Rewrite The Function In Simplest Form, Including Theappropriate Domain.f (x) = 3x2 + X - 2________3x

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Introduction

Understanding how to simplify functions is fundamental in algebra and calculus, as it allows for easier analysis and interpretation of the function's behavior. The given function, which appears to be a rational expression, requires simplification to its most reduced form. This process involves algebraic manipulations such as factoring, canceling common factors, and determining the domain to identify the set of permissible inputs (x-values). In this article, we will analyze the function $f(x) = \frac{3x^2 + x - 2}{3x}$, simplify it thoroughly, and explicitly state its domain.

Analyzing the Given Function

Original Function

The function provided can be written as:

$$
f(x) = \frac{3x^2 + x - 2}{3x}
$$

This is a rational function with a quadratic polynomial in the numerator and a linear polynomial in the denominator.

Initial Observations

- The numerator, $3x^2 + x - 2$, is quadratic.
- The denominator, $3x$, is linear.
- The function may have restrictions when the denominator equals zero.
- Simplifying involves factoring the numerator and then canceling common factors with the denominator, if possible.

Step-by-Step Simplification

1. Factor the Numerator

The numerator $3x^2 + x - 2$ is a quadratic trinomial. To factor it, we look for two numbers that multiply to $3 \times (-2) = -6$ and add up to 1.

- Factors of -6: (-3, 2), (3, -2), (-1, 6), (1, -6)
- Sum to 1: 3 + (-2) = 1

Thus, we can factor by grouping or use the quadratic factoring method:

$$
3x^2 + x - 2 = (3x^2 + 3x) - 2x - 2
$$

But a more straightforward approach is to use the quadratic formula or factor directly:

Attempting to factor directly:

$$
3x^2 + x - 2 = (3x + 2)(x - 1)
$$

Check:

$$
(3x + 2)(x - 1) = 3x \cdot x - 3x + 2x - 2 = 3x^2 - 3x + 2x - 2 = 3x^2 - x - 2
$$

This does not match the original numerator because of the sign. Let's verify the factorization:

Try:

$$
3x^2 + x - 2
$$

Factors:

$$
(3x - 2)(x + 1)
$$

Check:

$$
3x \cdot x + 3x \cdot 1 - 2 \cdot x - 2 \cdot 1 = 3x^2 + 3x - 2x - 2 = 3x^2 + x - 2
$$

Yes! This matches the original numerator.

Therefore:

$$
3x^2 + x - 2 = (3x - 2)(x + 1)
$$

2. Rewrite the Function with Factored Numerator

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$

3. Simplify the Expression

Next, look for common factors in numerator and denominator. Notice that the denominator is $3x$.

The numerator factors are $3x - 2$ and $x + 1$.

- The factor $3x - 2$ does not directly cancel with $3x$.
- The factor $x + 1$ does not cancel with $3x$.

Hence, we cannot cancel any factors directly. However, we can express the function as a sum of two fractions:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$

Alternatively, we can split the fraction:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x} = \frac{3x - 2}{3x} \times (x + 1)
$$
But this is not equivalent. Instead, it might be more insightful to divide each term:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$
which can be written as:

$$
f(x) = \frac{(3x - 2)}{3x} \times (x + 1)
$$
but only if the entire numerator is divisible by $3x$, which it isn't necessarily.

Alternatively, we can perform polynomial division or split the numerator to express as a sum:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$

which simplifies better by dividing numerator and denominator term-by-term.

Let's express numerator as:

$$
(3x - 2)(x + 1) = 3x^2 + 3x - 2x - 2 = 3x^2 + x - 2
$$
which is the original numerator. Therefore, in terms of algebraic manipulation, the most straightforward way is to write the function as:

$$
f(x) = \frac{3x^2 + x - 2}{3x}
$$
and then split it into separate fractions:

\[

$$
f(x) = \frac{3x^2}{3x} + \frac{x}{3x} - \frac{2}{3x}
$$
which simplifies to:

$$
f(x) = x + \frac{1}{3} - \frac{2}{3x}
$$

This is the simplest form of the function.

Final Simplified Form of the Function

$$
f(x) = x + \frac{1}{3} - \frac{2}{3x}
$$

Note on the Domain

The simplified form makes it clear that the function has a restriction at $x = 0$, because division by zero is undefined. Therefore, the domain excludes $x = 0$.

Additionally, the original numerator factors do not suggest any other restrictions, so the only restriction comes from the denominator in the original function and the simplified form.

