Please Give Me Answer Prove That,[tex](sec {}^{2}a - 1) \cos {}^2a = \sin {}^{2} A[/tex]answer Please

Please Give Me Answer Prove That, $(\sec^{2}a - 1) \cos^{2}a = \sin^{2}a$ answer Please

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Introduction to the Trigonometric Identity

Trigonometric identities are fundamental in mathematics, especially in the fields of geometry, calculus, and engineering. They help simplify complex expressions, solve equations, and understand the relationships among different trigonometric functions. The identity in question:

$$
(\sec^{2}a - 1) \cos^{2}a = \sin^{2}a
$$

appears straightforward but requires a methodical approach to prove its validity. The goal of this article is to rigorously prove this identity using core trigonometric principles, identities, and algebraic manipulations. To do so, we will explore the definitions and relationships among the basic trigonometric functions – sine, cosine, secant, and their squares.

Understanding the Basic Trigonometric Functions

Definitions

- Sine ($\sin a$): The ratio of the length of the side opposite angle a to the hypotenuse in a right triangle.
- Cosine ($\cos a$): The ratio of the length of the adjacent side to the hypotenuse.
- Secant ($\sec a$): The reciprocal of cosine, $\sec a = \frac{1}{\cos a}$.

Fundamental Pythagorean Identity

One of the cornerstone identities in trigonometry is:

$$
\sin^{2}a + \cos^{2}a = 1
$$

This identity holds true for all angles where the functions are defined.

Step-by-Step Proof

Step 1: Express $\sec^{2}a - 1$ using known identities

Since $\sec a = \frac{1}{\cos a}$, then:

$$
\sec^{2}a = \frac{1}{\cos^{2}a}
$$

Subtracting 1:

$$
\sec^{2}a - 1 = \frac{1}{\cos^{2}a} - 1
$$

Expressed as a common denominator:

$$
\sec^{2}a - 1 = \frac{1 - \cos^{2}a}{\cos^{2}a}
$$

Step 2: Simplify numerator using Pythagorean identity

Recall that:

$$
\sin^{2}a + \cos^{2}a = 1 \Rightarrow 1 - \cos^{2}a = \sin^{2}a
$$

Substitute this into the previous expression:

$$
\sec^{2}a - 1 = \frac{\sin^{2}a}{\cos^{2}a}
$$

Step 3: Rewrite the original expression

Now, considering the original expression:

$$
(\sec^{2}a - 1) \cos^{2}a
$$

Substitute the simplified form of $\sec^{2}a - 1$:

$$
\left(\frac{\sin^{2}a}{\cos^{2}a}\right) \cos^{2}a
$$

Step 4: Simplify the expression

The $\cos^{2}a$ terms cancel out:

$$
\frac{\sin^{2}a}{\cancel{\cos^{2}a}} \times \cancel{\cos^{2}a} = \sin^{2}a
$$

Thus, the original expression simplifies exactly to $\sin^{2}a$.

Final Conclusion

By following the above steps, we've established that:

$$
(\sec^{2}a - 1) \cos^{2}a = \sin^{2}a
$$

which confirms that the identity holds true for all angles a where the functions are defined (excluding points where $\cos a = 0$, which would make $\sec a$ undefined).

Additional Insights and Applications

Significance of the Identity

- Simplification: This identity allows us to replace complex expressions involving secant and cosine with sine functions, simplifying calculations.
- Problem Solving: It is useful in solving integrals, differential equations, and in proving other identities.
- Geometric Interpretations: The relationship reflects the fundamental Pythagorean relationship between the sides of a right triangle.

Common Mistakes to Avoid

- Ignoring domain restrictions: Remember that $\sec a$ is undefined when $\cos a = 0$.
- Misapplication of identities: Always derive or recall identities carefully to avoid errors in substitution or simplification.

Summary

In this article, we demonstrated a rigorous proof of the identity:

$$
(\sec^{2}a - 1) \cos^{2}a = \sin^{2}a
$$

using fundamental trigonometric definitions and identities. The key steps involved recognizing the reciprocal nature of secant, applying the Pythagorean identity, and simplifying algebraic expressions. The process highlights the interconnectedness of trigonometric functions and showcases the elegance of their relationships.

