

Plot The Complex Number, Then Write The Complex Number In Polar Form. You May Express The Argument In

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Understanding complex numbers is fundamental in advanced mathematics, engineering, and physics. Whether you're analyzing signals, solving equations, or exploring geometric representations, mastering how to plot and convert complex numbers to their polar form is essential. In this comprehensive guide, we will walk through the process of plotting a complex number on the complex plane and then transforming it into its polar representation, including how to express the argument accurately.

What Is a Complex Number?

A complex number is a number that can be expressed in the form $a + bi$, where:

- a is the real part
- b is the imaginary part
- i is the imaginary unit, satisfying $i^2 = -1$

For example, $3 + 4i$ is a complex number with a real part of 3 and an imaginary part of 4.

Plotting a Complex Number on the Complex Plane

Understanding the Complex Plane

The complex plane, also known as the Argand plane, is a two-dimensional plane where:

- The horizontal axis (x-axis) represents the real part.
- The vertical axis (y-axis) represents the imaginary part.

Each complex number corresponds to a point (a, b) on this plane.

Steps to Plot a Complex Number

1. Identify the Real and Imaginary Parts

For a given complex number, determine its real part (a) and imaginary part (b).

2. Locate the Point

- Move a units along the real axis (positive to the right, negative to the left).
- Move b units along the imaginary axis (positive upward, negative downward).

3. Mark the Point

Plot the point (a, b). This point visually represents the complex number.

Example

Suppose you have the complex number $3 + 4i$:

- Real part (a): 3
- Imaginary part (b): 4

Plot the point (3, 4) on the complex plane. This visualizes the complex number as a point located 3 units to the right of the origin and 4 units above the real axis.

Converting a Complex Number to Polar Form

Once the complex number is plotted, the next step is to express it in polar form. The polar form is written as:

$$r(\cos \theta + i \sin \theta)$$

or more compactly using Euler's formula:

$$r e^{i \theta}$$

where:

- r is the modulus (magnitude) of the complex number
- θ (theta) is the argument (angle) of the complex number

Calculating the Modulus (r)

The modulus is the distance from the origin to the point (a, b):

$$r = \sqrt{a^2 + b^2}$$

Example: For $3 + 4i$,

$$r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Calculating the Argument (θ)

The argument θ is the angle between the positive real axis and the line connecting the origin to the point (a, b) . It is calculated using the inverse tangent function:

$$\theta = \tan^{-1} \left(\frac{b}{a} \right)$$

Important:

- Consider the signs of a and b to determine the correct quadrant.
- Use the two-argument arctangent function (often denoted as atan2) for accurate angle calculation.

Example: For $3 + 4i$,

$$\theta = \tan^{-1} \left(\frac{4}{3} \right) \approx 53.13^\circ$$

or in radians,

$$\theta \approx 0.927 \text{ radians}$$

Quadrant considerations:

- If $a > 0$ and $b > 0$, θ is in the first quadrant (0 to 90 degrees).
- If $a < 0$ and $b > 0$, θ is in the second quadrant (90 to 180 degrees).
- If $a < 0$ and $b < 0$, θ is in the third quadrant (180 to 270 degrees).
- If $a > 0$ and $b < 0$, θ is in the fourth quadrant (270 to 360 degrees).

Expressing the Complex Number in Polar Form

Now, combine the modulus and argument:

$$z = r (\cos \theta + i \sin \theta)$$

or using Euler's formula:

$$z = r e^{i \theta}$$

Continuing the example:

$$z = 5 (\cos 0.927 + i \sin 0.927)$$

or

$$z = 5 e^{i 0.927}$$

Additional Tips

- Use a calculator or software tool for precise inverse tangent calculations.
- Always specify the quadrant when reporting the argument, especially when working with negative or zero components.
- Remember that the principal value of the argument θ is typically taken between $-\pi$ and π (or -180° and 180°).

Expressing the Argument in Different Forms

The argument θ can be expressed in various formats depending on the context:

- Degrees: For example, 53.13°
- Radians: For example, 0.927 radians
- Principal value: Usually between $-\pi$ and π

When communicating the argument, clarify the units used to avoid confusion.

Summary of the Process

Here's a step-by-step summary:

1. Identify the real and imaginary parts of the complex number.
2. Plot the point (a, b) on the complex plane.
3. Calculate the modulus $(r = \sqrt{a^2 + b^2})$.
4. Calculate the argument $(\theta = \tan^{-1}(b/a))$, adjusting for the correct quadrant.
5. Express the complex number in polar form: $(r e^{i\theta})$ or $(r(\cos \theta + i \sin \theta))$.

