

Rectangle PQRS Is Rotated 90 Counterclockwise About The Origin. On A Coordinate Plane, Rectangle P Q R

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Understanding how geometric figures behave under rotations on the coordinate plane is fundamental in both pure and applied mathematics. In particular, analyzing the transformation of a rectangle—specifically rectangle PQRS—when rotated 90 degrees counterclockwise about the origin offers insightful perspectives into coordinate geometry, vector transformations, and spatial reasoning. This article explores the detailed process of rotating rectangle PQRS, the effects on its vertices, and the implications for related geometric properties.

Introduction to Rotation in Coordinate Geometry

Understanding Rotations About the Origin

Rotation is a type of rigid transformation that turns a figure around a fixed point, called the center of rotation. When the center is the origin $(0,0)$, the rotation can be described mathematically using coordinate transformations. Specifically, a rotation of 90 degrees counterclockwise around the origin involves transforming each point (x, y) based on the following rule:

- The new x-coordinate becomes $-y$.
- The new y-coordinate becomes x .

Mathematically, this is expressed as:

$$\begin{aligned} &\backslash[\\ &(x', y') = (-y, x) \\ &\backslash] \end{aligned}$$

This transformation preserves distances and angles, maintaining the shape's size and internal angles, hence the figure remains congruent but in a different orientation.

Significance of Rotating Rectangles

Rotating rectangles and other polygons helps in:

- Visualizing geometric transformations
- Solving coordinate geometry problems
- Understanding symmetry and congruence
- Preparing for advanced topics like transformations in computer graphics

Initial Coordinates of Rectangle PQRS

Defining the Rectangle in the Coordinate Plane

Suppose the rectangle PQRS is positioned on the coordinate plane with vertices as follows:

- $P(x_1, y_1)$
- $Q(x_2, y_2)$
- $R(x_3, y_3)$
- $S(x_4, y_4)$

To analyze the rotation, we need specific coordinates. For illustration, assume the rectangle has the following vertices:

- $P(2, 1)$
- $Q(6, 1)$
- $R(6, 4)$
- $S(2, 4)$

This configuration forms a rectangle with length 4 units (from $x=2$ to $x=6$) and height 3 units (from $y=1$ to $y=4$).

Properties of the Original Rectangle

- Opposite sides are parallel and equal in length.
- The vertices are ordered either clockwise or counterclockwise.
- The rectangle's center can be calculated as the average of its vertices' coordinates.

Calculating the center (midpoint):

\[

```
\text{Center} (C_x, C_y) = \left( \frac{x_1 + x_3}{2}, \frac{y_1 + y_3}{2} \right) = \left( \frac{2 + 6}{2}, \frac{1 + 4}{2} \right) = (4, 2.5)
\]
```

Performing the 90-Degree Counterclockwise Rotation

Step-by-Step Transformation of Vertices

To rotate rectangle PQRS 90 degrees counterclockwise about the origin, apply the transformation rule to each vertex:

```
\[
(x', y') = (-y, x)
\]
```

Vertex P(2, 1):

```
\[
x' = -1 = -1 \\
y' = 2 \\
\rightarrow P'(-1, 2)
\]
```

Vertex Q(6, 1):

```
\[
x' = -1 = -1 \\
y' = 6 \\
\rightarrow Q'(-1, 6)
\]
```

Vertex R(6, 4):

```
\[
x' = -4 = -4 \\
y' = 6 \\
\rightarrow R'(-4, 6)
\]
```

Vertex S(2, 4):

```
\[
x' = -4 = -4 \\
```

```

y' = 2 \\
\rightarrow S'(-4, 2)
\]

```

Summary of rotated vertices:

- P'(-1, 2)
- Q'(-1, 6)
- R'(-4, 6)
- S'(-4, 2)

Visualizing the Rotated Rectangle

The new vertices form a rectangle in the coordinate plane, rotated 90 degrees counterclockwise from its original position. Notice that the rectangle's orientation has changed: what was originally aligned along the axes is now rotated, and the shape's size and shape are preserved.

