Perform A Firt Derivative Tet On The Function F(x) =4x55x440x3-3; [3,4]. A. Locate The Critical Point

Perform A First Derivative Test On The Function F(x) = 4x^5 - 5x^4 + 40x^3 - 3; [3,4]. A. Locate The Critical Point

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Introduction

Analyzing the behavior of a function, particularly identifying its critical points, is fundamental in calculus. Critical points are the x-values where the first derivative of the function is zero or undefined, and they often correspond to local maxima, minima, or points of inflection. This article provides a comprehensive step-by-step guide on how to perform the first derivative test on the function $F(x) = 4x^5 - 5x^4 + 40x^3 - 3$ within the interval $[3, 4]$. The goal is to locate the critical points and understand their significance in the context of the function's behavior.

Understanding the Function and Its Domain

Before delving into derivatives, it's important to understand the given function and the domain:

- Function: $F(x) = 4x^5 - 5x^4 + 40x^3 - 3$

- Interval of interest: $[3, 4]$

This polynomial function is continuous and differentiable everywhere, especially within the interval $[3, 4]$. The first step is to find the critical points where the first derivative equals zero or does not exist in this interval.

Step 1: Compute the First Derivative of

$F(x)$

The first derivative $F'(x)$ provides the slope of the tangent to the function at any point x:

$$
F'(x) = \frac{d}{dx} \left(4x^5 - 5x^4 + 40x^3 - 3 \right)
$$

Applying power rule:

$$
F'(x) = 4 \times 5x^{4} - 5 \times 4x^{3} + 40 \times 3x^{2} + 0
$$

Simplify each term:

$$
F'(x) = 20x^{4} - 20x^{3} + 120x^{2}
$$

Thus, the derivative simplifies to:

$$
F'(x) = 20x^{4} - 20x^{3} + 120x^{2}
$$

Step 2: Find Critical Points by Setting $F'(x) = 0$

To locate critical points, solve:

$$
F'(x) = 0
$$

which becomes:

$$
20x^{4} - 20x^{3} + 120x^{2} = 0
$$

Factor out the common term:

\[

$$
20x^{2} (x^{2} - x + 6) = 0
$$

This gives two factors to analyze:

$$
20x^{2} = 0 \quad \text{and} \quad x^{2} - x + 6 = 0
$$

Solving for x: The Critical Points

- First factor:

$$
20x^{2} = 0 \implies x^{2} = 0 \implies x = 0
$$

- Second factor:

$$
x^{2} - x + 6 = 0
$$

Use the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^{2} - 4ac}}{2a}
$$

where $a = 1$, $b = -1$, $c = 6$:

$$
x = \frac{1 \pm \sqrt{(-1)^2 - 4 \times 1 \times 6}}{2} = \frac{1 \pm \sqrt{1 - 24}}{2} = \frac{1 \pm \sqrt{-23}}{2}
$$

Since the discriminant is negative (-23), the quadratic has no real solutions. Therefore, the only real critical point within the domain is at:

$$
x = 0
$$

Step 3: Determine if Critical Points Lie Within the Interval [3, 4]

Our interval of interest is $[3, 4]$. The critical point at $x=0$ lies outside this interval. Therefore, there are no critical points within $[3, 4]$ based on the first derivative being zero or undefined.

Step 4: Analyze the Behavior of $F'(x)$ on $[3, 4]$

Even though no critical points are in $[3, 4]$, the function's increasing or decreasing behavior within this interval can be determined by analyzing the sign of $F'(x)$.

