Parallelogram ABCD With Vertices A(6,8) B(10,9) C(9,3) And D(5,2): 90 Counterclockwise Rotation About

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Understanding how geometric figures like parallelograms transform under rotations is essential in both pure and applied mathematics. In this comprehensive guide, we explore the specific case of parallelogram ABCD with vertices A(6,8), B(10,9), C(9,3), and D(5,2), focusing on performing a 90-degree counterclockwise rotation about a specified point—most often the origin or a vertex. This process involves applying coordinate transformation principles, vector operations, and geometric reasoning, which are crucial skills in coordinate geometry, computer graphics, physics, and engineering.

Introduction to Parallelogram ABCD and Rotation Concepts

Vertices of Parallelogram ABCD

The given vertices define a parallelogram in the Cartesian plane:

- A(6,8)
- B(10,9)
- C(9,3)

- D(5,2)

Understanding the shape’s properties, such as side lengths, slopes, and diagonals, provides a foundation for transformations.

Rotation in Coordinate Geometry

Rotation is a fundamental transformation that turns figures around a fixed point by a specified angle:

- Counterclockwise rotations are considered positive.
- The rotation about a point involves translating the figure so that the point becomes the origin, rotating, then translating back.

A 90-degree rotation is a common case because it results in a simple coordinate transformation.

Determining the Center of Rotation

Common Choices for Rotation Center

The center of rotation can be:

1. The origin (0,0)
2. A specific vertex of the parallelogram (e.g., point A)
3. The centroid of the figure

For this guide, we focus on rotating around the origin, which simplifies calculations and is a typical scenario.

Calculating the Rotation About the Origin

The process involves:

1. Translating the vertices so that the origin is the pivot point.

2. Applying the rotation transformation to each translated coordinate.

3. Translating back to the original coordinate system if necessary.

Mathematical Foundations of 90-Degree Counterclockwise Rotation

Rotation Matrix for 90 Degrees Counterclockwise

The rotation of a point (x, y) around the origin by 90 degrees counterclockwise can be expressed as:

\[

\begin{bmatrix}

x' \\

y' \\

\end{bmatrix}

=

\begin{bmatrix}

0 & -1 \\

1 & 0 \\

\end{bmatrix}

\begin{bmatrix}

x \\

y \\

\end{bmatrix}

\]

which results in:

\[

x' = -y,\quad y' = x

\]

Step-by-Step Transformation

For each point:

1. Identify the original coordinates (x, y).

2. Apply the rotation matrix to find (x', y').

Note: If rotating about a point other than the origin, translate the point so that the pivot is at the origin, perform the rotation, then translate back.

Applying the Rotation to Parallelogram ABCD

Rotation About the Origin

Step 1: List the vertices

Vertex	Coordinates (x, y)
A	(6,8)
B	(10,9)
C	(9,3)
D	(5,2)

Step 2: Apply the rotation to each vertex

Using the rotation formulas:

- For A(6,8):

$$
x' = -8 = -8,\quad y' = 6 = 6
$$

So, A' = (-8, 6)

- For B(10,9):

$$
x' = -9 = -9,\quad y' = 10 = 10
$$

So, B' = (-9, 10)

- For C(9,3):

$$
x' = -3 = -3,\quad y' = 9 = 9
$$

So, C' = (-3, 9)

- For D(5,2):

$$
x' = -2 = -2,\quad y' = 5 = 5
$$

So, D' = (-2, 5)

Step 3: New vertices after rotation

Vertex	Coordinates (x', y')
A'	(-8, 6)
B'	(-9, 10)
C'	(-3, 9)
D'	(-2, 5)

Visualizing the Result

Plotting the rotated vertices helps visualize the new position of the parallelogram. It would appear rotated 90 degrees counterclockwise around the origin, with the shape maintaining its parallelogram properties.

Alternative Rotation Center: About a Vertex

If the rotation is about a specific vertex, such as point A(6,8), the process involves:

1. Translating all points so that A becomes the origin.

2. Applying the rotation to the translated points.

3. Translating back to the original coordinate system.

Example: Rotation about A(6,8):

- Translate points:

- B: (10,9) becomes (10-6, 9-8) = (4,1)
- C: (9,3) becomes (3,-5)
- D: (5,2) becomes (-1,-6)

- Rotate translated points:

Using the rotation matrix:

- For (x, y):

$$
x' = -y,\quad y' = x
$$

- B(4,1):

$$
x' = -1 = -1,\quad y' = 4 = 4
$$

Translated back: (6 + (-1), 8 + 4) = (5, 12)

- C(3,-5):

\[

x' = 5,\quad y' = 3

\]

Translated back: (6 + 5, 8 + 3) = (11, 11)

- D(-1,-6):

\[

x' = 6,\quad y' = -1

\]

Translated back: (6 + 6, 8 + (-1)) = (12, 7)

- Resulting vertices:

- A remains at (6,8).
- B' at (5,12).
- C' at (11,11).
- D' at (12,7).

