Line Y = 3x-1 Is Transformed By A Dilation With A Scale Factor Of 2 And Centered At (3,8). The Line's

Line Y = 3x-1 Is Transformed By A Dilation With A Scale Factor Of 2 And Centered At (3,8). The Line's transformation process involves a series of geometric operations that alter its position and size on the coordinate plane. Specifically, applying a dilation centered at a point with a certain scale factor affects the entire line, changing its length and possibly its slope depending on the transformation's nature. Understanding how line transformations work, especially dilations, is essential in coordinate geometry and helps in visualizing geometric concepts more clearly.

In this article, we will explore the transformation of the line y = 3x - 1 under a dilation with a scale factor of 2, centered at the point (3,8). We will discuss what a dilation entails, how to perform this transformation step-by-step, and interpret the resulting line's properties. Additionally, we will analyze the impact of this dilation on the line's slope, position, and equation.

Understanding Dilation in Coordinate Geometry

What Is a Dilation?

A dilation is a transformation that enlarges or reduces a figure in the plane relative to a fixed point called the center of dilation. The size of the figure changes proportionally, but its shape remains similar.

Key features of dilation:
- It is characterized by a scale factor (k):
- If $k > 1$, the figure is enlarged.
- If $0 < k < 1$, the figure is reduced.
- The center of dilation is the fixed point relative to which the figure is expanded or contracted.

How Dilation Affects Lines

When a line undergoes a dilation:
- Its slope remains unchanged if the dilation is centered on a point on the line.
- Its length is scaled by the absolute value of the scale factor.
- Its position shifts depending on the location of the center relative to the line.

Applying Dilation to the Line y = 3x - 1

Initial Line Equation

The original line:
$$ y = 3x - 1 $$
has:
- Slope $m = 3$
- Y-intercept $b = -1$

This line passes through points such as:
- When $x=0$, $y=-1$
- When $x=1$, $y=2$

Center of Dilation: (3,8)

The dilation is centered at the point $C(3,8)$. To perform the dilation, we need to:
1. Pick points on the original line.
2. Find their images after dilation relative to the center.
3. Derive the equation of the new line.

Step-by-Step Transformation Process

1. Select Points on the Original Line

Choose two convenient points:
- Point A: $(0, -1)$ (from the original line)
- Point B: $(1, 2)$

2. Find the Images of these Points After Dilation

For each point, use the dilation formula:
$$ P' = C + k \times (P - C) $$
where:
- P is the original point,
- C is the center of dilation,
- k is the scale factor (here, 2).

Calculations:

- For Point A $(0, -1)$:
- $\Delta x = 0 - 3 = -3$
- $\Delta y = -1 - 8 = -9$
- $P_A' = (3 + 2 \times -3, 8 + 2 \times -9) = (3 - 6, 8 - 18) = (-3, -10)$

- For Point B $(1, 2)$:
- $\Delta x = 1 - 3 = -2$
- $\Delta y = 2 - 8 = -6$

- $P_B' = (3 + 2 \times -2, 8 + 2 \times -6) = (3 - 4, 8 - 12) = (-1, -4)$

3. Determine the Equation of the Transformed Line

Now, find the slope of the new line passing through points $P_A'(-3, -10)$ and $P_B'(-1, -4)$:

$$
m' = \frac{-4 - (-10)}{-1 - (-3)} = \frac{6}{2} = 3
$$

Notice that the slope remains 3, the same as the original line, which aligns with the fact that dilations centered on points on the line do not change the slope.

Using point-slope form with point $P_A'(-3, -10)$:

$$
y - (-10) = 3(x - (-3))
$$
$$
y + 10 = 3(x + 3)
$$
$$
y + 10 = 3x + 9
$$
$$
y = 3x - 1
$$

Surprisingly, the equation of the transformed line is identical to the original line, indicating that the line maps onto itself under this dilation centered on a point on the line.

Implications of the Transformation

Line Equation After Dilation

The analysis shows that the line $y = 3x - 1$ remains unchanged after the dilation:

$$
\boxed{
\text{Line after dilation: } y = 3x - 1
}
$$

This is because the dilation's center at $(3,8)$ lies on the line, and the transformation scales points along lines passing through the center, resulting in the same line.

What Happens When the Center Is Not on the Line?

If the center of dilation is not on the line, the transformation would:
- Not necessarily preserve the line's equation.
- Scale distances from the center, thus changing the line's position.
- Potentially alter the slope if the center is off the line, especially in more complex transformations.

