

Now, Rewrite The Given Integral As Follows And Use The General Exponential Rule. $7e^{7x} dx = e^{7x} (7) dx$

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Understanding how to evaluate integrals involving exponential functions is a fundamental aspect of calculus, essential for students and professionals working in fields such as engineering, physics, mathematics, and economics. In this article, we will explore the process of rewriting integrals involving exponential functions, specifically focusing on the integral of $7e^{7x} dx$, and demonstrate how to apply the general exponential rule to evaluate it efficiently. By the end, you will gain a comprehensive understanding of the techniques involved in handling exponential integrals, along with practical examples and tips for mastering this vital calculus concept.

Introduction to Exponential Integrals

Exponential functions are among the most important functions in calculus due to their unique properties, especially their derivatives and integrals. The basic exponential function e^x has the distinctive property that its derivative and integral are both e^x , making it a fundamental building block for more complex functions.

When dealing with integrals involving exponential functions with coefficients or shifts, such as e^{ax} where a is a constant, it's crucial to understand how to manipulate these expressions to simplify integration.

The General Exponential Rule

The general exponential rule provides a systematic way to evaluate integrals of exponential functions with linear arguments. It states that:

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C,$$

where:

- a is a non-zero constant,
- C is the constant of integration.

This rule applies because the derivative of (e^{ax}) with respect to (x) is $(a e^{ax})$. Therefore, when integrating (e^{ax}) , we reverse the differentiation process, dividing by (a) to compensate for the chain rule.

Rewriting the Integral: Step-by-Step Approach

Let's focus on the specific integral:

$$\int 7e^{7x} \, dx.$$

The goal is to rewrite this integral to facilitate applying the exponential rule effectively.

Step 1: Recognize the structure of the integral

Observe that the integrand is a product of a constant (7) and an exponential function (e^{7x}). Since constants can be factored out of the integral, we can write:

$$\int 7e^{7x} \, dx = 7 \int e^{7x} \, dx.$$

This simplifies the problem to evaluating $\int e^{7x} \, dx$.

Step 2: Apply the General Exponential Rule

Using the rule:

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax} + C,$$

we identify $(a = 7)$. Substituting into the rule gives:

$$\int e^{7x} \, dx = \frac{1}{7} e^{7x} + C,$$

where (C) is the constant of integration.

Step 3: Incorporate the constant factor

Recall the earlier step:

$$\int 7e^{7x} \, dx = 7 \times \left(\frac{1}{7} e^{7x} + C \right).$$

Since the constant 7 multiplies the entire integral, we can distribute it:

$$7 \int \frac{1}{7} e^{7x} + 7 C = e^{7x} + C',$$

where $(C' = 7C)$ is just a redefined constant of integration.

Important note: The constant of integration encompasses all constant terms, so multiplying by 7 simply results in a new constant.

Final Result and Interpretation

The integral evaluates to:

$$\boxed{\int 7 e^{7x} \, dx = e^{7x} + C.}$$

This is a straightforward application of the general exponential rule, demonstrating how recognizing the structure of the integral allows for quick and accurate evaluation.

Practical Applications of Exponential Integrals

Understanding how to evaluate exponential integrals is crucial in various real-world contexts, such as:

- Modeling Growth and Decay: In biology, finance, and physics, exponential functions model populations, radioactive decay, and interest calculations.
- Solving Differential Equations: Many differential equations involve exponential functions; integrating these functions is essential for finding solutions.
- Engineering Signal Processing: Exponentials are fundamental in analyzing systems' responses and filters.

Advanced Techniques for Handling Exponential Integrals

While the basic exponential rule covers many cases, some integrals require additional techniques:

Method 1: Substitution

When the exponential function involves a more complex argument, substitution can simplify the integral.

Example:

$$\int 3e^{5x+2} dx$$

Set $u = 5x + 2$, then $du = 5 dx$, so:

$$dx = \frac{du}{5}$$

Rewritten integral:

$$3 \int e^u \times \frac{du}{5} = \frac{3}{5} \int e^u du = \frac{3}{5} e^u + C = \frac{3}{5} e^{5x+2} + C.$$

Method 2: Integration by Parts

Although less common for exponential functions alone, this technique is useful when integrals involve products of exponential and polynomial functions.

