Nth Term Of An AP Is Given By An Expression 7n-2,find The Common Difference Of This Sequence If Fisrt

Nth Term Of An AP Is Given By An Expression 7n-2, find The Common Difference Of This Sequence If First

Understanding arithmetic progressions (AP) is fundamental in mathematics, especially in the study of sequences and series. When the nth term of an AP is expressed explicitly, it provides valuable insights into the nature of the sequence, including its common difference, which is a key parameter that defines the progression. In this article, we will explore how to determine the common difference of an AP given its nth term expression, specifically focusing on the sequence where the nth term is given by $7n - 2$. We will delve into the concepts, step-by-step calculations, and applications to deepen your understanding.

What Is an Arithmetic Progression (AP)?

Definition of AP

An arithmetic progression (AP) is a sequence of numbers in which the difference between any two consecutive terms is constant. This constant difference is known as the common difference, denoted by d.

For example, the sequence 3, 7, 11, 15, 19, ... is an AP because the difference between each pair of consecutive terms is 4.

Properties of AP

- The nth term of an AP can be expressed explicitly as $a_n = a_1 + (n-1)d$, where:
- a_1 is the first term
- d is the common difference
- n is the term number

- The sum of the first n terms of an AP can be calculated using the formula:

$$
S_n = \frac{n}{2} \left(2a_1 + (n - 1)d \right)
$$

Knowing these properties helps in analyzing sequences and solving related problems efficiently.

Understanding the Given Nth Term Expression: 7n - 2

Explicit Formula of the Sequence

The problem states that the nth term of the sequence is given by:

$$
a_n = 7n - 2
$$

This means for any positive integer n, the term in the sequence can be calculated directly using this formula.

Examples of Terms

Let's compute the first few terms to understand the sequence better:

- When $n = 1$:

$$
a_1 = 7(1) - 2 = 7 - 2 = 5
$$

- When $n = 2$:

$$
a_2 = 7(2) - 2 = 14 - 2 = 12
$$

- When $n = 3$:

$$
a_3 = 7(3) - 2 = 21 - 2 = 19
$$

- When $n = 4$:

$$
a_4 = 7(4) - 2 = 28 - 2 = 26
$$

- When $n = 5$:

$$
a_5 = 7(5) - 2 = 35 - 2 = 33
$$

The sequence generated is: 5, 12, 19, 26, 33, ...

Determining the Common Difference of the Sequence

What Is the Common Difference?

The common difference d in an AP is the constant amount by which each term increases (or decreases) to get the next term. In mathematical terms:

$$
d = a_{n+1} - a_n
$$

for any value of n.

Calculating the Common Difference from the nth Term Expression

Given the explicit nth term formula $a_n = 7n - 2$, we can find the common difference by computing the difference between consecutive terms:

$$
d = a_{n+1} - a_n
$$

Let's substitute the expressions:

$$
a_{n+1} = 7(n+1) - 2
$$

$$
a_n = 7n - 2
$$

Therefore:

$$
d = [7(n+1) - 2] - [7n - 2]
$$

Simplify:

$$
d = (7n + 7 - 2) - (7n - 2)
$$

$$
d = (7n + 5) - (7n - 2)
$$

$$
d = 7n + 5 - 7n + 2
$$

$$
d = (7n - 7n) + (5 + 2) = 0 + 7 = 7
$$

Thus, the common difference d of the sequence is 7.

Summary of Findings

- The sequence defined by $a_n = 7n - 2$ is an arithmetic progression.
- The common difference d is 7.
- The first term $a_1 = 5$.

Additional Insights and Applications

First Term of the Sequence

From the explicit formula:

$$
a_1 = 7(1) - 2 = 5
$$
The sequence begins at 5.

General Term of the Sequence

The explicit formula for the nth term is:
$$
a_n = a_1 + (n-1)d
$$
where:
- $a_1 = 5$
- $d = 7$

Rewriting the explicit formula in the standard form:
$$
a_n = 5 + (n-1) \times 7 = 7n - 2
$$
which confirms our earlier calculation.

Sum of the First n Terms

Using the sum formula:
$$
S_n = \frac{n}{2} \left(2a_1 + (n-1)d \right)
$$
Substitute $a_1=5$ and $d=7$:
$$
S_n = \frac{n}{2} \left(2 \times 5 + (n-1) \times 7 \right)
$$
$$
S_n = \frac{n}{2} \left(10 + 7n - 7 \right) = \frac{n}{2} (7n + 3)
$$
This expression allows you to find the sum of the first n terms for any positive integer n.

Practical Examples and Problem-Solving

Example 1: Find the 10th term of the sequence

Using the explicit formula:
$$
a_{10} = 7(10) - 2 = 70 - 2 = 68
$$
The 10th term is 68.

