive; no ambiguity once the system is set up.
- Demonstrates the importance of checking the consistency of the linear system.

Cons:
- Relies on assumptions about the matrix A, which are not explicitly provided.
- The initial ambiguity about the vector U and matrix A complicates interpretation.

Implications and Broader Context

Understanding Subspace Inclusion

In linear algebra, determining whether a vector belongs to a subspace spanned by a set of vectors involves solving a linear system. If the system is consistent, the vector lies in the span; otherwise, it does not. This process is fundamental in many applications, from computer graphics to data science, where understanding the structure of data and the relationships between vectors is crucial.

Features:
- The process hinges on solving linear equations.
- The span of a set of vectors forms a subspace of the vector space.

Pros:
- This approach is systematic and reliable.
- It generalizes well to higher dimensions and more complex systems.

Cons:
- Complexity increases with the number of vectors and dimensions.
- Incomplete or ambiguous data complicates the analysis.

Further Considerations

If the data about the matrix A were different, or if U were a different vector, the conclusion could change. For instance, if U were aligned with one of the columns, it would trivially be in the span. Conversely, if the columns of A were linearly independent and span the entire R^3, then any vector in R^3 would be in the span, but that is not the case here.

Summary and Final Remarks

Based on the interpretation that U is the vector (-5, -5, -5), and assuming the columns of A are as specified (or similar), the analysis shows that U does not belong to the subspace spanned by the columns of A. This conclusion is drawn from solving the linear system and finding it inconsistent, indicating that U cannot be expressed as a linear combination of those columns.

Key points:
- The inclusion of a vector in a span depends on the solvability of the linear system.
- Ambiguity in data presentation underscores the importance of clear notation and complete information.
- The methodology demonstrated is fundamental in linear algebra and applies broadly across various mathematical and engineering disciplines.

Final thoughts:
Understanding whether a vector lies within a subspace provides essential insights into the structure of vector spaces and the relationships between vectors. Although the specific problem encountered ambiguities, the core

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