Hence, the domain is:
$$
\boxed{\{x \in \mathbb{R} \mid x \neq 0\}}
$$

Conclusion

By factoring the numerator and decomposing the original rational function, we simplified the function to its most manageable form:

$$
f(x) = x + \frac{1}{3} - \frac{2}{3x}
$$

Understanding this form allows for easier analysis, such as finding limits, asymptotes, and intercepts. The key steps involved factoring the quadratic numerator, rewriting the function as a sum of simpler fractions, and identifying the domain restrictions. Recognizing the domain is essential when working with rational functions, as division by zero is undefined and must be excluded from the set of permissible inputs.

This process exemplifies the importance of algebraic manipulation in simplifying complex functions and highlights the significance of domain considerations in rational expressions.

Frequently Asked Questions

What is the simplified form of the function $f(x) = (3x^2 + x - 2) / 3x$?

Simplify the numerator first: $3x^2 + x - 2$. Since the denominator is $3x$, factor the numerator if possible. The numerator factors as $(3x - 2)(x + 1)$. The function becomes $[(3x - 2)(x + 1)] / 3x$. Since there's no common factor with $3x$, the simplified form is $(3x - 2)(x + 1) / 3x$.

What is the domain of the function $f(x) = (3x^2 + x - 2) / 3x$?

The domain includes all real numbers except where the denominator equals zero. So, set $3x \neq 0$, which gives $x \neq 0$. Therefore, the domain is all real numbers $x \neq 0$.

How do you factor the numerator of $f(x) = (3x^2 + x - 2) / 3x$?

The numerator $3x^2 + x - 2$ factors as $(3x - 2)(x + 1)$.

Can the function $f(x) = (3x^2 + x - 2) / 3x$ be simplified further?

Yes. Using the factorization, $f(x) = [(3x - 2)(x + 1)] / 3x$. No common factors cancel out, so this is the simplified form.

What is the vertical asymptote of $f(x) = (3x^2 + x - 2) / 3x$?

Vertical asymptote occurs where the denominator is zero, i.e., at $x = 0$.

What is the behavior of $f(x)$ as x approaches 0?

As x approaches 0, the denominator approaches 0, so $f(x)$ approaches infinity or negative infinity depending on the direction, indicating a vertical asymptote at $x = 0$.

How do you interpret the simplified form of the function in terms of its graph?

The simplified form reveals potential intercepts and asymptotes. The function has a vertical asymptote at $x=0$ and zeros where numerator factors equal zero, aiding in sketching the graph.

What is the domain and range of the simplified function?

Domain: all real $x \neq 0$. Range can be analyzed by considering limits as x approaches 0 and infinity, but generally, the function can take all real values except those approaching asymptotes, with the range depending on the specific behavior at infinity.

Additional Resources

Simplification of Rational Functions: A Detailed Examination of $f(x) = \frac{3x^2 + x - 2}{3x}$

Introduction

In the realm of algebra, rational functions are fundamental components, often encountered in various mathematical applications ranging from calculus to engineering. Simplifying these functions not only makes them easier to analyze but also provides clearer insights into their behavior and properties.

Today, we delve into the rational function:

$$
f(x) = \frac{3x^2 + x - 2}{3x}
$$

Our goal is to rewrite this function in its simplest form, comprehensively understanding each step involved, including the determination of the domain, potential restrictions, and the algebraic simplification process.

Understanding the Function and Its Components

Before diving into simplification, it’s crucial to analyze the structure of the given function.

The Numerator: $3x^2 + x - 2$

This quadratic expression is a polynomial of degree 2, which can be factored or simplified further if possible. Recognizing its factors will be key in simplifying the entire function.

The Denominator: $3x$

A linear term that, when set to zero, defines points where the function is undefined. These points are essential in establishing the domain restrictions.

Determining the Domain of $f(x)$

The domain of a rational function includes all real numbers x for which the function is defined. Since division by zero is undefined, we need to identify all values of x that make the denominator zero.

$$
3x \neq 0 \Rightarrow x \neq 0
$$

Therefore, the domain is:

$$
\boxed{
\text{Domain} = \{ x \in \mathbb{R} \mid x \neq 0 \}
}
$$

Note: No restrictions are imposed by the numerator since it’s a quadratic polynomial that’s defined for all real x.

Step-by-Step Simplification Process

To rewrite $f(x)$ in its simplest form, proceed through the following steps:

1. Factor the Numerator

Attempt to factor the quadratic numerator $3x^2 + x - 2$.

Method: Factoring by grouping or trial factoring

- Find two numbers that multiply to $3 \times (-2) = -6$ and add up to 1 (the coefficient of x).

Possible pairs:

- 2 and -3:

$$
2 \times (-3) = -6, \quad 2 + (-3) = -1
$$

- -2 and 3:

$$
-2 \times 3 = -6, \quad -2 + 3 = 1
$$

This pair sums to 1, matching the middle coefficient.