Final Remarks

Understanding and proving trigonometric identities enhances mathematical problem-solving skills and deepens comprehension of geometric relationships. The identity discussed here is a perfect example of how foundational identities like the Pythagorean theorem underpin more complex expressions, enabling mathematicians and students alike to manipulate and simplify diverse mathematical expressions confidently.

Frequently Asked Questions

How can I prove that $(\sec^2 a - 1) \cos^2 a = \sin^2 a$?

Start with the identity $\sec^2 a = 1 + \tan^2 a$. Rewrite $\sec^2 a - 1$ as $\tan^2 a$, so the left side becomes $\tan^2 a \cos^2 a$. Since $\tan a = \sin a / \cos a$, then $\tan^2 a = \sin^2 a / \cos^2 a$. Multiplying by $\cos^2 a$ yields $\sin^2 a$. Therefore, $(\sec^2 a - 1) \cos^2 a = \sin^2 a$, as required.

What identities are used to prove $(\sec^2 a - 1) \cos^2 a = \sin^2 a$?

The key identities are $\sec^2 a = 1 + \tan^2 a$ and $\tan a = \sin a / \cos a$. Using $\sec^2 a - 1 = \tan^2 a$ and substituting $\tan^2 a = \sin^2 a / \cos^2 a$ allows simplification of the expression.

Is the equation $(\sec^2 a - 1) \cos^2 a = \sin^2 a$ valid for all angles?

Yes, the equation holds true for all angles 'a' where the functions are defined, specifically where $\cos a \neq 0$ to avoid division by zero.

Can you show step-by-step how to prove $(\sec^2 a - 1) \cos^2 a = \sin^2 a$?

Certainly. First, recall $\sec^2 a = 1 + \tan^2 a$, so $\sec^2 a - 1 = \tan^2 a$. Then, the left side becomes $\tan^2 a \cos^2 a$. Since $\tan a = \sin a / \cos a$, $\tan^2 a = \sin^2 a / \cos^2 a$. Multiplying by $\cos^2 a$ gives $\sin^2 a$. Hence, the expression simplifies to $\sin^2 a$, proving the identity.

What is the significance of this identity in trigonometry?

This identity helps relate secant and cosine functions to sine, enabling simplification of complex trigonometric expressions and solving equations involving these functions.

How can I use this identity to simplify other trigonometric expressions?

You can substitute $(\sec^2 a - 1)$ with $\tan^2 a$ or rewrite the expression to express everything in terms of sine and cosine, simplifying calculations in problem-solving.

Are there any common mistakes to avoid when proving this identity?

Yes. A common mistake is to forget the domain restrictions (like $\cos a \neq 0$) and to incorrectly substitute or simplify the identities without proper algebraic steps. Always verify each step carefully.

How does this identity relate to the Pythagorean identity $\sin^2 a + \cos^2 a = 1$?

While not directly the same, it is derived from or related to the Pythagorean identity. The identity uses $\sec^2 a = 1 / \cos^2 a$ and $\tan^2 a = \sin^2 a / \cos^2 a$, which are based on the Pythagorean relation.

Can this identity be used to solve trigonometric equations?

Yes. Recognizing that $(\sec^2 a - 1) \cos^2 a = \sin^2 a$ allows you to transform and simplify equations involving sec, sin, and cos, making it easier to find solutions.

Additional Resources

Please Give Me Answer Prove That,
$$ (\sec^2 a - 1) \cos^2 a = \sin^2 A $$

In the realm of trigonometry, identities serve as fundamental tools for simplifying complex expressions and solving problems efficiently. Among these identities, some are derived from basic principles, while others involve intricate transformations. Today, we focus on a specific trigonometric identity and aim to prove that:

$$ (\sec^2 a - 1) \cos^2 a = \sin^2 A $$

This proof not only reinforces our understanding of core trigonometric concepts but also exemplifies the interconnectedness of various identities. Let’s delve into the mathematical journey step by step, elucidating each transformation and providing clarity for readers at all levels.

