On A Coordinate Plane, A Parabola Opens Up. It Goes Through (negative 8, Negative 2), Has A Vertex At

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Understanding the behavior and properties of parabolas on a coordinate plane is fundamental in algebra and coordinate geometry. When analyzing a parabola that opens upward, passes through a specific point such as (–8, –2), and has a known vertex, students and enthusiasts can develop a comprehensive understanding of its characteristics and equations. In this article, we will explore the key concepts related to such a parabola, including its vertex form, how to find its equation, and how the given point influences its shape and position.

What Is a Parabola and How Does It Open?

A parabola is a symmetric, U-shaped curve that is a graph of a quadratic function. Its general form can be written as:

- $y = ax^2 + bx + c$

where:
- $a \neq 0$,
- b and c are real numbers.

The direction in which a parabola opens depends on the sign of the coefficient a:

- If $a > 0$, the parabola opens upward.
- If $a < 0$, the parabola opens downward.

In our case, since the parabola opens up, it indicates that a is positive.

Understanding the Vertex of a Parabola

Definition of the Vertex

The vertex of a parabola is its highest or lowest point, depending on the direction it opens. For an upward-opening parabola, the vertex is the minimum point.

Vertex Form of a Parabola

The most useful form for analyzing and graphing parabolas with known vertices is the vertex form:

- $y = a(x - h)^2 + k$

where:
- (h, k) is the vertex,
- a determines the parabola's width and the direction it opens.

Knowing the vertex allows us to write the equation directly if we also find a.

Given Data: Passing Through (–8, –2) and Known Vertex

Suppose the parabola has a vertex at a point (h, k), which we need to find or is given elsewhere. Additionally, we know that the parabola passes through the point $(–8, –2)$.

The key steps are:

1. Identify the vertex: The problem states "Has a Vertex At," but the specific coordinates are missing here. For the purpose of this article, we will assume the vertex is at (h, k).

2. Use the vertex form equation: $y = a(x - h)^2 + k$.

3. Apply the point $(–8, –2)$ to find a once the vertex is known.

Let's explore scenarios depending on the vertex's position.

Determining the Vertex Coordinates

Since the problem states "Has A Vertex At" but does not specify the point, we will examine possible cases:

Case 1: Vertex at (h, k) with unknown coordinates

Case 2: Suppose the vertex is at $(–4, y_v)$

In many problems, the vertex's x-coordinate can be guessed or derived. For example, if the parabola passes through $(–8, –2)$ and the vertex is at $(–4, y_v)$, then:

- The axis of symmetry is at $x = –4$.

- The point $(–8, –2)$ is symmetric with respect to the axis $x = –4$. The symmetric point across the vertical line $x = –4$ is $(–0, –2)$.

- Since the parabola passes through both $(–8, –2)$ and the symmetric point $(–0, –2)$, and the vertex is at $(–4, y_v)$, the parabola's minimum or maximum is at $(–4, y_v)$.

Now, to find y_v, we consider the symmetry:

- The two points are equidistant from the vertex's x-coordinate $(–4)$. The points $(–8, –2)$ and $(–0, –2)$ are both 4 units away from $–4$.

- Since y-values are both $–2$, the parabola's vertex must have y-coordinate less than or equal to $–2$ for an upward-opening parabola passing through these points.

- The vertex is at $(–4, y_v)$, and the parabola passes through $(–8, –2)$.

Using the vertex form:

- $y = a(x + 4)^2 + y_v$.

Plugging in $(–8, –2)$:

$$
–2 = a(–8 + 4)^2 + y_v \\
–2 = a(–4)^2 + y_v \\
–2 = 16a + y_v
$$

Similarly, the point $(–0, –2)$:

$$
–2 = a(–0 + 4)^2 + y_v \\
–2 = 16a + y_v
$$

Both equations are identical, confirming symmetry.

From the first:

$$
16a + y_v = –2
$$

If we assume the vertex is at $(–4, y_v)$, then the parabola's vertex is at $(–4, y_v)$. To find y_v, we need more data points or assumptions.

Case 3: Vertices at $(–4, y_v)$ with a specific value

Suppose the vertex is at $(–4, 3)$. Then the parabola opens upward, passes through $(–8, –2)$, and has its vertex at $(–4, 3)$.

Using the vertex form:

\[

y = a(x + 4)^2 + 3
\]

Plug in \((–8, –2) \):

\[
–2 = a(–8 + 4)^2 + 3 \\
–2 = a(–4)^2 + 3 \\
–2 = 16a + 3 \\
16a = –5 \\
a = –\frac{5}{16}
\]

Since \(a \) is negative, the parabola opens downward, which contradicts our initial statement that it opens up. Therefore, the vertex cannot be at \((–4, 3) \).

Instead, if the parabola opens upward, \(a > 0 \), and \(y_v \) must be less than \(–2 \).

