# Point P Is Located At (2, 7), And Point R Is Located At (1, 0). Find The Y Value For The Point Q That

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Understanding the coordinate plane is fundamental in mathematics, especially in geometry and algebra. When given specific points, such as Point P at (2, 7) and Point R at (1, 0), a common problem involves determining the coordinates of an unknown point Q based on certain conditions. In this article, we will explore how to find the y-value of Point Q, given various scenarios involving points P and R. This comprehensive guide aims to improve your understanding of coordinate geometry and provide step-by-step methods to solve similar problems.

## Understanding Coordinates and Points in the Plane

### What Are Coordinates?

Coordinates are numerical values that determine the position of a point in the Cartesian plane. Each point is represented as an ordered pair (x, y), where:

- x-coordinate (abscissa): The horizontal position
- y-coordinate (ordinate): The vertical position

### Plotting Points on the Graph

Given points P(2, 7) and R(1, 0), you can plot these on the graph:

- Point P: Located 2 units to the right of the origin, and 7 units up.
- Point R: Located 1 unit to the right of the origin, and 0 units vertically (on the x-axis).

Understanding the placement of these points helps in visualizing the problem and applying geometric concepts such as slopes, lines, and distances.

## Possible Scenarios for Finding the Y-Value of Point Q

There are several common scenarios where you might be asked to find the y-coordinate of a point Q based on given points P and R:

1. Q lies on the line segment between P and R
2. Q lies on the line passing through P and R (collinear points)
3. Q forms a specific geometric figure with P and R (e.g., midpoint, vertex of a shape)
4. Q satisfies a particular condition (e.g., equidistant from P and R, or on a specific line)

Let's explore each case in detail.

# Finding the Y-Value When Q Is the Midpoint of P and R

## What Is a Midpoint?

The midpoint of a segment is the point exactly halfway between its endpoints. To find the midpoint Q of segment PR:

Midpoint formula:

$$
Q(x, y) = \left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
$$

Applying the formula with P(2, 7) and R(1, 0):

$$
x_Q = \frac{2 + 1}{2} = \frac{3}{2} = 1.5
$$
$$
y_Q = \frac{7 + 0}{2} = \frac{7}{2} = 3.5
$$

Result:

$$
\boxed{
Q = \left( 1.5, 3.5 \right)
}
$$

So, the y-value of Q as the midpoint is 3.5.

# Finding the Y-Value When Q Lies on the Line Segment Between P and R

# Determining the Equation of Line PR

If Q lies on the line segment connecting P and R, then Q's coordinates satisfy the equation of the line passing through P and R.

Step 1: Calculate the slope (m) of line PR

$$
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 7}{1 - 2} = \frac{-7}{-1} = 7
$$

Step 2: Write the equation of line PR in point-slope form

Using point P(2, 7):

$$
y - y_1 = m (x - x_1)
$$
$$
y - 7 = 7(x - 2)
$$
$$
y - 7 = 7x - 14
$$
$$
y = 7x - 14 + 7
$$
$$
y = 7x - 7
$$

Step 3: Find y for a specific x-value

If the x-coordinate of Q is known, substitute into the equation to find y.

- For example, if Q has x-value $x_q$, then

$$
y_q = 7x_q - 7
$$

Note: If Q is on the line segment between P and R, then the x-value must be between 1 and 2.

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Example:

Suppose Q has an x-value of 1.5, then:

$$

$$
y_q = 7(1.5) - 7 = 10.5 - 7 = 3.5
$$

which matches the midpoint calculation.

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# Finding the Y-Value When Q Is Equidistant From P and R

## Understanding Equidistance

If Q is equidistant from P and R, then:

$$
\text{Distance}(Q, P) = \text{Distance}(Q, R)
$$

Distance formula:

$$
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Setting up the equation:

$$
\sqrt{(x - 2)^2 + (y - 7)^2} = \sqrt{(x - 1)^2 + (y - 0)^2}
$$

Squaring both sides to eliminate the square root:

$$
(x - 2)^2 + (y - 7)^2 = (x - 1)^2 + y^2
$$

Expanding:

$$
(x^2 - 4x + 4) + (y^2 - 14y + 49) = x^2 - 2x + 1 + y^2
$$

Simplify both sides:

$$
x^2 - 4x + 4 + y^2 - 14y + 49 = x^2 - 2x + 1 + y^2
$$

\]

Subtract \(x^2\) and \(y^2\) from both sides:

\[
-4x + 4 + -14y + 49 = -2x + 1
\]

Combine like terms:

\[
-4x - 14y + 53 = -2x + 1
\]

Bring all to one side:

\[
-4x + 14y + 53 - (-2x + 1) = 0
\]
\[
-4x + 14y + 53 + 2x - 1 = 0
\]
\[
(-4x + 2x) + 14y + (53 - 1) = 0
\]
\[
-2x + 14y + 52 = 0
\]

Divide through by 2:

\[
- x + 7y + 26 = 0
\]

Express y in terms of x:

\[
7y = x - 26
\]
\[
y = \frac{x - 26}{7}
\]

Interpretation:

- The locus of points Q equidistant from P and R is the line \( y = \frac{x - 26}{7} \).
- To find a specific y-value, choose an x-value within the segment's range (say, between 1 and 2).

Example:

- For $x = 1.5$:

$$
y = \frac{1.5 - 26}{7} = \frac{-24.5}{7} \approx -3.5
$$

Thus, the point Q at x=1.5 and equidistant from P and R has y approximately -3.5.

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# Additional Geometric Considerations

## Line Equations and Their Use

Line equations are powerful tools in coordinate geometry. They allow you to:

- Find points lying on a line
- Determine intersections
- Calculate distances along the line

By understanding the slope-intercept form $y = mx + b$, you can quickly identify the y-value for any given x-value on a line.

## Using Geometric Constructions

Depending on the problem, you might need to:

- Find points forming specific angles
- Construct perpendicular bisectors
- Determine points at particular distances

These geometric tools often involve using the coordinate formulas discussed earlier.

# Summary and Key Takeaways

- To find the y-value of point Q, understand the context: midpoint, on a line, equidistant, etc.
- Use the midpoint formula for the middle point between P and R.
- Derive the equation of the line passing through P and R to find points on the line.
- Apply the distance formula for points equidistant from two given points.
- Always verify that your point Q lies within the relevant segment or satisfies the problem's constraints.