This method allows for flexible rotation about any point, which is especially useful in geometric constructions and computer graphics.

Practical Applications of Rotating Parallelograms

In Computer Graphics

Rotations are fundamental in rendering objects, animations, and transformations in digital environments. The principles covered here underpin algorithms in graphics programming.

In Engineering and Physics

Understanding rotations aids in analyzing mechanical linkages, structural deformations, and motion dynamics.

In Geometry and Mathematics Education

Visualizing and performing rotations reinforce understanding of coordinate systems, transformation matrices, and geometric properties.

Conclusion: Mastering Rotation Transformations

Rotating a parallelogram about a point involves a systematic approach:

1. Identify the center of rotation and vertices.
2. Translate the figure if necessary.
3. Apply the rotation matrix for 90 degrees counterclockwise.
4. Translate back to the original coordinate system.

The example of parallelogram ABCD with vertices A(6,8), B(10,9), C(9,3), and D(5,2) demonstrates these steps clearly. Whether rotating about the origin or a specific vertex, the process enables precise and predictable transformations, essential for various scientific and artistic applications.

By mastering these techniques, students and professionals can confidently perform and analyze rotations, enriching their understanding of geometric transformations and their real-world relevance.

Frequently Asked Questions

What are the coordinates of parallelogram ABCD after a 90° counterclockwise rotation about the origin?

The rotated vertices are A'(-8, 6), B'(-9, 10), C'(-3, 9), D'(-2, 5).

How do you find the image of a point after a 90° counterclockwise rotation about the origin?

To rotate a point (x, y) 90° counterclockwise about the origin, replace it with (-y, x).

What is the process to rotate a parallelogram about a point other than the origin?

First, translate the figure so the pivot point becomes the origin, perform the rotation, then translate back to the original position.

Are the side lengths of the parallelogram preserved after a 90° rotation?

Yes, rotations are rigid transformations, so the side lengths and angles of the parallelogram remain unchanged.

How can I verify that the rotated figure is still a parallelogram?

Check that the vectors representing opposite sides are equal or that the midpoints of diagonals coincide after rotation.

What is the significance of the 90° counterclockwise rotation in coordinate geometry?

It demonstrates how figures can be transformed while preserving shape and size, useful for symmetry and problem-solving in coordinate geometry.

Can the rotation be performed about any arbitrary point, and how?

Yes, to rotate about a point other than the origin, translate the figure so that the point becomes the origin, perform the rotation, then translate back.

Additional Resources

Parallelogram ABCD With Vertices A(6,8), B(10,9), C(9,3), and D(5,2): 90° Counterclockwise Rotation About a Point

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Introduction to Parallelogram ABCD and Transformation Concepts

Understanding geometric transformations is fundamental in coordinate geometry, especially when analyzing shapes like parallelograms. A parallelogram ABCD, with vertices at A(6,8), B(10,9), C(9,3), and D(5,2), offers a rich context for exploring rotation about a point.

In this detailed review, we focus on the 90-degree counterclockwise rotation of parallelogram ABCD about a specified point—commonly the origin or a specific vertex—and examine the implications on the shape's orientation, position, and properties. We will analyze the process step-by-step, including the mathematical formulas, geometric intuition, and potential applications.

Understanding the Parallelogram ABCD

Vertices and Coordinates

The given vertices of the parallelogram are:

- A(6,8)
- B(10,9)
- C(9,3)
- D(5,2)

Plotting these points on the coordinate plane reveals the shape's structure. The shape’s properties are:

- Opposite sides are parallel and equal in length.
- The diagonals bisect each other.

Verifying the Parallelogram Properties

Before discussing rotation, it's essential to confirm the shape's properties:

- Check if ABCD is a parallelogram:

1. Calculate vectors for sides:
- AB = (10 - 6, 9 - 8) = (4, 1)
- AD = (5 - 6, 2 - 8) = (-1, -6)

- BC = (9 - 10, 3 - 9) = (-1, -6)

- DC = (5 - 9, 2 - 3) = (-4, -1)

2. Compare opposite sides:

- AB and DC are (-4, -1) and (4, 1) but note the signs; they are negatives of each other, indicating parallel and equal in length.
- AD and BC are (-1, -6) and (-1, -6), identical vectors.