Summary and Key Takeaways

- Dilation with a scale factor of 2 centered at $(3,8)$ affects points on the line by scaling their distances from the center.
- When the center of dilation lies on the line, the line remains unchanged after the transformation.
- The slope of the line, $m=3$, remains the same because dilations centered on a point on the line do not alter its slope.
- The equation of the line after dilation, in this particular case, stays $y=3x-1$.

Additional Considerations and Applications

Transformations in Coordinate Geometry

Understanding how lines and figures transform under dilations helps in:
- Analyzing similarity and congruence.
- Solving problems involving scale models.
- Visualizing geometric transformations for design and engineering.

Real-World Applications

- Urban Planning: Scaling maps and models to real-world sizes.
- Computer Graphics: Enlarging or shrinking objects while maintaining proportions.
- Physics and Engineering: Analyzing stress and strain by scaling models.

Conclusion

The transformation of the line $y=3x-1$ under a dilation with a scale factor of 2 centered at $(3,8)$ reveals interesting properties about how geometric figures behave under such transformations. In this specific case, because the center lies on the line, the line remains unchanged, maintaining its equation and slope. This underscores the importance of the center's position relative to the figure when analyzing geometric transformations. Understanding these concepts enhances your ability to manipulate and interpret geometric figures in various mathematical and practical contexts.

Remember: The key to mastering transformations is to consider the position of the figure relative to the center of dilation and to systematically perform each step. This process deepens comprehension and improves problem-solving skills in coordinate geometry.

Frequently Asked Questions

What is the original equation of the line before any transformations?

The original line is given by the equation $y = 3x - 1$.

How does a dilation with a scale factor of 2 centered at (3,8) affect the line?

The dilation scales all distances from the center point (3,8) by a factor of 2, stretching the line away from the center, which results in a new, larger line that is a scaled version of the original.

What is the effect of the dilation on the slope of the line?

Since dilation is a similarity transformation centered at a point, the slope of the line remains unchanged at 3; only the position and length are affected, not the slope.

How do you find the equation of the transformed line after dilation?

First, identify a point on the original line, find its image after dilation centered at (3,8) with scale factor 2, then use the slope to write the new line's equation passing through the image point.

What are the coordinates of a point on the original line, and how are they transformed?

For example, when $x=0$, $y=-1$; after dilation, the point (0, -1) is transformed to $((0-3)2 + 3, (-1-8)2 + 8) = (-3, -9)$.

What is the equation of the line after the dilation transformation?

Using the image of the point (0, -1) as the point and slope 3, the new line passes through (-3, -9). The equation is $y + 9 = 3(x + 3)$, which simplifies to $y = 3x + 0$, or $y = 3x$.

Additional Resources

Line Y = 3x - 1 Is Transformed By A Dilation With A Scale Factor Of 2 And Centered At (3,8): An In-Depth Analysis

In the realm of coordinate geometry, transformations serve as fundamental tools that alter the position, size, or orientation of geometric figures while preserving certain properties. Among these, dilation (or scaling) stands out as a particularly intriguing transformation. When applied to lines, dilations modify their length and position relative to a fixed point called the center of dilation, often revealing deeper insights into geometric relationships.

This article embarks on an investigative journey to understand the transformation of the line defined by the equation Y = 3x - 1 under a dilation with a scale factor of 2, centered at the point (3,8). We will explore the mathematical principles underpinning this transformation, analyze the resulting line, and interpret the geometric implications in detail.

Understanding the Original Line: Y = 3x - 1

Before delving into the transformation, it is essential to comprehend the properties of the original line.

Equation and Characteristics

- Slope (m): 3
- Y-intercept (b): -1
- Graphical behavior:

The line crosses the y-axis at (0, -1) and has a steep incline, rising 3 units vertically for every 1 unit horizontally.

Points on the Line

- When x = 0, y = -1 → point (0, -1)
- When x = 1, y = 2 → point (1, 2)
- When x = 2, y = 5 → point (2, 5)
- When x = -1, y = -4 → point (-1, -4)

These points serve as reference markers for visualizing the line and understanding how they will transform under dilation.

Principles of Dilation in Coordinate Geometry

Dilation is a transformation that enlarges or reduces a figure relative to a fixed point called the center of dilation, using a specified scale factor.

Definition and Formula

Given a point P(x, y) and a center of dilation C(x_c, y_c), the image P' after dilation with scale factor k is found via:

$$
P' = (x_c + k(x - x_c), \, y_c + k(y - y_c))
$$

This formula indicates that:

- The vector from C to P is scaled by k.
- The point P' lies along the line segment from C to P, extending or contracting depending on k.

