Line Q Passes Through (5, 5) And Is Parallel To The Line 2x + Y + 1 = 0? What Is The Equation Of Line

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Understanding how to find the equation of a line given certain conditions is a fundamental skill in algebra and coordinate geometry. In this article, we will explore the process of determining the equation of a line, specifically when it passes through a particular point and is parallel to a given line. Using the example where Line Q passes through the point (5, 5) and is parallel to the line with the equation $2x + y + 1 = 0$, we will demonstrate step-by-step how to derive the equation of Line Q. This comprehensive guide is designed to enhance your understanding of line equations, slopes, and the principles of parallel lines, all optimized for clarity and SEO relevance.

Understanding the Basics of Line Equations

Before delving into the specific problem, it's essential to review some foundational concepts related to line equations.

Standard Form of a Line Equation

- The general form of a line's equation is $Ax + By + C = 0$.
- For example, the given line: $2x + y + 1 = 0$.

Slope-Intercept Form

- The slope-intercept form is $y = mx + b$, where m is the slope and b is the y-intercept.
- To convert from standard form to slope-intercept form, solve for y.

Understanding Parallel Lines

- Parallel lines are lines in the same plane that never intersect.
- They have identical slopes but different y-intercepts.
- The key property: If two lines are parallel, their slopes are equal.

Step-by-Step Solution to Find the Equation of Line Q

Given:
- Point P(5, 5) through which Line Q passes.
- Line L with equation $2x + y + 1 = 0$, which Line Q is parallel to.

Our goal:
- Find the equation of Line Q.

Step 1: Find the slope of the given line

To find the slope of line L:
- Convert $2x + y + 1 = 0$ to slope-intercept form.

Conversion:
- $2x + y + 1 = 0$
- $y = -2x - 1$

Result:
- Slope of line L, $m_L = -2$

Step 2: Determine the slope of Line Q

- Since Line Q is parallel to line L:
- Slope of Line Q, $m_Q = m_L = -2$

Step 3: Use point-slope form to find the equation of Line Q

The point-slope form of a line's equation is:
- $y - y_1 = m(x - x_1)$

Where:
- $(x_1, y_1) = (5, 5)$
- $m = -2$

Substitute:
- $y - 5 = -2(x - 5)$

Step 4: Simplify the equation

Distribute:
- y - 5 = -2x + 10

Add 5 to both sides:
- y = -2x + 10 + 5
- y = -2x + 15

Step 5: Write the equation in standard form

Bring all terms to one side:
- y + 2x - 15 = 0

Or, rearranged:
- 2x + y - 15 = 0

Final Equation of Line Q:
- 2x + y - 15 = 0

Key Points to Remember

- The slope of a line in standard form Ax + By + C = 0 is -A/B (when B ≠ 0).
- Parallel lines have identical slopes.
- To find the equation of a line passing through a point with a given slope, use the point-slope form.
- Convert to the desired form (slope-intercept or standard) for clarity or specific applications.

Additional Insights on Line Equations and Parallelism

How to Find the Slope from Standard Form

- For a line in standard form Ax + By + C = 0, the slope m = -A / B.
- Example: For 2x + y + 1 = 0, m = -2 / 1 = -2.

Why Parallel Lines Have the Same Slope

- Parallel lines maintain a constant distance apart.
- Having the same slope ensures they never intersect.

Converting Between Forms of Line Equations

- Standard form: $Ax + By + C = 0$.
- Slope-intercept form: $y = mx + b$.
- To convert, solve for y.

Real-World Applications of Line Equations and Parallel Lines

- Architectural design: ensuring walls are parallel.
- Navigation: plotting parallel routes.
- Engineering: designing components with parallel features.
- Data analysis: fitting parallel trend lines.

Summary and Conclusion

In summary, when given a line's equation and a specific point through which another line passes, determining the new line's equation involves understanding the slope and leveraging the point-slope form. In our example, since Line Q passes through (5, 5) and is parallel to $2x + y + 1 = 0$, we first find the slope of the given line, which is -2. Then, using the point (5, 5), we apply the point-slope form to derive $y - 5 = -2(x - 5)$, simplifying to $y = -2x + 15$, and finally expressing it in standard form as $2x + y - 15 = 0$. This process highlights the critical steps in solving line equations and understanding the geometric relationship between parallel lines.