Example 2: Find the sum of the first 15 terms

Using the sum formula:

$$
S_{15} = \frac{15}{2} (7 \times 15 + 3) = \frac{15}{2} (105 + 3) = \frac{15}{2} \times 108
$$

$$
S_{15} = 15 \times 54 = 810
$$

The sum of the first 15 terms is 810.

Example 3: Find the 20th term and verify the common difference

Calculate:

$$
a_{20} = 7(20) - 2 = 140 - 2 = 138
$$

Verify the common difference:

$$
a_{21} = 7(21) - 2 = 147 - 2 = 145
$$

Difference:

$$
a_{21} - a_{20} = 145 - 138 = 7
$$

Confirmed that the common difference is 7.

Conclusion

In summary, given the nth term of an arithmetic progression expressed as $7n - 2$, the common difference of this sequence is 7. This is derived by analyzing the difference between consecutive terms, which remains constant, confirming the sequence's arithmetic nature. Understanding how to extract the common difference from an explicit nth term formula is crucial for solving various problems related to sequences, such as finding specific terms, sum of terms, and analyzing sequence behavior.

Mastering these concepts allows students and mathematicians to handle a wide range of applications, from simple sequence calculations to complex series problems, with confidence and precision.

Key Takeaways

- The explicit nth term of the sequence is $a_n = 7n - 2$.
- The common difference d is obtained by calculating $a_{n+1} - a_n$, which equals 7.

- The sequence is an arithmetic progression with first term 5 and common difference 7.
- Formulas for nth term and sum of the first n terms can be directly applied for problem-solving.

By understanding these principles, you can analyze and work with similar sequences effectively, whether in academic contexts or real-world applications

Frequently Asked Questions

What is the general form of the nth term of the given AP?

The nth term of the AP is given by the expression 7n - 2.

How do you find the first term of the sequence when the nth term is 7n - 2?

Substitute n = 1 into the nth term: 7(1) - 2 = 5. So, the first term is 5.

What is the formula for the common difference in an AP given the nth term?

The common difference d is the difference between consecutive terms, which can be found by subtracting the (n-1)th term from the nth term.

How do we calculate the common difference for the sequence with nth term 7n - 2?

Subtract the (n-1)th term from the nth term: (7n - 2) - [7(n-1) - 2] = 7n - 2 - (7n - 7 - 2) = 7n - 2 - 7n + 7 + 2 = 7.

What is the common difference of the sequence 7n - 2?

The common difference is 7.

Is the sequence 7n - 2 an arithmetic progression?

Yes, because the difference between consecutive terms is constant (7), making it an arithmetic progression.

How would you verify the sequence's common difference if the sequence starts with the first term?

Calculate the difference between any two consecutive terms, for example, T2 - T1 = (72 - 2) - (71 - 2) = 12 - 5 = 7, confirming the common difference is 7.

Additional Resources

Nth Term Of An AP Is Given By An Expression 7n-2, Find The Common Difference Of This Sequence If Fisrt

Understanding arithmetic progressions (AP) is fundamental in the study of sequences and series in mathematics. They form the building blocks for more complex topics such as geometric progressions, algebraic series, and mathematical analysis. In this article, we delve into a specific AP characterized by its nth term expression: 7n - 2. Our goal is to determine the common difference of this sequence, assuming the sequence begins with its first term. This exploration combines theoretical insights with practical problem-solving approaches, making it accessible for learners and enthusiasts alike.

Deciphering the nth Term of an AP: The Expression 7n - 2

To understand the sequence, we first analyze the given nth term expression:

nth term, $a_n = 7n - 2$

This formula means that for any positive integer n, the term of the sequence can be calculated by substituting n into 7n - 2. For example:

- When n = 1: $a_1 = 7(1) - 2 = 7 - 2 = 5$
- When n = 2: $a_2 = 7(2) - 2 = 14 - 2 = 12$
- When n = 3: $a_3 = 7(3) - 2 = 21 - 2 = 19$

This sequence begins with 5, then 12, 19, and so forth.

Key observations:

- The sequence is linear, with each term increasing by a fixed amount.
- The expression 7n - 2 is linear in n, indicative of an arithmetic progression.

Why is this important?
Because the linear nature of the nth term confirms that the sequence is an AP, and the coefficient of n (which is 7) plays a crucial role in determining the common difference.

Understanding the Concept of Common Difference in an AP

Before calculating the common difference, let's clarify what it signifies:

Definition:
The common difference (denoted as d) in an arithmetic progression is the constant amount added to

each term to obtain the next term. Formally, for consecutive terms a_n and a_{n+1}:

$a_{n+1} = a_n + d$

Properties of the common difference:

- It remains constant throughout the sequence.
- Determines the progression's nature: increasing if $d > 0$, decreasing if $d < 0$.
- Is directly related to the nth term formula, especially when that formula is linear.

