

One Number Is 6 More Than Twice Another. If Their Sum Is 51, Find The Numbers. Which Of The Following

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Understanding basic algebra problems is essential for students and learners aiming to improve their problem-solving skills. One such intriguing problem involves finding two numbers based on given conditions. In this article, we will explore how to solve the problem: "One number is 6 more than twice another. If their sum is 51, find the numbers." We will also discuss related concepts, methods to approach such problems, and step-by-step solutions to enhance your mathematical reasoning.

Understanding the Problem

Before diving into the solution, it's important to understand what the problem is asking.

Restating the problem

- One number is 6 more than twice another number.
- The sum of these two numbers is 51.
- The goal is to find the two numbers.

Breaking down the problem

The problem involves two unknown numbers, which we can denote as:

- Let the smaller number be x .
- Let the larger number be y .

Based on the description:

- The larger number (y) is 6 more than twice the smaller number (x).

Mathematically:

$$y = 2x + 6$$

- The sum of the two numbers is 51:

$$x + y = 51$$

Our task is to find the values of x and y .

Setting Up the Equations

To solve the problem, we need to translate the word problem into algebraic equations.

Equation 1: Relationship between the two numbers

From the statement: "One number is 6 more than twice another," we get:

$$y = 2x + 6$$

Equation 2: Sum of the numbers

From "their sum is 51," we have:

$$x + y = 51$$

Solving the Equations Step-by-Step

The primary goal is to find the value of x , then use it to find y .

Step 1: Substitute Equation 1 into Equation 2

Since $y = 2x + 6$, substitute this into the sum equation:

$$x + (2x + 6) = 51$$

Step 2: Simplify the equation

Combine like terms:

$$x + 2x + 6 = 51$$

$$3x + 6 = 51$$

Step 3: Isolate x

Subtract 6 from both sides:

$$3x = 51 - 6$$

$$3x = 45$$

Divide both sides by 3:

$$x = 15$$

Step 4: Find y

Use the value of x in the equation $y = 2x + 6$:

$$y = 2(15) + 6 = 30 + 6 = 36$$

Final Answer

- The smaller number (x) is 15.
- The larger number (y) is 36.

Additional Insights and Related Problems

Understanding how to approach this problem provides a foundation for solving various algebraic word problems involving relationships between numbers.

Common Variations of Such Problems

- If the problem states the difference between two numbers instead of their sum.
- If the numbers are related through multiplication or division.
- Problems where the sum or difference is a different value.

Examples of Related Problems

1. Two numbers where one is 8 less than three times the other, and their sum is 70. Find the numbers.

2. Find two numbers if one is 4 times the other, and their difference is 20.
3. If the sum of two numbers is 100, and one number is 10 more than twice the other, find the numbers.

Strategies for Solving Similar Algebraic Word Problems

Employing effective strategies can simplify complex problems.

1. Define Variables Clearly

- Assign variables to unknown quantities.
- Use descriptive variable names if possible.

2. Translate Words into Equations

- Break down the problem into parts.
- Write equations based on relationships described.

3. Substitute and Simplify

- Use substitution to reduce the number of variables.
- Simplify equations step-by-step.

4. Solve the Equations

- Use algebraic methods such as substitution or elimination.
- Check solutions by substituting back into original conditions.

5. Verify the Results

- Confirm that the solutions satisfy all conditions in the problem.
- Consider whether the solutions make sense contextually.

Practical Applications of Such Problems

Problems like these are not just academic exercises—they have real-world applications:

- Budgeting and financial planning.
- Engineering and design calculations.
- Coding and algorithm development.
- Logical reasoning and decision-making.

Conclusion

Solving problems involving relationships between numbers, such as "One number is 6 more than twice another," coupled with sum conditions, is fundamental to mastering algebra. By translating word problems into equations, systematically solving them, and verifying solutions, learners can build a strong foundation in mathematical reasoning. Remember, practice with similar problems enhances problem-solving skills and prepares you for more complex mathematical challenges.

Additional Resources

- Algebra textbooks and workbooks.
- Online algebra tutorials and interactive problem solvers.
- Math forums and discussion groups for collaborative learning.

By understanding the principles outlined in this article, students and enthusiasts can confidently approach and solve similar algebraic problems with clarity and precision.

Frequently Asked Questions

If one number is 6 more than twice another and their sum is 51, what are the two numbers?