Understanding the Components of the Identity

Before embarking on the proof, it’s essential to dissect the components involved in the expression:

- $\sec^2 a$: The square of the secant function, which is defined as $\sec a = \frac{1}{\cos a}$.
- $\cos^2 a$: The square of the cosine function.
- $\sin^2 A$: The square of the sine function, where A appears as a different variable, possibly related to a.

The key to proving the identity lies in expressing everything in terms of fundamental functions and known identities, primarily sine and cosine functions.

Step 1: Recall Fundamental Trigonometric Identities

The starting point is to remember well-known identities:

- Pythagorean Identity:
$$
\sin^2 x + \cos^2 x = 1
$$
- Secant Identity:
$$
\sec^2 x = 1 + \tan^2 x
$$

Using these, we can relate $\sec^2 a$ to $\cos a$ and $\sin a$.

Step 2: Simplify $\sec^2 a - 1$

Using the identity:

$$
\sec^2 a = 1 + \tan^2 a
$$

we have:

$$
\sec^2 a - 1 = \tan^2 a
$$

Thus, the original expression becomes:

$$
(\sec^2 a - 1) \cos^2 a = \tan^2 a \times \cos^2 a
$$

Step 3: Express $\tan^2 a$ in terms of sine and cosine

Recall that:

$$
\tan a = \frac{\sin a}{\cos a}
$$

Therefore,

$$
\tan^2 a = \frac{\sin^2 a}{\cos^2 a}
$$

Substituting into the expression:

$$
\frac{\sin^2 a}{\cos^2 a} \times \cos^2 a
$$

The $\cos^2 a$ terms cancel out:

$$
\frac{\sin^2 a}{\cos^2 a} \times \cos^2 a = \sin^2 a
$$

Result so far:

$$
(\sec^2 a - 1) \cos^2 a = \sin^2 a
$$

Step 4: Connect $\sin^2 a$ to $\sin^2 A$

The original goal involves proving that:

$$
(\sec^2 a - 1) \cos^2 a = \sin^2 A
$$

From the previous step, we have shown:

$$
(\sec^2 a - 1) \cos^2 a = \sin^2 a
$$

For the identity to hold true as $\sin^2 A$, it suggests that:

$$
\sin^2 A = \sin^2 a
$$

which implies:

$$
A = a \quad \text{or} \quad A = \pi - a
$$

since $\sin^2$ is symmetric about these angles.

Step 5: Final Interpretation and Conditions

The proof demonstrates that:

$$
(\sec^2 a - 1) \cos^2 a = \sin^2 a
$$

and if A is such that:

$$
A = a \quad \text{or} \quad A = \pi - a
$$

then:

$$
\sin^2 A = \sin^2 a
$$

Thus, the original identity can be written as:

$$
(\sec^2 a - 1) \cos^2 a = \sin^2 A
$$

under the condition that A relates to a in the manner described.

Conclusion: The Proof in Summary

- Starting from the left-hand side, we utilized the secant identity to express $\sec^2 a - 1$ as $\tan^2 a$.
- Replacing $\tan^2 a$ with $\frac{\sin^2 a}{\cos^2 a}$, we simplified the expression to $\sin^2 a$.
- Recognizing the relationship between $\sin^2 a$ and $\sin^2 A$, we concluded that the identity holds when the angles A and a are related via the properties of the sine function.

This proof elegantly demonstrates how fundamental identities in trigonometry interconnect, providing a clear pathway from complex expressions to simple, well-understood relationships. It underscores the importance of mastering these identities for problem-solving in mathematics, physics, and engineering contexts.

Final notes:
- Always verify the conditions under which identities hold, especially involving angle relationships.
- Recognize that many identities are interconnected; understanding one can illuminate others.
- Practice simplifying complex expressions step by step to build confidence and clarity in problem-solving.

This exploration not only confirms the validity of the original identity but also deepens appreciation for the beauty and consistency inherent in trigonometric functions.

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