Practice Example

Suppose you are given the complex number $-1 + i$:

- Step 1: Real part (a): -1, Imaginary part (b): 1
- Step 2: Plot point (-1, 1) on the complex plane.
- Step 3: Calculate modulus:

$$[r = \sqrt{(-1)^2 + 1^2} = \sqrt{1 + 1} = \sqrt{2} \approx 1.414]$$

- Step 4: Calculate argument:

$$[\theta = \tan^{-1}\left(\frac{1}{-1}\right) = \tan^{-1}(-1) \approx -45^\circ \text{ or } -0.785 \text{ radians}]$$

Since the point is in the second quadrant (a negative, b positive), adjust:

$\theta = 180^\circ - 45^\circ = 135^\circ$
or in radians:

$\theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \approx 2.356 \text{ radians}$

- Step 5: Write in polar form:

$z = \sqrt{2} \left(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)$

or

$z = \sqrt{2} e^{i \frac{3\pi}{4}}$

Conclusion

Plotting complex numbers and converting them into their polar form is a powerful technique in mathematics and engineering. It offers a geometric perspective that simplifies many operations such as multiplication, division, and finding powers or roots of complex numbers. Remember to carefully determine the modulus and argument, considering the signs of the real and imaginary parts, to accurately represent the complex number in polar form. With practice, these steps become intuitive, enhancing your understanding of complex analysis and its applications.

Frequently Asked Questions

How do you plot a complex number on the complex plane?

To plot a complex number, identify its real part (x-axis) and imaginary part (y-axis). Mark the point at (Re, Im) on the complex plane, where Re is the real component and Im is the imaginary component.

What is the first step in converting a complex number to polar form?

Calculate the magnitude (r) of the complex number using $r = \sqrt{\text{Re}^2 + \text{Im}^2}$.

How do you determine the argument (angle) when converting to polar form?

Find the argument θ using $\theta = \arctan(\text{Im} / \text{Re})$, adjusting based on the quadrant where the point lies.

In what units can the argument be expressed, and which is more common?

The argument can be expressed in radians or degrees. Radians are more common in mathematical contexts.

Can you write the complex number in polar form if the argument is given in degrees?

Yes, first convert the degrees to radians by multiplying by $\pi/180$, then express the complex number as $r(\cos \theta + i \sin \theta)$.

What is the polar form of the complex number $3 + 4i$?

First, find $r = \sqrt{3^2 + 4^2} = 5$. Then, $\theta = \arctangent(4/3) \approx 0.93$ radians or 53.13° . The polar form is $5(\cos 0.93 + i \sin 0.93)$ or $5\angle 53.13^\circ$.

Why is it important to determine the correct quadrant when finding the argument?

Because the arctangent function alone cannot distinguish between certain quadrants, adjusting the angle based on the signs of Re and Im ensures an accurate argument.

How do you express the complex number in exponential form after converting to polar form?

Use Euler's formula to write the complex number as $r e^{i\theta}$, where r is the magnitude and θ is the argument in radians.

What are common mistakes to avoid when plotting and converting complex numbers to polar form?

Common mistakes include miscalculating the magnitude or argument, forgetting to adjust the argument for the correct quadrant, or mixing degrees and radians. Always verify the signs and units.

Additional Resources

Plot The Complex Number, Then Write The Complex Number In Polar Form. You May Express The Argument In

Introduction

In the realm of complex numbers, understanding how to graphically interpret and

algebraically manipulate these entities is fundamental to advanced mathematics, engineering, and physics. The process of plotting a complex number and then converting it into its polar form is not only a core skill but also a gateway to deeper insights into the behavior of complex functions, signals, and systems. This article explores this process comprehensively, providing a detailed examination suitable for students, educators, or professionals seeking clarity on the topic.

The Significance of Complex Numbers

Before delving into plotting and conversion techniques, it is essential to recognize why complex numbers are vital. They extend the real number system to include solutions to equations like $(x^2 + 1 = 0)$, which has no real solutions. Complex numbers are expressed as $(z = a + bi)$, where:

- (a) is the real part
- (b) is the imaginary part
- (i) is the imaginary unit, satisfying $(i^2 = -1)$

Understanding their geometric representation as points or vectors in the complex plane enhances intuition and enables the utilization of geometric operations in algebraic computations.