Implications for Lines

- When applying dilation to a line, each point on the line moves along a ray passing through the center.
- The overall line is "stretched" or "shrunk" relative to the center.
- The transformed line remains a straight line, but its orientation and position may change depending on the center and scale factor.

Applying Dilation to the Line Y = 3x - 1

Our task is to understand how the line transforms under a dilation with:

- Scale factor (k): 2
- Center of dilation (C): (3, 8)

We will analyze the transformation by considering key points on the original line, transforming them individually, and then deducing the equation of the new line.

Step 1: Select Representative Points on the Original Line

For clarity, choose points that are easy to work with:

- Point A: (0, -1)
- Point B: (1, 2)
- Point C: (2, 5)

Step 2: Transform Each Point Using the Dilation Formula

Transforming Point A (0, -1):

$$
x' = x_c + k(x - x_c) = 3 + 2(0 - 3) = 3 + 2(-3) = 3 - 6 = -3
$$

\]
\[
y' = y_c + k(y - y_c) = 8 + 2(-1 - 8) = 8 + 2(-9) = 8 - 18 = -10
\]
- Image A': (-3, -10)

Transforming Point B (1, 2):

\[
x' = 3 + 2(1 - 3) = 3 + 2(-2) = 3 - 4 = -1
\]
\[
y' = 8 + 2(2 - 8) = 8 + 2(-6) = 8 - 12 = -4
\]
- Image B': (-1, -4)

Transforming Point C (2, 5):

\[
x' = 3 + 2(2 - 3) = 3 + 2(-1) = 3 - 2 = 1
\]
\[
y' = 8 + 2(5 - 8) = 8 + 2(-3) = 8 - 6 = 2
\]
- Image C': (1, 2)

Step 3: Derive the Equation of the Transformed Line

Using the transformed points:

- (-3, -10)
- (-1, -4)
- (1, 2)

Calculate the slope of the new line:

\[
m' = \frac{y_2' - y_1'}{x_2' - x_1'} = \frac{-4 - (-10)}{-1 - (-3)} = \frac{6}{2} = 3
\]

The slope remains 3, identical to the original line's slope. This is a crucial observation indicating that dilation with a center not on the line can preserve the slope of the original line, provided the points are correctly transformed.

Next, find the equation using point-slope form with point (-3, -10):

\[

$$
y - y_1' = m'(x - x_1') \Rightarrow y + 10 = 3(x + 3)
$$

$$
y + 10 = 3x + 9
$$

$$
y = 3x - 1
$$

Result: The transformed line is $Y = 3x - 1$.

Interpreting the Result: Preservation of Line Equation

At first glance, this suggests that the line's equation remains unchanged after the dilation. However, this conclusion warrants a deeper geometric interpretation.

Why Does the Equation Remain the Same?

- The key reason is that the points used for the transformation lie along the same line as the original, and their transformations preserve the slope and intercept relative to the coordinate axes.
- The dilation, centered at a point not on the line, effectively "scales" the entire line in a manner that preserves its slope and intercept, especially when considering the specific points chosen.

Geometric Implications

- The original line $Y = 3x - 1$ is mapped onto itself in terms of its equation.
- The points on the line are mapped to new points along the same line, scaled appropriately relative to the center.
- The line's position relative to the dilation center shifts, but the line's equation remains the same.

Additional Perspectives and Geometric Insights

While the algebraic approach suggests the line's equation remains invariant, the geometric reality is more nuanced.

Visualizing the Transformation

- The dilation with center at (3,8) "pushes" points away from or toward the center depending on their original position.
- For points on the line, this results in some points moving along rays passing through (3,8), but the

overall line, as a set, preserves its slope.

Effect on the Line's Position

- The line Y = 3x - 1 is not passing through the dilation center (3,8); hence, it is scaled in a manner that preserves its slope but may shift its position relative to the axes.
- The key is that the points used in the transformation lie on the same line as the original, confirming that the line's equation appears unchanged after dilation centered at (3,8).

Implication for Other Lines and Shapes

- Lines passing through the center of dilation are invariant under the transformation—they map onto themselves.
- Lines not passing through the center are scaled and shifted, but their equations can often preserve their form depending on the points chosen.

Conclusions and Broader Implications

This investigation reveals several important insights:

1. Dilation can preserve the equation of a line if the points used in the transformation are chosen appropriately, especially when the line's points are scaled relative to the center.

2. Transformation of Key Points: Applying the dilation formula to select points confirms that the line Y = 3x - 1 remains invariant under a dilation with center (3,8) and scale

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