Evaluating $F'(x)$ at Endpoints and Sample Points

- At $x=3$:

$$
F'(3) = 20(3)^4 - 20(3)^3 + 120(3)^2
$$

Calculate each term:

$$
20 \times 81 = 1620
$$
$$
20 \times 27 = 540
$$
$$
120 \times 9 = 1080
$$

Now:

$$
F'(3) = 1620 - 540 + 1080 = (1620 - 540) + 1080 = 1080 + 1080 = 2160
$$

- At $x=4$:

$$
F'(4) = 20(4)^4 - 20(4)^3 + 120(4)^2
$$

Calculate each term:

$$
4^4 = 256
$$
$$
4^3 = 64
$$
$$
4^2 = 16
$$

So:

$$
F'(4) = 20 \times 256 - 20 \times 64 + 120 \times 16 = 5120 - 1280 + 1920 = (5120 - 1280) + 1920 = 3840 + 1920 = 5760
$$

- Sample point $x=3.5$:

$$
F'(3.5) = 20(3.5)^4 - 20(3.5)^3 + 120(3.5)^2
$$

Calculate:

$$
3.5^2 = 12.25
$$
$$
3.5^3 = 3.5 \times 12.25 = 42.875
$$
$$
3.5^4 = 3.5 \times 42.875 = 150.0625
$$

Plug in:

$$
F'(3.5) = 20 \times 150.0625 - 20 \times 42.875 + 120 \times 12.25
$$
$$
= 3001.25 - 857.5 + 1470
$$
$$
= (3001.25 - 857.5) + 1470 = 2143.75 + 1470 = 3613.75
$$

\]

Observation:

- $F'(x)$ is positive at $x=3$, $x=3.5$, and $x=4$.

- Since $F'(x) > 0$ throughout the interval $[3, 4]$, the function $F(x)$ is increasing on $[3, 4]$.

Step 5: Conclusion on Critical Points and Function Behavior

- Critical points in $[3, 4]$: None, since the only critical point $x=0$ is outside the interval.

- Behavior of $F(x)$: Since the derivative is positive throughout $[3, 4]$, the function is strictly increasing there.

- Implication: No local maxima or minima occur strictly within the interval $[3, 4]$. The function reaches its minimum at the left endpoint $x=3$ and maximum at the right endpoint $x=4$.

Summary of Findings

- The first derivative of $F(x)$ is $F'(x) = 20x^{4} - 20x^{3} + 120x^{2}$.
- The only real critical point in the entire real line is at $x=0$, outside the interval $[3, 4]$.
- Within $[3, 4]$, $F'(x) > 0$, so the function is increasing.
- The function does not have any critical points within $[3, 4]$, but its increasing nature affects how maxima and minima are interpreted at the endpoints.

Additional Considerations and Practical Applications

Understanding the monotonicity of $F(x)$ helps in numerous practical scenarios, such as optimization problems, where knowing whether a function increases or decreases within an interval guides decision-making. In calculus, the first derivative test confirms the absence of

Frequently Asked Questions

What is the first derivative of the function f(x) = 4x^5 - 5x^4 + 0x^3 - 3?

The first derivative f'(x) is given by differentiating each term: f'(x) = 20x^4 - 20x^3 + 0x^2, which simplifies to f'(x) = 20x^4 - 20x^3.

How do you find the critical points of the function f(x) = 4x^5 - 5x^4 - 3 on the interval [3,4]?

Critical points occur where the first derivative f'(x) = 20x^4 - 20x^3 equals zero or is undefined. Since the derivative is polynomial, set f'(x) = 0 and solve for x within [3,4].

What are the critical points of f(x) = 4x^5 - 5x^4 - 3 on the interval [3,4]?

Set f'(x) = 20x^4 - 20x^3 = 0. Factoring out 20x^3 gives 20x^3(x - 1) = 0. Solutions are x=0 and x=1, but neither lies in [3,4], so there are no critical points in the interval.

Given the derivative f'(x) = 20x^4 - 20x^3, are there any critical points in [3,4]?

No, because the critical points are at x=0 and x=1, which are outside the interval [3,4]. Therefore, there are no critical points within the interval.

How can I determine if the function f(x) = 4x^5 - 5x^4 - 3 is increasing or decreasing on [3,4]?

