Select The Correct Answer. A Circle With Center O (0,0) Has Point B (4,5) On Its Circumference, Which

Select The Correct Answer. A Circle With Center O (0,0) Has Point B (4,5) On Its Circumference, Which introduces a fundamental problem in coordinate geometry that involves understanding the properties of circles, the coordinate plane, and how to determine key attributes such as the radius, the equation of the circle, and related geometrical concepts. This article provides a comprehensive explanation of how to analyze such a problem, interpret the given data, and select the correct answer based on mathematical principles.

Understanding the Basics: The Coordinate Plane and Circles

The Coordinate Plane

The coordinate plane, also known as the Cartesian plane, consists of two perpendicular axes:

- The x-axis (horizontal axis)
- The y-axis (vertical axis)

Any point on this plane can be represented as an ordered pair (x, y), where:

- x is the horizontal distance from the origin (0,0)
- y is the vertical distance from the origin

Equation of a Circle in the Coordinate Plane

A circle centered at point O (0,0), with radius r, has the equation:

- **Standard form:** $x^2 + y^2 = r^2$

If the circle is centered at a point other than the origin, the equation becomes:

- **General form:** $(x - h)^2 + (y - k)^2 = r^2$

where (h, k) is the center of the circle.

Given Data and Its Significance

The problem states:

- Center of the circle: O (0,0)
- Point B: (4,5) lies on the circumference of the circle

This information allows us to determine the radius of the circle, as the distance from the center O to point B is exactly the radius.

Calculating the Radius

The radius r is the distance between points O (0,0) and B (4,5), which can be found using the distance formula:

$$
r = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
$$

Substituting the known values:

$$
r = \sqrt{(4 - 0)^2 + (5 - 0)^2} = \sqrt{16 + 25} = \sqrt{41}
$$

Thus, the radius of the circle is $\sqrt{41}$.

Equation of the Circle

Knowing the radius, we can write the equation of the circle centered at O (0,0):

$$
x^2 + y^2 = r^2 = (\sqrt{41})^2 = 41
$$

Therefore, the equation of the circle is:

Standard form:

- **$x^2 + y^2 = 41$**

This equation summarizes all points (x, y) that lie on the circle with center at (0,0) and passing through B (4,5).

Implications and Further Geometric Concepts

Understanding this setup allows us to explore various properties and answer related questions, such as:

1. The Length of Chords

- For any chord passing through the circle, its length can be calculated if the perpendicular distance from the center to the chord is known.
- The general formula for the length of a chord with a perpendicular distance d from the center is:
$$
\text{Chord length} = 2 \sqrt{r^2 - d^2}
$$
- Example: If a chord is at a distance d from the center, its length can be found using this formula.

2. Coordinates of Points on the Circle

- Any point (x, y) satisfying the circle's equation is on the circumference.
- For example, point B (4, 5) satisfies the equation:
$$
4^2 + 5^2 = 16 + 25 = 41
$$
which confirms it lies on the circle.

3. Geometric Constructions and Problems

- Finding the equations of tangents, secants, and other chords.
- Calculating the area of sectors or segments of the circle.
- Determining the coordinates of other points that satisfy certain conditions.

Common Questions and How to Approach Them

When solving multiple-choice questions related to circles with given centers and points on the circumference, consider the following:

Question 1: What is the radius of the circle?

- As shown, compute the distance from the center to the given point on the circumference.
- In this case, radius r = $\sqrt{41}$.

Question 2: What is the equation of the circle?

- Use the standard form with the calculated radius:

$$
x^2 + y^2 = 41
$$

Question 3: Find the length of a chord at a certain position

- Determine the perpendicular distance from the center to the chord.
- Use the chord length formula.

Question 4: Coordinates of points satisfying certain conditions

- Substitute conditions into the circle's equation to find possible coordinates.

Practical Applications and Real-World Contexts

Understanding circles and their properties is crucial in various fields:

- **Engineering:** Design of gears, circular tracks, and mechanical components.
- **Navigation:** Calculating distances and bearings.
- **Architecture:** Creating circular structures and understanding spatial relationships.
- **Computer Graphics:** Rendering circles and curved shapes.
- **Physics:** Analyzing orbital paths and circular motion.