Plotting a Complex Number

Step 1: Identify the Components

Given a complex number $(z = a + bi)$, identify:

- The real part (a)
- The imaginary part (b)

For example, consider $(z = 3 - 4i)$:

- Real part $(a = 3)$
- Imaginary part $(b = -4)$

Step 2: Graph in the Complex Plane

The complex plane (also called the Argand plane) is a two-dimensional coordinate system:

- The horizontal axis (x-axis) represents the real part
- The vertical axis (y-axis) represents the imaginary part

Plot the point (a, b) :

- For $(3 - 4i)$, plot the point at $(3, -4)$

Visualizing the Point

- From the origin $(0,0)$, draw a point located 3 units along the x-axis and 4 units downward along the y-axis
- The position of this point visually represents the complex number

Step 3: Draw the Vector

- Draw a directed line segment (vector) from the origin to the point
- This vector visually encodes the magnitude and direction of the complex number

Converting to Polar Form

The polar form expresses a complex number in terms of its magnitude and argument, which provides a more natural way to analyze multiplication, division, and rotation.

Step 1: Calculate the Magnitude (r)

The magnitude (or modulus) (r) is the distance from the origin to the point:

$$r = |z| = \sqrt{a^2 + b^2}$$

Using the previous example:

$$r = \sqrt{3^2 + (-4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Step 2: Determine the Argument (θ)

The argument (θ) (also called the angle or phase) is the angle between the positive real axis and the line segment representing the complex number.

$$\theta = \arctan\left(\frac{b}{a}\right)$$

However, care must be taken with the quadrant:

- If $(a > 0)$, $(\theta = \arctan\left(\frac{b}{a}\right))$
- If $(a < 0)$ and $(b \geq 0)$, $(\theta = \arctan\left(\frac{b}{a}\right) + \pi)$
- If $(a < 0)$ and $(b < 0)$, $(\theta = \arctan\left(\frac{b}{a}\right) - \pi)$
- If $(a = 0)$ and $(b \neq 0)$, $(\theta = \pm \frac{\pi}{2})$

For $(z = 3 - 4i)$:

$\backslash$

$$\theta = \arctan\left(\frac{-4}{3}\right) \approx \arctan(-1.333) \approx -0.927 \text{ radians}$$

Since $(a > 0)$ and $(b < 0)$, the point lies in the fourth quadrant. It's customary to express the argument as a positive angle:

$$\theta = 2\pi - 0.927 \approx 5.356 \text{ radians}$$

Alternatively, in degrees:

$$\theta \approx -53.13^\circ$$

or

$$\theta \approx 306.87^\circ$$

depending on the convention used.

Expressing in Polar Form

The polar form of a complex number is:

$$z = r (\cos \theta + i \sin \theta)$$

or, using Euler's formula:

$$z = r e^{i\theta}$$

For $(z = 3 - 4i)$, the polar form is:

$$z = 5 \left(\cos 5.356 + i \sin 5.356 \right)$$

or

$$z = 5 e^{i \times 5.356}$$

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Advantages of Polar Representation

Converting to polar form offers several benefits:

- Simplifies multiplying and dividing complex numbers
- Facilitates raising complex numbers to powers or extracting roots
- Enhances understanding of rotational transformations in the plane

Practical Applications and Examples

Example 1: Multiply Two Complex Numbers

Suppose you want to multiply $(z_1 = 2 e^{i \pi/4})$ and $(z_2 = 3 e^{i \pi/6})$:

- Magnitudes multiply: $(r_1 \times r_2 = 2 \times 3 = 6)$
- Arguments add: $(\pi/4 + \pi/6 = (3\pi/12 + 2\pi/12) = 5\pi/12)$

Result:

$$z = 6 e^{i \times 5\pi/12}$$

This illustrates how the polar form simplifies complex multiplication.

Example 2: Find the Roots of a Complex Number

To find the (n) -th roots of $(z = r e^{i \theta})$:

$$z^{1/n} = r^{1/n} e^{i (\theta + 2\pi k)/n}, \quad k=0,1,\dots,n-1$$

This method is particularly useful in solving polynomial equations with complex roots.

Challenges and Considerations

- Quadrant determination: Properly identifying the correct quadrant for (θ) is critical for accurate conversion.
- Principal value: Usually, the principal value of (θ) is taken in $(-\pi, \pi]$ or $[0, 2\pi)$, depending on the context.
- Multiple representations: A single complex number can have infinitely many polar forms differing by multiples of (2π) .

Conclusion

The process of plotting a complex number and converting it into its polar form is foundational in complex analysis and related fields. By visualizing the complex number as a point and then translating that into magnitude and argument, mathematicians and engineers can leverage geometric intuition to simplify complex operations. This duality between algebraic and geometric representations underscores the richness of complex numbers and their applications across science and technology.

Understanding this process thoroughly enables practitioners to perform operations more efficiently, analyze oscillations and rotations, and develop a deeper appreciation of the complex plane's geometry. Whether for educational purposes or practical applications, mastering plotting and polar conversion of complex numbers is an indispensable skill in the mathematical toolkit.

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Note: This article assumes familiarity with basic trigonometry, algebra, and complex number operations.

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