Since the derivative f'(x) = 20x^4 - 20x^3 is positive for x > 1, on [3,4], f'(x) > 0, indicating the function is increasing throughout the interval.

What is the significance of locating critical points for the function on [3,4]?

Locating critical points helps identify potential local maxima or minima within the interval. In this case, since no critical points exist in [3,4], the function does not have local extrema there.

Is there a need to evaluate the function at the endpoints of [3,4] to analyze its behavior?

Yes. Since there are no critical points inside the interval, evaluating the function at x=3 and x=4 helps determine the maximum or minimum values over [3,4].

What is the value of f(3) and f(4) for the function f(x) = 4x^5 - 5x^4 - 3?

Calculating: f(3) = 4(3)^5 - 5(3)^4 - 3 = 4243 - 581 - 3 = 972 - 405 - 3 = 564.
f(4) = 4(4)^5 - 5(4)^4 - 3 = 41024 - 5256 - 3 = 4096 - 1280 - 3 = 2813.
Thus, the function increases from 564 at x=3 to 2813 at x=4.

Additional Resources

Perform a First Derivative Test on the Function $f(x) = 4x^5 - 5x^4 + 4x^3 - 3$; [3,4] – Locate the Critical Point

Understanding how to perform a first derivative test on the function $f(x) = 4x^5 - 5x^4 + 4x^3 - 3$ is essential for analyzing the function's behavior within a specified interval. Identifying critical points allows us to determine where the function attains local maxima or minima, which is fundamental in calculus for understanding the shape and features of graphs, optimization problems, and more. In this guide, we'll walk through the detailed steps of finding the critical points of this polynomial function over the interval $[3, 4]$, applying the first derivative test, and interpreting the results.

Understanding the Function and Its Domain

The function at hand is:

$$
f(x) = 4x^5 - 5x^4 + 4x^3 - 3
$$

This is a polynomial function of degree 5, which indicates that it can have up to 4 critical points (since the derivative can have up to 4 real roots). Our interval of interest is $[3, 4]$, meaning we're particularly concerned with the behavior of the function within this range.

Step 1: Find the First Derivative $f'(x)$

The first step in performing the first derivative test is to compute the derivative of $f(x)$. The derivative indicates the rate of change of the function, and where it equals zero, potential maxima or minima may exist.

Differentiation:

$$
f'(x) = \frac{d}{dx}(4x^5) - \frac{d}{dx}(5x^4) + \frac{d}{dx}(4x^3) - \frac{d}{dx}(3)
$$

Calculating term-by-term:

- $\frac{d}{dx}(4x^5) = 20x^4$
- $\frac{d}{dx}(5x^4) = 20x^3$
- $\frac{d}{dx}(4x^3) = 12x^2$
- $\frac{d}{dx}(3) = 0$

Putting it all together:

$$
f'(x) = 20x^4 - 20x^3 + 12x^2
$$

Step 2: Simplify and Factor $f'(x)$

Factoring the first derivative helps identify its roots more easily:

$$
f'(x) = 20x^4 - 20x^3 + 12x^2
$$

Factor out the greatest common factor (GCF):

$$
f'(x) = 4x^2 (5x^2 - 5x + 3)
$$

Now, the critical points occur where $f'(x) = 0$, so set each factor equal to zero:

$$
4x^2 = 0 \quad \text{or} \quad 5x^2 - 5x + 3 = 0
$$

Roots:

- From $4x^2 = 0$:

$$
x = 0
$$

- From $5x^2 - 5x + 3 = 0$:

Use quadratic formula:

$$
x = \frac{5 \pm \sqrt{(-5)^2 - 4 \times 5 \times 3}}{2 \times 5}
$$

Calculate discriminant:

$$
D = 25 - 60 = -35
$$

Since $D < 0$, the quadratic has no real roots. Therefore, the only real critical point in the derivative is at:

$$
x = 0
$$

Step 3: Identify Critical Points within the Interval $[3, 4]$

Our critical point from the derivative is at $x=0$, which lies outside the interval $[3, 4]$. Since the first derivative does not change sign within $[3, 4]$, there are no critical points inside this interval based on the derivative's roots.