Let the smaller number be x . Then the larger number is $2x + 6$. Setting up the equation: $x + (2x + 6) = 51$. Simplifying: $3x + 6 = 51$. Subtract 6: $3x = 45$. Divide by 3: $x = 15$. The larger number is $2(15) + 6 = 36$. So, the numbers are 15 and 36.

In the problem where one number is 6 more than twice another and their sum is 51, how do you set up the equations?

Let the smaller number be x . The larger number is $2x + 6$. The sum equation is $x + (2x + 6) = 51$. Simplify to $3x + 6 = 51$, then solve for x .

What is the importance of defining variables in solving this type of problem?

Defining variables helps to translate word problems into algebraic equations, making it easier to find the unknown numbers systematically.

Can this problem be solved using substitution or elimination methods?

Yes, since there are two expressions involving the same variables, substitution is used here by expressing one variable in terms of the other and solving accordingly.

What are common mistakes to avoid when solving this problem?

Common mistakes include misinterpreting the relationship between the numbers, incorrectly setting up the equations, or algebraic errors during simplification and solving.

How can you verify your solution in this type of problem?

Plug the found numbers back into the original conditions to check if their sum is 51 and if one number is 6 more than twice the other, ensuring the solution is correct.

Which option correctly represents the two numbers in this problem?

The correct pair of numbers is 15 and 36, where 36 is 6 more than twice 15, and their sum is 51.

Additional Resources

One Number Is 6 More Than Twice Another. If Their Sum Is 51, Find The Numbers. Which Of The Following?

Mathematics, often regarded as the language of the universe, offers countless intriguing problems that challenge our reasoning and problem-solving skills. Among these, algebraic word problems stand out for their ability to translate real-world situations into mathematical expressions, fostering critical thinking. One such problem that has captured the attention of students and educators alike involves two numbers with a specific relationship: one number is six more than twice another, and their combined total is 51. This problem not only tests one's understanding of algebraic concepts but also exemplifies the importance of translating word problems into manageable equations. In this comprehensive review, we delve into the details of this problem, exploring the methods to solve it, analyzing potential solutions, and discussing its significance in the broader context of mathematical education.

Understanding the Problem

Restating the Problem

The problem states: "One number is 6 more than twice another. If their sum is 51, find the numbers. Which of the following?"

Essentially, the problem involves two unknown numbers, say x and y , with a specific relationship between them. The key points are:

- One number (say x) is 6 more than twice the other number (y).
- The sum of the two numbers ($x + y$) is 51.
- The goal is to find the specific values of x and y .

This is a classic algebraic word problem that requires translating verbal descriptions into algebraic equations.

Identifying Variables and Relationships

To approach such problems systematically, it is vital to assign variables appropriately:

- Let x be the first number.
- Let y be the second number.

From the problem, we understand that x is related to y by the phrase "6 more than twice another," which can be mathematically expressed as:

$$x = 2y + 6$$

This equation captures the relationship between the two numbers.

Furthermore, their sum is given:

$$x + y = 51$$

This provides the second equation needed to form a system of equations.

Setting Up Equations and Mathematical Model

Formulating the Equations

Based on the problem statement, the two equations are:

1. $x = 2y + 6$ – from "One number is 6 more than twice another."
2. $x + y = 51$ – from "their sum is 51."

These two equations form a system that can be solved simultaneously.

The System of Equations

The system is:

```
\[
\begin{cases}
x = 2y + 6 \\
x + y = 51
\end{cases}
\]
```

The goal is to find the values of x and y that satisfy both equations.

Solving the System of Equations

Method 1: Substitution

The substitution method involves solving one equation for one variable and

substituting that expression into the other equation.

Step 1: Express x from the first equation:

$$\begin{aligned} & \backslash[\\ x &= 2y + 6 \\ & \backslash] \end{aligned}$$

Step 2: Substitute into the second equation:

$$\begin{aligned} & \backslash[\\ (2y + 6) + y &= 51 \\ & \backslash] \end{aligned}$$

Step 3: Combine like terms:

$$\begin{aligned} & \backslash[\\ 3y + 6 &= 51 \\ & \backslash] \end{aligned}$$

Step 4: Isolate y:

$$\begin{aligned} & \backslash[\\ 3y &= 51 - 6 \\ & \backslash] \\ & \backslash[\\ 3y &= 45 \\ & \backslash] \\ & \backslash[\\ y &= \frac{45}{3} = 15 \\ & \backslash] \end{aligned}$$

Step 5: Find x using the first equation:

$$\begin{aligned} & \backslash[\\ x &= 2(15) + 6 = 30 + 6 = 36 \\ & \backslash] \end{aligned}$$

Solution: The numbers are $x = 36$ and $y = 15$.