By mastering these concepts, students and professionals can efficiently handle various problems involving line equations, whether in academic settings, engineering projects, or everyday problem-solving scenarios. Remember, the key is understanding the relationship between slope, points, and the different forms of line equations to accurately describe and analyze geometric figures.

SEO Keywords for Optimization:
- Line Q passes through (5, 5) and is parallel to $2x + y + 1 = 0$
- Equation of line passing through point and parallel to given line
- How to find the equation of parallel lines
- Standard form of line equation
- Slope of a line from standard form
- Point-slope form of a line

- Converting line equations between forms
- Parallel lines in coordinate geometry
- Applications of line equations in real life

Frequently Asked Questions

What is the slope of the line 2x + y + 1 = 0?

The slope of the line $2x + y + 1 = 0$ is -2, since it can be rewritten as $y = -2x - 1$.

How do you find the slope of a line parallel to 2x + y + 1 = 0?

A line parallel to $2x + y + 1 = 0$ will have the same slope, which is -2.

What is the general form of the equation of a line passing through point (5, 5) with slope -2?

The equation can be written as $y - 5 = -2(x - 5)$, which simplifies to $y = -2x + 15$.

How do you determine the equation of a line passing through (5, 5) and parallel to a given line?

Use the point-slope form $y - y_1 = m(x - x_1)$, substituting the point (5, 5) and the slope $m = -2$.

What is the equation of the line passing through (5, 5) and parallel to 2x + y + 1 = 0?

The equation is $y = -2x + 15$.

Why is the line passing through (5, 5) and parallel to the given line?

Because it has the same slope (-2) as the original line, ensuring they are parallel.

How can I verify that the line y = -2x + 15 passes through (5, 5)?

Substitute $x = 5$ into the equation: $y = -2(5) + 15 = -10 + 15 = 5$, confirming it passes through (5, 5).

What are some real-world applications of finding lines parallel to a given

line passing through a specific point?

Applications include designing parallel roads or tracks, optimizing layouts in engineering, and graphical data analysis where maintaining consistent slopes is important.

Additional Resources

Line Q Passes Through (5, 5) And Is Parallel To The Line 2x + y + 1 = 0? What Is The Equation Of Line?

Understanding the geometric relationship between lines in the coordinate plane is fundamental to algebra, analytic geometry, and numerous applications across science and engineering. When asked to find a line passing through a specific point and parallel to another given line, the problem involves translating geometric conditions into algebraic equations, leveraging concepts like slopes, point-slope form, and standard form of linear equations. This article provides a comprehensive, detailed exploration of this problem, starting from the basic principles to deriving the explicit equation of the required line, with a focus on clarity, depth, and analytical insight.

Fundamental Concepts in Line Equations and Parallelism

Before tackling the specific problem, it is essential to revisit some core concepts related to line equations, slope, and the criteria for lines to be parallel.

Linear Equations in the Coordinate Plane

A straight line in the xy-plane can be represented algebraically in various forms, with the most common being:

- Slope-Intercept Form: $y = mx + b$
Here, m represents the slope, and b is the y-intercept.

- Standard Form: $Ax + By + C = 0$
This form is useful for various algebraic manipulations and for understanding geometric properties.

- Point-Slope Form: $y - y_1 = m(x - x_1)$
Used when a point on the line and its slope are known.

Understanding these forms allows us to analyze lines' properties systematically.

Slope of a Line and Its Significance

The slope (m) measures the steepness of the line and is calculated as:

$$ m = \frac{\Delta y}{\Delta x} $$

for two distinct points (x_1, y_1) and (x_2, y_2).

- Parallel Lines: Two lines are parallel if and only if they have the same slope, provided they are not coincident.
- Perpendicular Lines: They have slopes that are negative reciprocals of each other.

This criterion simplifies the task of identifying lines that are parallel once their slopes are known.

Analyzing the Given Line: 2x + y + 1 = 0

The first step is to interpret the given line and determine its slope, as the parallel line must share this same slope.

Converting to Slope-Intercept Form

Starting from the standard form:

$$ 2x + y + 1 = 0 $$

Rearranged to slope-intercept form:

$$ y = -2x - 1 $$

This form explicitly reveals the slope:

$$ m_{original} = -2 $$

Thus, the original line has a slope of -2.