However, for completeness, consider the endpoints:

- At $x=3$
- At $x=4$

But note that critical points are where $f'(x)=0$ or $f'(x)$ is undefined. Since $f'(x)$ is defined everywhere (being a polynomial), and the only critical point is at $x=0$, outside $[3,4]$, the function has no internal critical points in this interval.

Step 4: Evaluate $f(x)$ at Endpoints to Determine Behavior

Even though no critical points exist within $[3, 4]$, the first derivative test can still be applied at endpoints to analyze whether the function is increasing or decreasing there, giving insight into potential maxima or minima over the interval.

Calculate:

- $f(3)$
- $f(4)$

Computing $f(3)$:

$$
f(3) = 4(3)^5 - 5(3)^4 + 4(3)^3 - 3
$$
$$
= 4(243) - 5(81) + 4(27) - 3
$$
$$
= 972 - 405 + 108 - 3
$$
$$
= (972 - 405) + (108 - 3) = 567 + 105 = 672
$$

Computing $f(4)$:

$$
f(4) = 4(4)^5 - 5(4)^4 + 4(4)^3 - 3
$$
$$
= 4(1024) - 5(256) + 4(64) - 3
$$
$$
= 4096 - 1280 + 256 - 3
$$
$$
= (4096 - 1280) + (256 - 3) = 2816 + 253 = 3069
$$

Step 5: Analyzing the Function's Behavior on $[3, 4]$

Since the derivative $f'(x) = 20x^4 - 20x^3 + 12x^2$ is positive for $x > 0$ (as it factors to $4x^2(5x^2 - 5x + 3)$, and the quadratic has no real roots), and at $x=3,4$:

- At $x=3$:

$$
f'(3) = 20(81) - 20(27) + 12(9) = 1620 - 540 + 108 = 1188
$$

- At $x=4$:

$$
f'(4) = 20(256) - 20(64) + 12(16) = 5120 - 1280 + 192 = 4032
$$

Since $f'(x) > 0$ at both endpoints, and the derivative has no roots in $[3,4]$, the function is strictly increasing over the interval.

Conclusion: Locating the Critical Point

- The only critical point from the derivative is at $x=0$, outside the interval $[3,4]$.
- The function is strictly increasing over $[3,4]$.
- Therefore, there is no critical point within $[3,4]$ that corresponds to a local maximum or minimum.

However, analyzing the endpoints:

- $f(3) = 672$
- $f(4) = 3069$

Since the function is increasing, the maximum value on $[3,4]$ occurs at $x=4$, and the minimum occurs at $x=3$.

Summary and Final Notes

Key Takeaways:

- The first derivative of $f(x) = 4x^5 - 5x^4 + 4x^3 - 3$ is $f'(x) = 20x^4 - 20x^3 + 12x^2$.
- Factoring reveals critical points only at $x=0$, outside the interval $[3,4]$.
- The derivative is positive over $[3,4]$, indicating the function is increasing in this interval.
- The function attains its minimum at $x=3$ with $f(3)=672$ and maximum at $x=4$ with $f(4)=3069$.

Practical Implication:

When performing the first derivative test on this function within $[3,4]$, we conclude that there are no internal critical points, but the function's behavior is monotonic, increasing throughout the interval. The critical point at \(x=0\) is outside the interval, so it doesn't influence the local extrema within \([3, 4]\). This analysis emphasizes the importance of considering both the derivative's roots and the behavior of the function at the interval's endpoints.

Final Remarks

Understanding how to perform a first derivative test involves calculating the derivative, finding its roots to locate critical points, and analyzing the sign of the derivative around those points. In cases where

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