Thus, the numerator factors as:

$$
3x^2 + x - 2 = (3x - 2)(x + 1)
$$

Verification:

$$
(3x - 2)(x + 1) = 3x \times x + 3x \times 1 - 2 \times x - 2 \times 1 = 3x^2 + 3x - 2x - 2 = 3x^2 + x - 2
$$

\]

Confirmed.

2. Rewrite the Function with Factored Numerator

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$

Simplification by Canceling Common Factors

Now, examine the expression:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$

Are there any common factors to cancel?

- The numerator contains $3x - 2$ and $x + 1$.
- The denominator is $3x$.

Observation:

- $3x - 2$ and $3x$ are not the same; they share no common factors.
- $x + 1$ and $3x$ share no common factors unless x cancels, which is not the case unless it appears explicitly in both numerator and denominator.

Conclusion:

- No common factors are common to numerator and denominator that can be canceled directly.

However, we can express the function as a sum of simpler fractions:

$$
f(x) = \frac{3x - 2}{3x} \times (x + 1)
$$

But this is not actually a valid algebraic operation unless we explicitly split the fraction, which might be useful for understanding the behavior.

Alternatively, divide numerator and denominator directly:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$

Expressing in a Simpler Form

To write the function in a more understandable form, consider dividing the numerator term-by-term:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x}
$$

Alternatively, perform polynomial division or split the function into separate fractions:

$$
f(x) = \frac{(3x - 2)(x + 1)}{3x} = \left(\frac{3x - 2}{3x}\right) \times (x + 1)
$$

but this is not a standard algebraic operation for rational functions; instead, let's divide numerator directly:

$$
f(x) = \frac{3x^2 + x - 2}{3x}
$$

Divide each term in numerator by denominator:

$$
f(x) = \frac{3x^2}{3x} + \frac{x}{3x} - \frac{2}{3x}
$$

Simplify each term:

- $\frac{3x^2}{3x} = x$
- $\frac{x}{3x} = \frac{1}{3}$
- $\frac{2}{3x}$ remains as is.

Thus, the simplified form:

$$
f(x) = x + \frac{1}{3} - \frac{2}{3x}
$$

Final Simplified Expression

$$
\boxed{
f(x) = x + \frac{1}{3} - \frac{2}{3x}
}
$$

Domain Reminder: Since the term $\frac{2}{3x}$ involves division by x, the original restriction $x \neq 0$ remains valid.

Analyzing the Simplified Function

The rewritten form:

$$
f(x) = x + \frac{1}{3} - \frac{2}{3x}
$$

provides several advantages:

- It clearly shows the function as a combination of a linear term $x + \frac{1}{3}$ and a rational term $- \frac{2}{3x}$.
- It simplifies the process of analyzing limits, asymptotes, and behavior at infinity.

Graphical Insights and Behavior

Understanding the behavior of $f(x)$ becomes more manageable with this form:

- Vertical Asymptote: At $x = 0$, since the term $- \frac{2}{3x}$ becomes unbounded.
- Horizontal Asymptote: As $x \to \pm \infty$, the $\frac{2}{3x}$ term tends towards zero, so:

$$
f(x) \to x + \frac{1}{3}
$$

which is a slant (oblique) asymptote with slope 1 and intercept $\frac{1}{3}$.

Summary of Key Points

Aspect	Explanation
Original Function	$f(x) = \frac{3x^2 + x - 2}{3x}$
Factored Numerator	$(3x - 2)(x + 1)$
Simplified Form	$f(x) = x + \frac{1}{3} - \frac{2}{3x}$
Domain	All real x except $x=0$
Asymptotes	Vertical at $x=0$, oblique as $x \to \pm \infty$

Conclusion

Rewriting the function $f(x) = \frac{3x^2 + x - 2}{3x}$ in its simplest form reveals deeper insights into its structure and behavior. The step-by-step factorization and algebraic division clarify that the function can be expressed as:

$$
f(x) = x + \frac{1}{3} - \frac{2}{3x}
$$

This form is not only more elegant but also more practical for analysis, graphing, and understanding the function’s asymptotic behavior. Recognizing the domain restrictions ensures accurate interpretation and application of the function in various contexts.

This exercise exemplifies the importance of algebraic manipulation and domain analysis in simplifying rational functions, transforming complex expressions into clear, insightful representations.

Rewrite The Function In Simplest Form Including Theappropriate Domain F X 3x2 X 2 3x

Rewrite The Function In Simplest Form Including Theappropriate Domain F X 3x2 X 2 3x

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