One Pipe Can Fill A Pool In 6 Hours. The Second Pipe Can Drain The Pool In 18 Hours. How Long Will It

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Understanding the dynamics of filling and draining a swimming pool using multiple pipes involves interesting mathematical concepts and problem-solving skills. When one pipe fills a pool in 6 hours, and another drains it in 18 hours, a common question arises: how long will it take to fill the pool if both pipes are operated simultaneously? This problem combines rates, time, and the principle of combined work, making it an engaging scenario for learners and professionals alike. In this article, we will explore this problem in detail, breaking down the concepts step-by-step to understand how to determine the total time required for the pool to be filled under these conditions.

Understanding the Problem: Filling and Draining Pipes

What Are Filling and Draining Pipes?

In the context of pool management, a filling pipe is a conduit through which water flows into the pool, gradually increasing its volume. Conversely, a draining pipe removes water from the pool, decreasing its volume. When both pipes are open simultaneously, their effects combine to determine the net change in the pool's water level over time.

Given Data and Objective

The problem provides two key pieces of information:
- Filling Pipe: Can fill the pool in 6 hours.
- Draining Pipe: Can drain the pool in 18 hours.

The primary objective is to determine how long it will take to fill the pool when both pipes are operating at the same time, considering the drain rate of the second pipe.

Calculating the Rates of Filling and Draining

Understanding Rates in Terms of Pool Volume

To analyze the problem mathematically, it’s helpful to think in terms of rate of work—how much of the pool each pipe can fill or drain in one hour.

- Filling Pipe Rate: Since it fills the pool in 6 hours, it fills $\frac{1}{6}$ of the pool per hour.

- Draining Pipe Rate: Since it drains the pool in 18 hours, it drains $\frac{1}{18}$ of the pool per hour.

Expressing Rates Mathematically

Let’s denote:
- R_f = rate of filling pipe = $\frac{1}{6}$ (pool/hour)
- R_d = rate of draining pipe = $\frac{1}{18}$ (pool/hour)

When both pipes are open simultaneously, their combined rate R_c is:
$$R_c = R_f - R_d$$

Because the draining pipe reduces the volume of water in the pool, we subtract its rate from the filling rate to find the net rate of filling.

Calculating the Net Rate of Filling

Determining the Combined Rate

Using the rates provided:
$$R_c = \frac{1}{6} - \frac{1}{18}$$

To subtract these fractions, find a common denominator:
$$\frac{1}{6} = \frac{3}{18}$$
$$R_c = \frac{3}{18} - \frac{1}{18} = \frac{2}{18} = \frac{1}{9}$$

This means the combined rate of the pipes operating simultaneously is $\frac{1}{9}$ of the pool per hour.

Interpreting the Combined Rate

A combined rate of $\frac{1}{9}$ indicates that in one hour, the net amount of water added to the pool is $\frac{1}{9}$ of its total volume.

Therefore:
- It takes 9 hours to fill the entire pool when both pipes are open simultaneously.

Final Calculation: Total Time to Fill the Pool

Answer to the Main Question

Based on the rate calculations, the total time required to fill the pool when both pipes are open simultaneously is 9 hours.

Summary of Calculation Steps

- Determine the individual rates of filling and draining: $\frac{1}{6}$ and $\frac{1}{18}$ pool/hour respectively.
- Calculate the combined rate: $\frac{1}{6} - \frac{1}{18} = \frac{1}{9}$ pool/hour.
- Find the total time to fill the pool: reciprocal of combined rate = 9 hours.

Additional Insights and Variations

What If The Drain Pipe Drains Faster?

Suppose the drain pipe could empty the pool faster, say in 12 hours. Then its rate would be $\frac{1}{12}$ pool/hour, and the combined rate would be:

$$
\frac{1}{6} - \frac{1}{12} = \frac{2}{12} - \frac{1}{12} = \frac{1}{12}
$$

In this case, it would take 12 hours to fill the pool with both pipes operating simultaneously.

What If Both Pipes Work Independently?

If only the filling pipe is working, it takes 6 hours. If only the draining pipe is working, it would empty the pool in 18 hours. The combined operation significantly influences the total filling time.