Common Mistakes and Tips

To ensure accurate integration, keep in mind:

- Always factor out constants before integrating.
- Remember that the integral of e^{ax} involves dividing by a .
- When substituting, carefully change the differential dx .
- Keep track of the constant of integration throughout.

Summary and Key Takeaways

- The integral $\int 7e^{7x} dx$ can be efficiently evaluated by factoring out constants and applying the general exponential rule.
- The rule $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$ simplifies the process.
- Proper substitution techniques can handle more complex exponential integrals.

- Mastery of these techniques is essential for solving a wide range of problems in calculus and its applications.

Conclusion

Rewriting integrals involving exponential functions and applying the general exponential rule are fundamental skills in calculus. Recognizing the structure of the integrand, factoring constants, and applying the rule systematically can greatly simplify the evaluation process. Whether dealing with straightforward integrals like $\int 7e^{7x} dx$ or more complex cases involving substitution, understanding these core concepts provides a solid foundation for tackling exponential integrals confidently. Practice with diverse examples will enhance familiarity and efficiency, making exponential integrals an accessible and manageable part of your calculus toolkit.

Frequently Asked Questions

What is the main goal when rewriting the integral $7e^{7x} dx$ using the exponential rule?

The goal is to express the integral in a form that simplifies integration by recognizing the derivative of the exponential function, typically rewriting it as a product involving e^{7x} and a constant.

How do you apply the general exponential rule to the integral of $7e^{7x} dx$?

You factor out constants and recognize that the derivative of e^{ax} is $a e^{ax}$. So, the integral becomes E^{ax} divided by a , with appropriate constants considered.

What is the rewritten form of the integral $7e^{7x} dx$ according to the exponential rule?

The integral can be rewritten as $(7/7) e^{7x} + C$, which simplifies to $e^{7x} + C$.

Why is it useful to rewrite the integral as shown in the problem statement?

Rewriting the integral makes it easier to evaluate by directly applying the exponential rule, saving time and reducing errors.

What does the notation $E^{7x} (7) Dx$ represent in the

context of the integral?

It indicates the exponential function $e^{\{7x\}}$ multiplied by the constant 7, integrated with respect to x .

Can you explain the step-by-step process to evaluate the integral $7e^{\{7x\}} dx$ using the exponential rule?

Yes. First, recognize that the integral of $a e^{\{ax\}} dx$ is $(a/a) e^{\{ax\}} + C$. Here, $a=7$, so the integral becomes $7/7 e^{\{7x\}} + C$, which simplifies to $e^{\{7x\}} + C$.

Additional Resources

Exponential Integration Mastery: Unlocking the Power of the General Exponential Rule

Introduction: Demystifying the Integral of $7e^{\{7x\}} Dx$

When venturing into calculus, one of the foundational yet sometimes challenging concepts is integrating exponential functions. Among these, the integral of functions like $7e^{\{7x\}} dx$ stands out as a prime example where understanding the underlying rules can significantly simplify the process.

In this article, we will explore how to rewrite the integral $\int 7e^{\{7x\}} dx$ using the general exponential rule, a powerful technique that leverages the properties of exponential functions to streamline integration. We will delve into the derivation, applications, and best practices, providing a comprehensive guide that transforms a seemingly complex integral into an elegant solution.

Understanding the Integral: Why It Matters

Before diving into rewriting the integral, it's essential to grasp the significance of exponential functions in calculus.

- Exponential functions like $e^{\{ax\}}$ are fundamental in modeling growth, decay, and many natural phenomena.
- Integrating these functions often involves recognizing patterns and applying specific rules, notably the exponential rule.
- Mastering these techniques enhances problem-solving skills in calculus, physics, engineering, and beyond.

The specific integral in question:

$$\int 7e^{7x} dx$$

appears straightforward but invites a deeper exploration into the mechanics of exponential integration.

Rewriting the Integral: From Original to General Exponential Rule

Step 1: Factor Out Constants

The first step in simplifying the integral involves isolating constants:

$$\int 7e^{7x} dx = 7 \int e^{7x} dx$$

This step is crucial because it separates the constant coefficient, making the integral more manageable.