In our case, since the nth term is expressed as a linear function of n, the common difference can be directly derived from the coefficient of n.

Deriving the Common Difference from the nth Term Expression

Given the nth term formula:

$a_n = 7n - 2$

Let's examine the difference between consecutive terms:

$a_{n+1} - a_n = [7(n + 1) - 2] - [7n - 2]$

Calculating this:

$a_{n+1} - a_n = (7n + 7 - 2) - (7n - 2) = (7n + 5) - (7n - 2) = 7n + 5 - 7n + 2 = 7 + 2 = 9$

Result:
The difference between consecutive terms is 9.

This indicates that the sequence increases by 9 with each step, making the common difference:

$d = 9$

Conclusion:
The common difference of the sequence with nth term 7n - 2 is 9.

Verifying the Calculation Through Sequence Terms

To reinforce our conclusion, let's verify with actual terms:

n	$a_n = 7n - 2$	Difference with previous term
1	5	—
2	12	12 - 5 = 7

3	19	19 - 12 = 7
4	26	26 - 19 = 7

Wait — here, it appears that the difference between terms is 7, not 9. This suggests a discrepancy; let's examine the calculations carefully.

Correction:

Earlier, we computed $a_{n+1} - a_n$:

$a_{n+1} - a_n = (7(n + 1) - 2) - (7n - 2) = 7n + 7 - 2 - 7n + 2 = (7n + 5) - (7n - 2) = 7n + 5 - 7n + 2 = 7 + 2 = 9$

But this conflicts with the actual differences in sequence terms shown above, which are consistently 7.

Why the discrepancy?
Because in the initial calculation, the difference was computed as:

$a_{n+1} - a_n = (7(n + 1) - 2) - (7n - 2) = 7n + 7 - 2 - 7n + 2 = (7n + 5) - (7n - 2) = 7n + 5 - 7n + 2 = 7 + 2 = 9$

This calculation is flawed; the step where $7n + 7 - 2$ is simplified to $7n + 5$ is incorrect.

Let's do the calculation accurately:

$a_{n+1} = 7(n + 1) - 2 = 7n + 7 - 2 = 7n + 5$

$a_n = 7n - 2$

Subtracting:

a_{n+}_1 $- a_n = (7n + 5) - (7n - 2) = 7n + 5 - 7n + 2 = (7n - 7n) + (5 + 2) = 0 + 7 = 7$

Corrected Calculation:
The common difference is 7, matching the observed difference between sequence terms.

Therefore, the correct common difference of the sequence is 7.

Summary:
- The nth term is $a_n = 7n - 2$
- The difference between consecutive terms is $a_{n+1} - a_n = 7$
- Hence, the common difference $d = 7$

Implications of the Derived Common Difference

Having established that the common difference is 7, several implications follow:

- The sequence is an arithmetic progression with a consistent increase of 7 between terms.

- The first term, a_1, can be calculated directly: $a_1 = 7(1) - 2 = 5$.
- The sequence progresses as: 5, 12, 19, 26, 33, and so forth.

This consistent pattern has practical applications:

- Predicting future terms: For example, the 10th term $a_{10} = 7(10) - 2 = 70 - 2 = 68$.
- Calculating sums of terms: The sum of the first n terms, S_n, can be computed using the formula:

$S_n = n/2 (a_1 + a_n) = n/2 [a_1 + (a_1 + (n - 1)d)]$

- Modeling real-world scenarios: Such sequences appear in contexts like evenly spaced scheduling, financial calculations, and pattern analysis.

Conclusion: A Synthesis of Theory and Calculation

The journey to identifying the common difference of the sequence defined by the nth term 7n - 2 underscores the importance of meticulous calculation and understanding the properties of arithmetic progressions. Initially, the linear form of the nth term suggests a constant difference, and through precise computation, we confirm that this difference is indeed 7. This insight not only clarifies the structure of the sequence but also equips learners and professionals with the tools to analyze similar sequences efficiently.

In sum, whenever faced with an nth term of the form an = mn + c, the common difference d is simply the coefficient m. Recognizing this pattern simplifies the analysis of sequences and aids in broader mathematical problem-solving.

Final note:
Always verify calculations with actual sequence terms to ensure accuracy. As demonstrated, a small misstep can lead to incorrect conclusions, but careful checking helps maintain mathematical integrity.

In essence: The sequence generated by the nth term 7n - 2 has a common difference of 7, confirming its nature as an arithmetic progression with consistent, predictable growth.

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