Practical Applications and Tips for Solving Similar Problems

Key Concepts to Remember

- Convert rates to fractional parts of the pool per hour.
- Use the principle of combined work: when two processes work together, their rates add or subtract depending on whether they are filling or draining.
- The total time is the reciprocal of the combined rate.

Step-by-Step Approach for Similar Problems

1. Identify the individual rates of the processes involved.

2. Express these rates as fractions of the work done per unit time.

3. Determine whether the processes are additive or subtractive.

4. Calculate the combined rate accordingly.

5. Find the total time by taking the reciprocal of the combined rate.

Conclusion

The problem of how long it takes to fill a pool with a filling pipe and drain pipe operating simultaneously is a classic example of work rate problems. By converting the given times into rates, calculating the net rate, and then finding the reciprocal, we determine that the pool will be filled in 9 hours when both pipes are working together under the specified conditions. Mastering these types of problems enhances problem-solving skills and deepens understanding of rates, ratios, and algebraic concepts.

Remember:
- The key to solving such problems lies in understanding the individual rates and how they combine.
- Always check whether the processes add or subtract to get the net rate.
- Practice with different scenarios to strengthen your grasp of combined work

problems involving filling and draining processes.

Frequently Asked Questions

If one pipe fills a pool in 6 hours and another drains it in 18 hours, how long will it take to fill the pool if both pipes are open simultaneously?

When both pipes are open, the net filling rate is 1/6 - 1/18 = (3/18 - 1/18) = 2/18 = 1/9. Therefore, the pool will be filled in 9 hours.

What is the combined rate of filling and draining when both pipes are open?

The combined rate is the filling rate minus the draining rate: 1/6 (fill rate) minus 1/18 (drain rate), which equals 1/9 of the pool per hour.

If the pipes are opened together, how much of the pool is filled in 3 hours?

At a combined rate of 1/9 per hour, in 3 hours, 3 1/9 = 1/3 of the pool will be filled.

How long would it take to empty the pool completely if both pipes are open starting with a full pool?

Since the filling pipe adds water and the draining pipe removes it, with both open, the pool will never empty completely unless initially empty; instead, it reaches a steady state of filling in 9 hours if starting empty.

What is the significance of the times 6 hours and 18 hours in this problem?

The 6-hour time indicates how quickly the filling pipe can fill the pool alone, while the 18-hour time shows how slowly the draining pipe empties it, which are used to determine the net filling time when both are open.

Can the pool ever be completely filled if both pipes are open at the same time?

Yes, the pool can be completely filled starting from empty when both pipes are open, and it will take 9 hours to fill completely, since the net rate is 1/9 per hour.

Additional Resources

One Pipe Can Fill A Pool In 6 Hours. The Second Pipe Can Drain The Pool In 18 Hours. How Long Will It Take To Fill The Pool When Both Pipes Are Open?

Introduction

In the world of fluid mechanics and practical problem-solving, understanding how different pipes work together to fill or drain a pool is a classic scenario that blends physics with real-world applications. Imagine a situation where one pipe is tasked with filling a swimming pool at a steady rate, while another pipe simultaneously drains water from the pool at a different rate. The question naturally arises: how long will it take to fill the pool when both pipes are operating simultaneously? This problem, often encountered in engineering and everyday life, underscores the importance of rate calculations and the interplay between filling and draining processes.

In this article, we will explore this problem in detail, breaking down the concepts step-by-step. We will analyze the individual rates of both pipes, combine these rates to find the net filling rate, and then determine the total time required. Through clear explanations and methodical calculations, we will provide a comprehensive understanding of this classic problem.

Understanding the Problem

Let’s first restate the problem clearly:

- Pipe A fills the pool in 6 hours.
- Pipe B drains the pool in 18 hours.
- Both pipes are opened at the same time.
- The goal is to find out how long it will take to fill the pool under these conditions.

This problem involves rates of work—how much of the pool each pipe can complete per unit time—and how these rates combine when pipes are operating simultaneously, with one filling and the other draining.

Breaking Down the Individual Rates

To analyze the problem systematically, it’s essential to understand the individual rates at which each pipe operates.

Rate of Pipe A (Filling Rate)

Since Pipe A fills the entire pool in 6 hours, its rate of filling is:

\[
\text{Rate of Pipe A} = \frac{1 \text{ pool}}{6 \text{ hours}} = \frac{1}{6}
\text{ pool per hour}
\]

This means that in one hour, Pipe A adds \(\frac{1}{6}\) of the pool's
volume.