Method 2: Elimination

Alternatively, the elimination method involves manipulating the equations to cancel out one variable.

Step 1: Write the equations:

$$\begin{aligned} & \backslash[\\ & x = 2y + 6 \\ & \backslash] \\ & \backslash[\\ & x + y = 51 \\ & \backslash] \end{aligned}$$

Step 2: Subtract the second equation from the first (after appropriate adjustment), or substitute as before.

Since substitution is straightforward here, the previous method suffices. However, for completeness, eliminating variables in more complex systems follows similar steps.

Verifying the Solution

Verification ensures that the found values satisfy both original equations.

- Check the sum:

$$\begin{aligned} & \backslash[\\ & x + y = 36 + 15 = 51 \quad \checkmark \\ & \backslash] \end{aligned}$$

- Check the relationship:

$$\begin{aligned} & \backslash[\\ & x = 2y + 6 = 2(15) + 6 = 30 + 6 = 36 \quad \checkmark \\ & \backslash] \end{aligned}$$

Both conditions are satisfied, confirming the solution is correct.

Interpreting the Results and Choosing the Correct Option

Suppose the problem is part of a multiple-choice question with options such as:

1. 36 and 15
2. 30 and 21
3. 33 and 18
4. 40 and 11

Based on the calculations, the correct answer is option 1: 36 and 15.

This step emphasizes the importance of verifying whether the options align with the solution obtained through algebraic methods.

Broader Context and Educational Significance

The Role of Word Problems in Mathematics Education

Word problems like this serve as fundamental tools in mathematics education because they bridge the gap between abstract algebraic concepts and real-world applications. They compel students to:

- Read carefully and interpret verbal descriptions.
- Translate words into equations.
- Apply various algebraic methods to find solutions.
- Verify their answers for consistency.

Such problems enhance critical thinking, improve problem-solving skills, and foster a deeper understanding of algebra.

Common Challenges and Misconceptions

Students often encounter difficulties with word problems due to:

- Misreading or misinterpreting the language.
- Incorrectly assigning variables.
- Forming the wrong equations or relationships.
- Making arithmetic errors during calculations.

Addressing these challenges involves teaching strategies such as:

- Carefully analyzing the problem statement.
- Breaking down complex sentences into simpler components.
- Using diagrams or visual aids when applicable.
- Practicing with varied problems to build confidence.

Relevance in Standardized Testing and Competitive Exams

Problems of this nature frequently appear in standardized tests, such as SAT,

GRE, and various school-level assessments. Mastery of these problems is crucial for students aiming for high scores, as they often constitute a significant portion of the algebra section.

Advanced Analysis and Variations

Introducing Different Relationships

While the current problem involves "6 more than twice," variations could include:

- "One number is 4 less than three times another."
- "The difference between two numbers is 10, and their sum is 50."
- "One number is half of another plus 3."

Each variation requires similar techniques but different algebraic expressions, reinforcing understanding of translating words into equations.

Extending to Multiple Variables

More complex problems might involve three or more variables with multiple relationships, requiring systems of equations and methods like matrix algebra or substitution.

Applications in Real-Life Contexts

Such algebraic problems model real-life situations, including financial calculations, engineering problems, and scientific measurements, demonstrating the practical utility of algebra.

Conclusion: The Power of Algebra in Problem Solving

This exploration of the problem "One number is 6 more than twice another. If their sum is 51, find the numbers" underscores the importance of algebra as a fundamental tool in logical reasoning and problem-solving. By translating

verbal descriptions into algebraic equations, students can systematically approach and solve complex problems, fostering analytical skills essential across various disciplines. The solution—36 and 15—not only answers the question but also exemplifies the clarity and precision that algebra brings to understanding relationships and quantities. As educational tools, such problems serve to develop critical thinking, build confidence, and prepare learners for more advanced mathematical challenges that are ubiquitous in scientific and everyday contexts.

In summary, mastering these types of problems enhances mathematical literacy, enabling individuals to interpret and solve real-world problems effectively. Whether in academic settings, professional environments, or daily life, the ability to analyze relationships between quantities remains an invaluable skill—highlighted vividly through the solution of this classic algebraic problem.

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