Analyzing the Properties of the Rotated Rectangle

Verifying Congruence and Orientation

Since rotation is a rigid transformation, the dimensions of rectangle PQRS remain unchanged:

- Length of sides: both original and rotated rectangles have the same side lengths.
- Internal angles: all right angles are preserved.

Original rectangle sides:

- PQ: distance between P(2, 1) and Q(6, 1):

```

\[
\sqrt{(6-2)^2 + (1-1)^2} = \sqrt{4^2 + 0} = 4
\]

```

- QR: between Q(6, 1) and R(6, 4):

```

\[
\sqrt{(6-6)^2 + (4-1)^2} = \sqrt{0 + 3^2} = 3

```

\]

Similarly, the rotated rectangle's sides:

- P'(-1, 2) to Q'(-1, 6):

\[

$$\sqrt{(-1 - (-1))^2 + (6 - 2)^2} = \sqrt{0 + 4^2} = 4$$

\]

- Q'(-1, 6) to R'(-4, 6):

\[

$$\sqrt{(-4 - (-1))^2 + (6 - 6)^2} = \sqrt{(-3)^2 + 0} = 3$$

\]

This confirms the shape's congruence post-rotation.

Center of the Rotated Rectangle

The center of the rotated rectangle can be found similarly:

\[

$$C_{x'} = \frac{-1 + (-4)}{2} = -2.5 \quad \backslash \backslash$$

$$C_{y'} = \frac{2 + 6}{2} = 4$$

\]

Note that the rotation about the origin shifts the rectangle's position, but the center's coordinates depend on the vertices' positions. Since rotation about the origin is a rigid transformation, the original center (4, 2.5) is not necessarily the center of the rotated rectangle unless the rectangle is centered at the origin.

Implications of Rotation on Coordinates and Geometry

Coordinate Transformations and Their Effects

Rotating rectangle PQRS demonstrates key principles:

- Each vertex's position changes according to the rotation formula.
- The shape's size and internal angles are preserved.

- The rectangle's orientation shifts, affecting the direction of its sides.

Applications in Geometry and Real-World Problems

Understanding these transformations has numerous applications:

- Computer Graphics: Rotating images or shapes around a point.
- Robotics: Calculating the position of parts after rotation.
- Engineering: Analyzing how objects behave under rotation.
- Mathematics Education: Enhancing spatial reasoning skills.

Other Rotations and Transformations

Beyond 90-degree rotations, similar principles apply to:

- 180-degree rotations
- 270-degree rotations
- Arbitrary angles
- Translations, reflections, and dilations

Each transformation has its specific formula and effect on the figure.

Conclusion: Mastering Rotations in Coordinate Geometry

Rotating rectangle PQRS 90 degrees counterclockwise about the origin exemplifies a fundamental transformation in coordinate geometry. By applying the rotation rule systematically to each vertex, we transform the rectangle's position while preserving its shape and size. This process underscores the importance of understanding coordinate transformations, especially for solving complex geometric problems, designing computer graphics, or analyzing physical systems.

The detailed analysis of the original and rotated vertices reveals the congruence of the figures and highlights the geometric properties that remain invariant under rotation. Mastery of such transformations enhances spatial reasoning and provides a solid foundation for exploring more advanced topics in geometry, trigonometry, and vector calculus.

Whether you are a student, educator, or professional, grasping how to perform and analyze rotations about the origin equips you with a powerful toolset for tackling diverse mathematical challenges and real-world applications.

Keywords: rectangle rotation, coordinate geometry, 90-degree rotation, rotation about the origin, transformation of vertices, spatial reasoning, geometric properties, congruence, vector transformation.

Frequently Asked Questions

What happens to rectangle PQRS when it is rotated 90 degrees counterclockwise about the origin?

The rectangle rotates so that its vertices move to new positions, effectively swapping its axes and changing its orientation, with each point rotating 90 degrees counterclockwise around the origin.

How do you find the new coordinates of rectangle PQRS after a 90-degree counterclockwise rotation about the origin?

For each vertex (x, y) , the new coordinates become $(-y, x)$ after a 90-degree counterclockwise rotation about the origin.