Summary and Key Takeaways

To summarize, when given a circle with a known center and a point on its circumference:

- Calculate the radius using the distance formula.
- Write the circle's equation using the standard form.
- Use properties of circles for further problem-solving, such as finding chord lengths or other points.
- Apply these principles to answer multiple-choice questions effectively.

This problem exemplifies fundamental concepts in coordinate geometry that are essential for students and professionals alike. Mastery of these techniques enhances problem-solving skills and deepens understanding of geometric relationships in the plane.

Final Note

Always verify your calculations and ensure that your points satisfy the circle's equation before selecting answers. Remember, understanding the relationship between the center, radius, and points on the circle is key to solving most problems involving circles in the coordinate plane.

In conclusion, the problem of identifying the attributes of a circle given its center and a point on its circumference involves straightforward application of the distance formula, equations of circles, and geometric principles. By mastering these concepts, you can confidently approach similar problems and select the correct answers in exams or real-world scenarios.

Frequently Asked Questions

What is the radius of the circle with center O(0,0) passing through point B(4,5)?

The radius is $\sqrt{(4^2 + 5^2)} = \sqrt{(16 + 25)} = \sqrt{41}$.

Which of the following is the equation of the circle with center O(0,0) passing through B(4,5)?

The equation is $x^2 + y^2 = 41$.

If a point B(4,5) lies on the circle with center O(0,0), what is the length of the line segment OB?

The length OB is $\sqrt{(4^2 + 5^2)} = \sqrt{41}$.

Does point B(4,5) lie inside, on, or outside the circle centered at O(0,0) with radius √41?

Point B lies exactly on the circle since its distance from O is $\sqrt{41}$, which equals the radius.

What is the significance of point B(4,5) with respect to the circle centered at O(0,0)?

Point B is a point on the circumference of the circle, defining its radius.

If the circle has center O(0,0) and passes through B(4,5), what is the diameter of the circle?

The diameter is $2 \times \sqrt{41}$.

Additional Resources

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In the realm of coordinate geometry, understanding the properties and equations of circles is fundamental. When a circle is defined with a known center and a point lying on its circumference, numerous questions arise related to its radius, equation, and related geometric configurations. This article delves into the problem: A circle with center O (0,0) has point B (4,5) on its circumference, which, and explores the multiple-choice questions that typically follow such a problem statement, providing a comprehensive analysis of the underlying concepts, calculations, and reasoning processes.

Understanding the Basics: Circle Equation and Coordinates

Before diving into the specific problem, it's crucial to revisit the fundamental concepts of circles in coordinate geometry.

The Standard Equation of a Circle

A circle with center at (h, k) and radius r is described by the equation:

$$(x - h)^2 + (y - k)^2 = r^2$$

In our case, the center O is at $(0, 0)$, simplifying the equation to:

$$x^2 + y^2 = r^2$$

where r is the distance from the center to any point on the circle.

Determining the Radius from a Known Point

Given a point $B(x_b, y_b)$ on the circle's circumference, the radius is simply the distance from the center $O(0, 0)$ to B:

$$ r = \sqrt{(x_b - 0)^2 + (y_b - 0)^2} $$

which directly gives us the radius once the point's coordinates are known.

Calculating the Radius: Point B (4, 5)

Given the problem statement, point $B(4, 5)$ lies on the circumference of the circle centered at $O(0, 0)$.

Step-by-Step Calculation

1. Identify the coordinates:

- Center $O(0, 0)$

- Point $B(4, 5)$

2. Apply the distance formula:

$$
r = \sqrt{(4 - 0)^2 + (5 - 0)^2} = \sqrt{16 + 25} = \sqrt{41}
$$

3. Result:

$$
\boxed{r = \sqrt{41}}
$$

This value is central to understanding the circle's properties, including its equation and related geometric measures.

Formulating the Equation of the Circle

With the radius $r = \sqrt{41}$, the explicit equation of the circle is:

$$ x^2 + y^2 = 41 $$

This fundamental equation serves as the basis for answering multiple-choice questions, such as identifying points lying on the circle, calculating diameters, or determining the length of chords.