Suppose the vertex is at \((–4, –4) \). Repeating the process:

\[
–2 = a(–8 + 4)^2 + (–4) \\
–2 = 16a – 4 \\
16a = 2 \\
a = \frac{1}{8}
\]

Now, the parabola's equation:

\[
y = \frac{1}{8}(x + 4)^2 – 4
\]

This parabola opens upward (since \(a > 0 \)), passes through \((–8, –2) \), and has its vertex at \((–4, –4) \).

Summary:

- The vertex at \((–4, –4) \) with the point \((–8, –2) \) confirms an upward-opening parabola.
- Equation: \(y = \frac{1}{8}(x + 4)^2 – 4 \).

This example illustrates how the vertex's position influences the parabola's equation.

How To Find the Equation of the Parabola

Given the vertex \((h, k) \) and a point \((x_1, y_1) \) the parabola passes through, the steps are:

1. Write the vertex form: $y = a(x - h)^2 + k$.
2. Substitute (x_1, y_1) into the equation to solve for a: $y_1 = a(x_1 - h)^2 + k$.
3. Solve for a: $a = \frac{y_1 - k}{(x_1 - h)^2}$.
4. Write the complete equation with the calculated a.

Example:

Suppose the vertex is at $(-4, -4)$ and the point $(-8, -2)$ lies on the parabola.

- $a = \frac{-2 - (-4)}{(-8 - (-4))^2} = \frac{2}{(-4)^2} = \frac{2}{16} = \frac{1}{8}$.

- Equation: $y = \frac{1}{8}(x + 4)^2 - 4$.

Graphical Interpretation:

- The

Frequently Asked Questions

What is the general form of a parabola that opens upward on a coordinate plane?

The general form is $y = ax^2 + bx + c$, where $a > 0$ indicates the parabola opens upward.

How can I find the vertex of a parabola given its equation in vertex form?

In vertex form $y = a(x - h)^2 + k$, the vertex is at the point (h, k).

Given a point (-8, -2) on a parabola that opens upward, how can I determine its vertex?

Additional information, such as the parabola's equation or symmetry, is needed. If the point is not the vertex, you can use the symmetry property or find the vertex by completing the square or using derivatives if the equation is known.

What methods can be used to find the vertex of a

parabola passing through a known point?

Methods include converting the parabola to vertex form, completing the square, or using calculus to find the minimum point if the equation is known.

If a parabola passes through (-8, -2) and opens upward, what does this tell us about its shape?

It indicates the parabola is U-shaped and the point (-8, -2) lies somewhere on its curve, but without additional info, we can't determine the vertex exactly.

How does knowing a point on a parabola help in graphing it?

Knowing a point helps establish the shape and position of the parabola, especially when combined with the vertex or additional points for accurate plotting.

What additional information is needed to find the vertex of the parabola if it passes through (-8, -2)?

You need either the parabola's equation, another point, or the axis of symmetry to determine the vertex location.

Can you determine the vertex of a parabola with only one point and the knowledge that it opens upward?

No, you need at least one more piece of information, such as another point or the equation, to locate the vertex precisely.

What is the significance of the vertex in understanding the properties of a parabola?

The vertex represents the maximum or minimum point of the parabola, indicating its turning point and symmetry axis, essential for graphing and analyzing the parabola's behavior.

Additional Resources

Understanding the Behavior of a Parabola Opening Up on a Coordinate Plane

When exploring the fascinating world of quadratic functions and their graphs, one of the most fundamental shapes encountered is the parabola. Particularly, a parabola that opens upward possesses distinctive properties that can be analyzed through its vertex, intercepts, and other key features. In this discussion, we focus on a specific scenario: a parabola that opens upward, passes through the point \((-8, -2)\), and has a vertex at a certain point. We will thoroughly examine its characteristics, equations, and the implications of these features.

Defining the Parabola and Its Fundamental Properties

A parabola is the graph of a quadratic function of the form:

$$
y = ax^2 + bx + c
$$

where a, b, c are real numbers, and $a \neq 0$. The key features of a parabola include:

- Direction of Opening: Determined by the sign of a:
- If $a > 0$, the parabola opens upward.
- If $a < 0$, it opens downward.

- Vertex: The highest or lowest point on the parabola, depending on the direction it opens. For an upward-opening parabola, the vertex is the minimum point.

- Axis of Symmetry: A vertical line passing through the vertex, given by $x = -\frac{b}{2a}$.

- Intercepts:
- y-intercept: The point where the parabola crosses the y-axis ($x=0$).
- x-intercepts: The points where the parabola crosses the x-axis ($y=0$), if any exist.

Given Data and Its Significance

In our specific case, the parabola:

- Opens upward, implying $a > 0$.
- Passes through the point $(-8, -2)$.
- Has a vertex at an unspecified point (we will determine or assume it for illustration).

This information guides our understanding of the parabola's shape and positions.

Determining the Vertex of the Parabola

The vertex's position is central to understanding the parabola's overall shape. Since the problem statement says "Has a Vertex At" but doesn't specify the point, let's explore possible scenarios:

Scenario 1: The vertex is at $(-8, -2)$

If the vertex coincides with the point $(-8, -2)$, then:

- The parabola has its minimum at $(-8, -2)$.
- The parabola passes through $(-8, -2)$ and opens upward.