If the original rectangle PQRS has vertices $P(x_1, y_1)$, $Q(x_2, y_2)$, $R(x_3, y_3)$, and $S(x_4, y_4)$, what are the coordinates of the rotated rectangle?

The rotated vertices will be at $(-y_1, x_1)$, $(-y_2, x_2)$, $(-y_3, x_3)$, and $(-y_4, x_4)$, respectively.

Does the size or shape of rectangle PQRS change after a 90-degree rotation about the origin?

No, the size and shape of the rectangle remain the same; only its orientation and position relative to the axes change.

Is the rotation of rectangle PQRS about the origin a rigid transformation?

Yes, rotating a figure about a point (the origin) by 90 degrees is a rigid transformation, preserving distances and angles.

How can understanding the rotation of rectangle PQRS

help in coordinate geometry problems?

It helps in visualizing transformations, solving for new positions of figures, and understanding symmetry and geometric properties related to rotations.

Additional Resources

Rectangle Rotation: A Deep Dive into Geometric Transformation

Introduction

Geometry, at its core, is an elegant dance of shapes and transformations. Among the many transformations that bring dynamism to static figures, rotation stands out as both fundamental and visually captivating. When we talk about a rectangle—specifically Rectangle PQRS—being rotated 90 degrees counterclockwise about the origin, we delve into a rich exploration of coordinate geometry, transformation matrices, and spatial reasoning. This article aims to dissect this transformation comprehensively, offering insights not just for students and educators but also for enthusiasts seeking to understand the nuances of geometric manipulation.

The Significance of Rotation in Coordinate Geometry

Rotation, unlike translation or reflection, involves turning a figure around a fixed point, known as the center of rotation. In our case, the center is the origin (0,0). Such transformations are crucial in various applications:

- Computer graphics: Rotating images and objects.
- Robotics: Path planning and orientation.
- Engineering: Designing mechanical parts with specific angular alignments.
- Mathematics education: Understanding symmetry and spatial relationships.

Specifically, a 90-degree counterclockwise rotation is one of the fundamental rotations, often used to introduce students to rotation matrices and coordinate transformation techniques.

Defining Rectangle PQRS on the Coordinate Plane

Before analyzing the effects of rotation, it's essential to establish the initial position of Rectangle PQRS. Typically, such a rectangle can be described by its vertices in coordinate form:

- $P(x_1, y_1)$

- $Q(x_2, y_2)$
- $R(x_3, y_3)$
- $S(x_4, y_4)$

For clarity, assume the rectangle is positioned in the coordinate plane with specific coordinates. For example:

- $P(2, 3)$
- $Q(6, 3)$
- $R(6, 1)$
- $S(2, 1)$

This configuration forms a rectangle with length 4 units along the x-axis and width 2 units along the y-axis.

Key Properties of Rectangle PQRS

Understanding the properties of the initial rectangle is vital:

- Vertices: Four points with right angles.
- Opposite sides: Parallel and equal in length.
- Diagonals: Equal in length, bisecting each other.
- Center of the rectangle: Midpoint of diagonals, which can be calculated as the average of the x and y coordinates of opposite vertices.

In our example:

- Center (midpoint of diagonals):

$$C_x = \frac{x_1 + x_3}{2} = \frac{2 + 6}{2} = 4$$

$$C_y = \frac{y_1 + y_3}{2} = \frac{3 + 1}{2} = 2$$

So, $C(4, 2)$.

Mathematical Framework for Rotation

Rotation About the Origin

The key to understanding the rotation process lies in the rotation matrix for 2D space:

$$R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix}$$

$$\begin{bmatrix} \sin \theta & \cos \theta \\ \cos \theta & -\sin \theta \end{bmatrix}$$

Where:

- θ is the angle of rotation.
- For a 90-degree counterclockwise rotation, $\theta = 90^\circ$ or $\frac{\pi}{2}$ radians.