Common Multiple-Choice Questions and Their Analysis

Problems related to this configuration often include multiple-choice questions focusing on various properties:

1. What is the length of the diameter?

Options:

- A) $2\sqrt{41}$

- B) 2×41

- C) $\sqrt{82}$

- D) 41

Analysis:

The diameter D is twice the radius:

$$
D = 2r = 2 \times \sqrt{41} = \boxed{2\sqrt{41}}
$$

Correct answer: A

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2. Which of the following points lie on the circle?

Options:

- A) $(2, 4)$

- B) $(0, \pm \sqrt{41})$

- C) $(\pm 1, \pm 6)$

- D) $(\pm \sqrt{41}, 0)$

Analysis:

To verify, substitute each point into the circle's equation $x^2 + y^2 = 41$:

- A) $2^2 + 4^2 = 4 + 16 = 20 \neq 41$

- B) $0^2 + (\pm \sqrt{41})^2 = 0 + 41 = 41$ → lies on the circle.

- C) $1^2 + 6^2 = 1 + 36 = 37 \neq 41$

- D) $(\pm \sqrt{41})^2 + 0^2 = 41 + 0 = 41$ → lies on the circle.

Correct answers: B and D

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3. What is the length of the chord passing through point $B(4, 5)$ and the origin?

Options:

- A) $\sqrt{41}$

- B) $2r$

- C) $\sqrt{82}$

- D) $\sqrt{41}$

Analysis:

Since the points are $O(0, 0)$ and $B(4, 5)$, the chord's length between these points is simply the distance:

$$
\text{Chord length} = \sqrt{(4 - 0)^2 + (5 - 0)^2} = \sqrt{16 + 25} = \sqrt{41}
$$

Correct answer: A

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4. What is the equation of the circle in standard form?

Options:

- A) $x^2 + y^2 = 41$

- B) $x^2 + y^2 = 20$

- C) $x^2 + y^2 = 25$

- D) $x^2 + y^2 = 16$

Analysis:

As calculated earlier, the radius is $\sqrt{41}$, so the equation is:

$$
x^2 + y^2 = 41
$$

Correct answer: A

Extended Geometric Insights and Applications

The problem extends beyond mere calculations, inviting a deeper exploration of circle properties and their implications in geometric constructions.

Application in Coordinate Geometry and Trigonometry

Knowing the radius and the position of points on the circle allows for:

- Calculation of angles subtended by chords at the center.

- Determining the length of tangents from external points.

- Analyzing the position of other points relative to the circle.

Understanding the Significance of the Center at the Origin

Having the circle centered at $(0, 0)$ simplifies many calculations, as the radius directly corresponds to the distance from the origin to the point B. This symmetry facilitates easier derivations and problem-solving.

Implications for Coordinate Geometry Problems

Students and researchers encountering such problems are encouraged to:

- Use the distance formula as a primary tool.

- Recall the standard form of the circle equation for quick verification.

- Visualize the geometric configuration to understand relationships between points, lines, and circles.

Conclusion: Selecting the Correct Answer and Its Significance

In conclusion, the problem of a circle centered at $O(0, 0)$ passing through $B(4, 5)$ exemplifies key concepts in coordinate geometry, particularly the calculation of radius and the formulation of the circle's equation. The correct answer, based on thorough analysis, confirms that:

- The radius $r = \sqrt{41}$.

- The standard equation of the circle is $x^2 + y^2 = 41$.

- The diameter is $2\sqrt{41}$.

- Points satisfying the equation include $(0, \pm \sqrt{41})$ and $(\pm \sqrt{41}, 0)$.

This problem underscores the importance of understanding fundamental formulas, geometric reasoning, and their applications in solving complex questions. Whether for academic assessments, competitive exams, or research purposes, mastering these concepts enhances one's ability to analyze and interpret coordinate geometric figures accurately.

In summary, the correct answer to the typical multiple-choice question derived from this problem is:

Select The Correct Answer:
A) $2\sqrt{41}$ for the diameter, or equivalently, the equation of the circle being $x^2 + y^2 = 41$, depending on the specific question asked.

This comprehensive analysis affirms the importance of precise calculations and conceptual understanding in coordinate geometry, providing a solid foundation for further exploration of geometric properties and their applications.

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