Implication: The vertex form of the parabola is:

$$
y = a(x + 8)^2 - 2
$$

where a is a positive real number (since the parabola opens upward).

Next Step: Find a using the point $(-8, -2)$. Since this point is on the parabola:

$$
-2 = a(-8 + 8)^2 - 2 \Rightarrow -2 = a(0)^2 - 2 \Rightarrow -2 = -2
$$

which holds for any a. So, this confirms the point lies at the vertex, but doesn't specify a. To determine a, we need an additional point.

Scenario 2: The vertex is at a different point

Suppose the vertex is at (h, k), distinct from $(-8, -2)$. Then, the parabola's equation in vertex form is:

$$
y = a(x - h)^2 + k
$$

Given that the parabola passes through $(-8, -2)$:

$$
-2 = a(-8 - h)^2 + k
$$

Without the explicit vertex point, we cannot determine a precisely. However, for illustration, let's assume the vertex is at $(-2, 4)$, a common choice that simplifies calculations.

Constructing the Equation of the Parabola in Vertex Form

Suppose the vertex is at $(-2, 4)$. Then, the parabola's equation is:

$$
y = a(x + 2)^2 + 4
$$

Because it opens upward, $a > 0$.

Using the point $(-8, -2)$ to find a:

$$
-2 = a(-8 + 2)^2 + 4
$$

Calculate the squared term:

$$
-8 + 2 = -6 \Rightarrow (-6)^2 = 36
$$

Substitute:

$$
-2 = 36a + 4
$$

Solve for a:

$$
36a = -6 \Rightarrow a = -\frac{6}{36} = -\frac{1}{6}
$$

Since $a = -\frac{1}{6}$, which is negative, the parabola opens downward, contradicting our initial assumption that it opens upward. Therefore, this choice of vertex at $(-2, 4)$ is incompatible with the upward opening.

Adjusting the vertex for an upward parabola:

Let's try a different vertex, say at $(-2, 4)$, but now ensure the parabola opens upward. Since the point $(-8, -2)$ is below the vertex, the parabola must be opening upward and passing through that point.

Rearranged, the equation is:

$$
-2 = a(-8 + 2)^2 + 4

\]
\[
-2 = 36a + 4
\]
\[
36a = -6
\]
\[
a = -\frac{1}{6}
\]

Again, negative. To get a positive \(a\), the \((-8, -2)\) point must be below the vertex for the parabola to open upward and include that point.

Suppose the vertex is at \((-8, 0)\). Then:

\[
y = a(x + 8)^2 + 0
\]

Using the point \((-8, -2)\):

\[
-2 = a(0)^2 + 0 \Rightarrow -2 = 0
\]

No, that doesn't work.

Alternatively, the vertex could be at \((-8, 2)\):

\[
y = a(x + 8)^2 + 2
\]

Plug in \((-8, -2)\):

\[
-2 = a(0)^2 + 2 \Rightarrow -2 = 2
\]

No, inconsistent.

Conclusion: Without additional specific information about the vertex, it's challenging to pin down the exact equation. However, the key takeaway is that the vertex's position relative to the point \((-8, -2)\) influences the parabola's shape and orientation.

General Form and Derivation of the Parabola Equation

Given the constraints, a common approach is to express the parabola in the vertex form:

$$
y = a(x - h)^2 + k
$$

where (h, k) is the vertex. To determine a, h, k, you need at least two points on the parabola.

Steps to derive the equation:

1. Identify the vertex: (h, k).
2. Use known point(s): For example, the point $(-8, -2)$.
3. Solve for a:

$$
-2 = a(-8 - h)^2 + k
$$

4. Express a:

$$
a = \frac{-2 - k}{(-8 - h)^2}
$$

5. Formulate the explicit equation once h, k are known.

Transforming to Standard Form and Graphical Interpretation

Once the vertex form is established, you can convert the equation into standard form:

$$
y = ax^2 + bx + c
$$

by expanding:

$$
y = a(x - h)^2 + k = a(x^2 - 2hx + h^2) + k
$$

\[
= a x^2 - 2a h x + a h^2 + k
\]

which corresponds to:

\[
a x^2 + b x + c
\]

with:

\[
b = -2 a h, \quad c = a h^2 + k
\]

Graphical implications:

- The vertex at \((h, k)\) acts as the parabola's minimum point.
- The parabola's width depends on \(|a|\). Larger \(|a|\) means a narrower parabola.
- The parabola's symmetry line is at \(x = h\).

Analyzing the Point \((-8, -2)\) and Its Role

This point's position relative to the vertex determines whether it lies:

- Above the parabola (if \(k > -2\)), or
- Below it (if \(k < -2\)).

Depending on the vertex's position, the parabola may or may not pass through \((-8, -2)\) for a given \(a\).

Example Calculation

On A Coordinate Plane A Parabola Opens Up It Goes Through Negative 8 Negative 2 Has A Vertex At

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