Rate of Pipe B (Draining Rate)

Similarly, Pipe B drains the pool in 18 hours, so its rate of draining is:

\[
\text{Rate of Pipe B} = \frac{1 \text{ pool}}{18 \text{ hours}} =
\frac{1}{18} \text{ pool per hour}
\]

This indicates that in one hour, Pipe B removes \(\frac{1}{18}\) of the
pool's volume.

Combining the Rates: Net Filling Rate

When both pipes are open simultaneously, the total change in the pool's
volume per hour depends on whether the pipes are filling or draining. Since
Pipe B drains the pool, its rate acts in the opposite direction to the
filling rate of Pipe A.

The net rate when both pipes are open is:

\[
\text{Net Rate} = \text{Filling Rate} - \text{Draining Rate}
\]

Substituting the values:

\[
\text{Net Rate} = \frac{1}{6} - \frac{1}{18}
\]

To combine these fractions, find a common denominator:

\[
\frac{1}{6} = \frac{3}{18}
\]

So,

\[
\text{Net Rate} = \frac{3}{18} - \frac{1}{18} = \frac{2}{18} = \frac{1}{9}

\]

This means that with both pipes open, the pool fills at a rate of $\frac{1}{9}$ of the pool per hour.

Calculating the Total Time to Fill the Pool

Given that the net filling rate is $\frac{1}{9}$ of the pool per hour, we can now determine the total time needed to fill the pool completely:

$$
\text{Time} = \frac{\text{Total volume of pool}}{\text{Net rate}} = \frac{1}{\frac{1}{9}} = 9 \text{ hours}
$$

Therefore, it will take 9 hours to fill the pool when both pipes are open simultaneously.

Important Considerations

While the calculations above provide a straightforward answer, it’s valuable to consider some additional factors:

- Initial Conditions: The problem assumes the pool starts empty, and both pipes are opened at the same time.
- Pipe Efficiency: The rates are consistent, and the pipes operate continuously without interruption.
- Potential Variations: If the rates change due to pressure, pipe diameter, or other factors, the calculation would need adjustment.
- Practical Implications: In real-world scenarios, pipes might not operate perfectly at constant rates, but for theoretical problems, these assumptions are standard.

Broader Applications and Related Problems

This problem serves as a foundation for understanding more complex rate-based scenarios in engineering and everyday life. Examples include:

- Tank Filling and Draining: Similar calculations are used when managing water supplies, fuel tanks, or chemical reactors.
- Work Rate Problems: The concept extends to tasks like completing projects, cleaning, or manufacturing processes where multiple agents work simultaneously.
- Flow Rate Optimization: Engineers optimize pipe sizes and flow rates to achieve desired filling or draining times efficiently.

Variations and Practice Problems

To deepen understanding, consider the following variations:

1. What if the pipes are operated sequentially? How long does each take individually, and what is the total combined time?
2. If the draining pipe is faster (say, drains in 12 hours), how does that affect the total filling time?
3. What if the pipes are operated intermittently rather than simultaneously? How would the total time change?

Final Thoughts

The problem of determining how long it takes to fill a pool with one pipe while another drains it is a classic example of rate problems in mathematics and physics. By breaking down the process into individual rates, combining them carefully, and understanding the net effect, we arrive at a clear, logical answer. In this case, operating both pipes together results in a net filling rate of \(\frac{1}{9}\) of the pool per hour, leading to a total fill time of 9 hours.

This problem not only illustrates the fundamental principles of rate and work but also demonstrates how mathematical reasoning can inform practical decision-making in real-world scenarios. Whether managing water systems, manufacturing processes, or project timelines, understanding how to combine rates effectively is an essential skill that finds application across numerous fields.

In summary:

- Pipe A fills the pool in 6 hours (\(\frac{1}{6}\) per hour).
- Pipe B drains the pool in 18 hours (\(\frac{1}{18}\) per hour).
- When both are open, the net fill rate is \(\frac{1}{9}\) per hour.
- Total time to fill the pool with both pipes operating simultaneously is 9 hours.

Understanding these basic principles provides a solid foundation for tackling more complex rate problems in science, engineering, and everyday life.

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