Step 2: Recognize the Inner Function and Its Derivative

The integral now takes the form:

$$7 \int e^{7x} dx$$

Notice that the exponent $7x$ is a linear function whose derivative is 7:

$$\frac{d}{dx} (7x) = 7$$

This insight hints at the possibility of substitution or applying the exponential rule directly.

Step 3: Applying the General Exponential Rule

The general exponential rule states:

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

where a is a constant coefficient in the exponent.

Why does this rule work?

- The derivative of (e^{ax}) with respect to (x) is $(a e^{ax})$.
- To invert this process (integration), we divide by a to compensate for this derivative.

Applying the General Exponential Rule: A Detailed Walkthrough

Step 4: Formal Application

Using the rule:

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

for our specific integral:

$$\int e^{7x} dx = \frac{1}{7} e^{7x} + C$$

Now, recall the earlier factorization:

$$7 \int e^{7x} dx$$

Substituting the result:

$$7 \times \left(\frac{1}{7} e^{7x} + C \right) = e^{7x} + C'$$

where (C) is the constant of integration (absorbing the constant 7 into the general constant).

Final Result:

$$\boxed{\int 7e^{7x} dx = e^{7x} + C}$$

Deeper Insights: Why the General Exponential Rule Is So Effective

Understanding the Mechanics

The exponential rule hinges on the chain rule from differentiation. When differentiating (e^{ax}) :

$$\frac{d}{dx} e^{ax} = a e^{ax}$$

To integrate (e^{ax}) , the reverse process, you divide by (a) :

$$\int e^{ax} dx = \frac{1}{a} e^{ax} + C$$

This concept is universal for all exponential functions where the exponent is a linear function of (x) .

Practical Advantages

- Simplifies complex integrals: When encountering functions like $(7e^{7x})$, recognizing the pattern avoids trial-and-error substitution.
- Reduces calculation errors: Applying the rule directly minimizes the chance of mistakes.
- Provides a blueprint for more complex functions: Extending to functions like (e^{kx^n}) involves similar principles but with additional substitution strategies.

Common Pitfalls and Best Practices

Pitfall 1: Forgetting to Factor Constants

Always factor out constants before applying the exponential rule. Failing to do so can lead to incorrect integrations.

Pitfall 2: Confusing the Derivative and Integral Rules

Remember, the derivative of e^{ax} is $a e^{ax}$, so the integral is $\frac{1}{a} e^{ax}$. Confusing these leads to errors.

Best Practice 1: Use Substitution When Necessary

While the exponential rule is straightforward here, more complex functions benefit from substitution:

- Set $u = 7x$
- Then $du = 7 dx$
- Rewrite the integral accordingly

Best Practice 2: Always Include the Constant of Integration

No matter how simple the integral, include $+ C$ at the end.

Extensions and Applications in Advanced Calculus

The understanding of rewriting integrals using the exponential rule extends beyond basic functions:

- Solving differential equations: Many linear differential equations have solutions involving exponential functions.
- Modeling exponential growth and decay: Calculating total accumulated growth or decay often involves integrating exponential functions.
- Laplace transforms: These transforms heavily rely on exponential integrals to convert differential equations into algebraic equations.

Summary: Key Takeaways

- The integral $\int 7e^{7x} dx$ can be elegantly rewritten and solved using the general exponential rule.

- The rule states: $\int e^{ax} dx = \frac{1}{a} e^{ax} + C$.
- Always factor constants before applying the rule.
- Recognize the structure of the exponent to determine the appropriate substitution or direct application.
- Understanding these principles simplifies complex calculus problems and enhances analytical skills.

Final Thoughts: Embracing the Power of Exponential Integration

Mastering the rewriting of integrals like $\int 7e^{7x} dx$ using the general exponential rule is a cornerstone of calculus proficiency. It transforms what seems like a daunting calculation into a straightforward process rooted in fundamental principles. Whether tackling advanced differential equations or modeling real-world phenomena, this technique empowers mathematicians, scientists, and engineers to approach exponential functions with confidence and clarity.

By internalizing these concepts and practicing their application, you'll unlock a powerful toolset for mathematical problem-solving that extends far beyond the basics—making complex integrals manageable and opening doors to deeper analytical understanding.

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