Geometric Interpretation

- The line passes through all points satisfying $y = -2x - 1$.
- Its slope indicates that for every increase of 1 unit in x, y decreases by 2 units.
- The y-intercept occurs at $(0, -1)$.

Understanding this slope is crucial because the new line, which we will find, must be parallel and therefore share this same slope.

Defining the Conditions for Line Q

The problem specifies that Line Q passes through the point (5, 5) and is parallel to the line 2x + y + 1 = 0.

This sets up two primary conditions:

1. Point condition: The line passes through $(5, 5)$.
2. Parallel condition: The line has the same slope as the original line, i.e., -2.

These conditions allow us to determine the explicit equation of Line Q.

Deriving the Equation of Line Q

Using the point-slope form, which is ideal given a point and a slope:

$$y - y_1 = m (x - x_1)$$

where:

- $(x_1, y_1) = (5, 5)$,
- $m = -2$.

Plugging in these values:

$$y - 5 = -2 (x - 5)$$

This is the point-slope form, which can be expanded into slope-intercept or standard form.

Expanding to Slope-Intercept Form

$$ y - 5 = -2x + 10 $$

Adding 5 to both sides:

$$ y = -2x + 15 $$

So, the equation of Line Q in slope-intercept form is:

$$ y = -2x + 15 $$

This line passes through $(5, 5)$ and shares the same slope as the original line, confirming parallelism.

Expressing in Standard Form

Rearranging to standard form:

$$ y = -2x + 15 $$

which becomes:

$$ 2x + y - 15 = 0 $$

This is the standard form of the line passing through $(5, 5)$ and parallel to $2x + y + 1 = 0$.

Verification and Geometric Consistency

Ensuring that the derived line meets the problem's criteria involves verifying two key aspects:

1. Point Inclusion: Substituting $(5, 5)$ into the derived equation:

$$ 2(5) + 5 - 15 = 10 + 5 - 15 = 0 $$

which confirms that $(5, 5)$ lies on the line.

2. Parallelism: Both lines have slope -2, which is confirmed by the slope-intercept form:

- Original line: $y = -2x - 1$
- Line Q: $y = -2x + 15$

Since their slopes are identical and their equations are distinct, the lines are parallel.

Additional Considerations and Insights

While the problem appears straightforward, several subtle points and extensions enrich the discussion:

Uniqueness of the Line

Given a point and a slope, there is exactly one line passing through that point with that slope, making the solution unique. This emphasizes the deterministic nature of line equations once these parameters are fixed.

Effect of Different Forms

- Slope-Intercept Form: Directly shows the slope and intercepts, useful for quick visualization.
- Standard Form: Useful for algebraic manipulations, such as intersection calculations.
- Point-Slope Form: Convenient when a point and slope are known.

Application in Coordinate Geometry and Real-World Contexts

- Navigation and Mapping: Determining paths or trajectories that are parallel to existing routes.
- Engineering: Designing components or systems where parallel lines ensure structural integrity.
- Data Analysis: Fitting lines through points with specific constraints.

Summary and Final Equation

Bringing all the analysis together, the problem's core solution is summarized as follows:

- The original line: $2x + y + 1 = 0$, slope -2.
- The point the new line passes through: $(5, 5)$.
- The new line, parallel to the original, shares the same slope: -2.

Final Equation of Line Q:

- In slope-intercept form:

$$\boxed{ y = -2x + 15 }$$

- In standard form:

$$\boxed{ 2x + y - 15 = 0 }$$

This equation precisely describes the line passing through $(5, 5)$ and parallel to the given line.

Conclusion

The task of finding a line passing through a specified point and parallel to another involves translating geometric conditions into algebraic equations, recognizing the importance of slope, and applying fundamental formulas like the point-slope form. The problem exemplifies the elegance and power of analytic geometry, where simple conditions lead to precise, unique solutions. Whether for academic purposes, practical applications, or theoretical exploration, understanding these principles enhances geometric intuition and algebraic proficiency.

In this case, the derived line $y = -2x + 15$ stands as a testament to the effective interplay between algebraic manipulation and geometric reasoning, providing a clear example of how constraints translate into concrete mathematical expressions.

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