- Since AB and DC are negatives of each other, and AD and BC are identical, the shape is confirmed to be a parallelogram.

Summary: ABCD is a parallelogram with sides AB parallel to DC and AD parallel to BC.

Mathematical Foundations of Rotation About a Point

Rotation in Coordinate Geometry

Rotation is a rigid transformation that turns a figure around a fixed point called the center of rotation. For a rotation by an angle □:

- Every point (x, y) in the figure moves to a new position (x', y').
- The transformation preserves distances and angles.

Rotation about the origin (0,0):

\[

\begin{cases}

x' = x \cos \theta - y \sin \theta \\

y' = x \sin \theta + y \cos \theta

\end{cases}

\]

Rotation about a point (h, k):

- The process involves translating the point so that (h, k) becomes the origin, performing the rotation, and then translating back.

\[

\begin{cases}

x' = h + (x - h) \cos \theta - (y - k) \sin \theta \\

y' = k + (x - h) \sin \theta + (y - k) \cos \theta

\end{cases}

\]

Performing a 90° Counterclockwise Rotation of ABCD

Choosing the Center of Rotation

The rotation can be about:

1. The origin (0,0)
2. A vertex (e.g., point A or B)
3. The centroid of the parallelogram
4. Any arbitrary point in the plane

For this analysis, we will consider two common centers:

- Rotation about the origin (0,0)
- Rotation about vertex A(6,8)

The choice significantly affects the resulting shape and position.

Case 1: Rotation About the Origin (0,0)

Step 1: Identify the rotation angle

- θ = 90° (counterclockwise)
- $\cos 90^\circ = 0$, $\sin 90^\circ = 1$

Step 2: Apply the rotation formulas to each vertex

- For each point (x, y), compute:

$$
x' = x \times 0 - y \times 1 = - y
$$

$$
y' = x \times 1 + y \times 0 = x
$$

Step 3: Calculate the rotated vertices

Original Vertex	Coordinates (x, y)	Rotated Coordinates (x', y')
A	(6, 8)	(-8, 6)

B	(10, 9)	(-9, 10)
C	(9, 3)	(-3, 9)
D	(5, 2)	(-2, 5)

Step 4: Plot and analyze the new shape

- The rotated parallelogram has vertices at:

- A'(-8, 6)
- B'(-9, 10)
- C'(-3, 9)
- D'(-2, 5)

- The figure maintains its parallelogram properties, with opposite sides parallel and equal in length, but it is now rotated 90° counterclockwise about the origin.

Implications:

- The shape's orientation has changed; what was previously a certain angle now appears rotated.
- The shape's position has shifted from the original coordinates, now located primarily in the left-upper quadrant.

Case 2: Rotation About Vertex A(6,8)

Step 1: Translate points so that A(6,8) becomes the origin

- For each point, subtract (6,8):

Point	Translated Coordinates (x - 6, y - 8)
A	(0, 0)
B	(10 - 6, 9 - 8) = (4, 1)
C	(9 - 6, 3 - 8) = (3, -5)
D	(5 - 6, 2 - 8) = (-1, -6)

Step 2: Rotate these points about the origin by 90° CCW

- Using the same rotation formulas:

$$
x' = - y
$$

$$
y' = x
$$

- Applying:

Translated Point	Calculation	Result (x', y')
(0, 0)	(0, 0)	(0, 0)
(4, 1)	(-1, 4)	(-1, 4)
(3, -5)	(5, 3)	(-5, 3)
(-1, -6)	(6, -1)	(6, -1)

Step 3: Translate back to the original coordinate system

- Add back the point A(6,8):

Point	Final Coordinates (x + 6, y + 8)
A'	(0 + 6, 0 + 8) = (6, 8)
B'	(-1 + 6, 4 + 8) = (5, 12)
C'	(-5 + 6, 3 + 8) = (1, 11)
D'	(6 + 6, -1 + 8) = (12, 7)

Analysis:

- The rotated vertices are:

- A'(6,8) (original point)
- B'(5,12)
- C'(1,11)
- D'(12,7)

- The shape has undergone a 90° counterclockwise rotation about vertex A, resulting in a shift that preserves the shape's parallelogram properties but changes its orientation and position.

Properties and Implications of the Rotation

Shape Preservation

- Congruence: The rotated figure remains congruent to the original, with identical side lengths and angles.
- Parallelism: Opposite sides stay parallel post-rotation.
- Area: The area

Parallelogram Abcd With Vertices A 6 8 B 10 9 C 9 3 And D 5 2 90 Counterclockwise Rotation About

Parallelogram Abcd With Vertices A 6 8 B 10 9 C 9 3 And D 5 2 90 Counterclockwise Rotation About

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