Calculating the sine and cosine values:

$$\begin{aligned} \cos 90^\circ &= 0, \quad \sin 90^\circ = 1 \end{aligned}$$

Thus, the rotation matrix simplifies to:

$$R(90^\circ) = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

Applying the Rotation to the Coordinates

Step 1: Translate vertices to origin

Since we're rotating about the origin, each vertex's coordinates are used directly. If rotation were about a different point, we'd first translate the figure to the origin.

Step 2: Apply rotation matrix

For each vertex (x, y) , the rotated coordinates (x', y') are:

$$\begin{aligned} x' &= x \cdot 0 - y \cdot 1 = -y \\ y' &= x \cdot 1 + y \cdot 0 = x \end{aligned}$$

Step 3: Write the transformed vertices

Using our example vertices:

Vertex	Original Coordinates	Rotated Coordinates
P(2, 3)	$((2, 3))$	$((-3, 2))$
Q(6, 3)	$((6, 3))$	$((-3, 6))$
R(6, 1)	$((6, 1))$	$((-1, 6))$
S(2, 1)	$((2, 1))$	$((-1, 2))$

Visualizing the Rotation

Graphically, the rectangle initially positioned with vertices at P(2,3), Q(6,3), R(6,1), S(2,1) is rotated 90 degrees counterclockwise about the origin to new positions. The figure now:

- Has vertices at $((-3, 2))$, $((-3, 6))$, $((-1, 6))$, and $((-1, 2))$.
- Is oriented differently, with the original width along the y-axis and length along the negative x-axis.

This transformation effectively swaps the rectangle's axes alignment, illustrating how rotation alters orientation and position while preserving shape and size.

Re-establishing the Rectangle's Properties Post-Rotation

After rotation, the shape retains its fundamental properties:

- Side lengths: Calculated using distance formula; remain unchanged.
- Angles: Still right angles.
- Area: Unchanged, since rotation is an isometric transformation.
- Vertices: Now located in the second and third quadrants, indicating a change in orientation.

The rotated rectangle is a perfect geometric congruence, just reoriented in space.

Practical Applications and Implications

Understanding such rotations has several practical implications:

- Design and Engineering: Visualizing how components fit together when rotated.
- Computer-Aided Design (CAD): Rotating parts precisely without distortion.
- Animation and Gaming: Rotating objects smoothly around a pivot point.
- Mathematical Proofs: Demonstrating symmetry and congruence through transformations.

In educational contexts, practicing rotation helps develop spatial intelligence and grasp the underlying principles of coordinate geometry.

Extending the Concept: Rotation About Arbitrary Points

While this article focuses on rotation about the origin, the same principles extend to rotations about any point $((a, b))$:

1. Translate the shape to the origin:

```
\[
(x - a, y - b)
\]
```

2. Rotate using the rotation matrix.

3. Translate back:

```
\[
(x' + a, y' + b)
\]
```

This flexibility allows for complex geometric constructions and transformations in various contexts.

Summary and Key Takeaways

- Rotation is a fundamental geometric transformation that preserves shape and size.
- A 90-degree counterclockwise rotation can be succinctly represented with a simple rotation matrix.
- Coordinates of each vertex are transformed by swapping axes and changing signs as dictated by the matrix.
- The center of rotation significantly influences the transformation; here, it's the origin.
- Post-rotation, the rectangle maintains its area, angles, and side lengths but changes orientation.
- Mastery of rotation transformations enhances understanding of symmetry, congruence, and spatial reasoning.

Final Thoughts

The rotation of Rectangle PQRS by 90 degrees counterclockwise about the origin exemplifies the elegance and power of coordinate geometry. It demonstrates how a simple matrix operation can systematically and predictably alter a figure's position, offering insights into both theoretical mathematics and practical applications. Whether for academic pursuits, engineering design, or computer graphics, grasping this transformation is a cornerstone in the study of shapes and their behavior under various

manipulations.

By exploring the step-by-step process—from defining vertices to applying the rotation matrix and visualizing the result—one gains a comprehensive understanding of rotational transformations. This knowledge serves as a foundation for more complex geometric operations, enriching one's spatial and mathematical toolkit.

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