Select The Equation That Is The Inverse Of The Given Function. F (x) = StartFraction X Minus 3 Over 5

Select The Equation That Is The Inverse Of The Given Function. F (x) = $\frac{x - 3}{5}$

Understanding inverse functions is a fundamental concept in algebra and mathematical analysis. When we talk about the inverse of a function, we are referring to a function that essentially "undoes" the original function's operation. In this article, we will explore how to find the inverse of the given function $F(x) = \frac{x - 3}{5}$, analyze its properties, and identify the correct inverse equation among potential options.

What Is an Inverse Function?

Definition of an Inverse Function

An inverse function, denoted as $F^{-1}(x)$, is a function that reverses the effect of the original function $F(x)$. Formally, if F is a function with domain D and range R, then its inverse F^{-1} is a function with domain R and range D such that:

$$
F^{-1}(F(x)) = x \quad \text{for all } x \in D
$$
$$
F(F^{-1}(x)) = x \quad \text{for all } x \in R
$$

In simple terms, applying F and then F^{-1} returns you to your original input, and vice versa.

Characteristics of Inverse Functions

- They "mirror" the original function across the line $y = x$.
- The inverse of a linear function is also linear.
- Not all functions have inverses; a function must be one-to-one (injective) to have an inverse.

Understanding the Given Function $F(x) = \frac{x - 3}{5}$

Type and Properties

- The function $F(x) = \frac{x - 3}{5}$ is a linear function.
- It has a slope of $\frac{1}{5}$ and a y-intercept at $-\frac{3}{5}$.
- Because it is linear with a non-zero slope, it is one-to-one and thus has an inverse.

Graphical Insight

- The graph of $F(x)$ is a straight line inclined at a gentle slope.
- Reflecting this line across the line $y = x$ gives the inverse function.

How to Find the Inverse of $F(x) = \frac{x - 3}{5}$

Step-by-Step Process

To find the inverse function $F^{-1}(x)$, follow these steps:

1. Replace $F(x)$ with y:
   $$
   y = \frac{x - 3}{5}
   $$

2. Interchange the roles of x and y:
   $$
   x = \frac{y - 3}{5}
   $$

3. Solve this equation for y:
   $$
   x = \frac{y - 3}{5}
   $$
   Multiply both sides by 5:
   $$
   5x = y - 3
   $$
   Add 3 to both sides:
   $$
   y = 5x + 3
   $$

Thus, the inverse function is:
$$
F^{-1}(x) = 5x + 3
$$

Verification of the Inverse

To verify that $F^{-1}(x) = 5x + 3$ is indeed the inverse, check the compositions:

- Compute $F(F^{-1}(x))$:
$$
F(F^{-1}(x)) = F(5x + 3) = \frac{(5x + 3) - 3}{5} = \frac{5x}{5} = x
$$
- Compute $F^{-1}(F(x))$:
$$
F^{-1}\left(\frac{x - 3}{5}\right) = 5 \times \frac{x - 3}{5} + 3 = (x - 3) + 3 = x
$$

Both compositions return x, confirming the inverse.

Matching the Inverse with Given Options

Suppose you are presented with multiple-choice options for the inverse function; your task is to select the correct one based on the derived formula.

- (A) $F^{-1}(x) = \frac{x + 3}{5}$
- (B) $F^{-1}(x) = 5x + 3$
- (C) $F^{-1}(x) = \frac{5x - 3}{1}$
- (D) $F^{-1}(x) = \frac{x - 3}{5}$

Analyzing these options:

- Option A: $F^{-1}(x) = \frac{x + 3}{5}$ — Incorrect, as the inverse should be $5x + 3$.
- Option B: $F^{-1}(x) = 5x + 3$ — Correct, matches the derived inverse.
- Option C: $F^{-1}(x) = 5x - 3$ — Incorrect.
- Option D: $F^{-1}(x) = \frac{x - 3}{5}$ — This is actually the original function, not the inverse.

Therefore, the correct choice is Option B.

Additional Insights and Applications

Understanding the Reflection Property

- The inverse function graph is a reflection of the original function's graph across the line $y = x$.
- Visualizing this reflection helps in comprehending the inverse's behavior.

Practical Applications

- Inverting functions is crucial in various fields such as engineering, physics, and computer science.
- It allows solving for input variables given output values, essential in reverse engineering and data analysis.

Summary

- The given function $F(x) = \frac{x - 3}{5}$ is a linear, one-to-one function with an inverse.
- The inverse is found by swapping x and y and solving for y, resulting in $F^{-1}(x) = 5x + 3$.
- Among given options, the inverse corresponds to option B.
- Understanding how to find and verify inverse functions enhances comprehension of algebraic transformations and their real-world applications.

Conclusion

Finding the inverse of a function involves understanding the fundamental concept of reversing the operations performed by the original function. For the specific case of $F(x) = \frac{x - 3}{5}$, the process is straightforward due to its linearity. After swapping variables and solving, we determine that the inverse function is $F^{-1}(x) = 5x + 3$. Recognizing this allows students and professionals to correctly identify the inverse equation among options, deepen their grasp of algebraic concepts, and apply this knowledge across various mathematical and practical contexts.

Frequently Asked Questions

What is the inverse of the function F(x) = (x - 3)/5?

The inverse function is $F^{-1}(x) = 5x + 3$.

How do you find the inverse of F(x) = (x - 3)/5?

Replace F(x) with y, swap x and y, then solve for y: y = (x - 3)/5 becomes x = (y - 3)/5, so y = 5x + 3.

Is the inverse of F(x) = (x - 3)/5 a linear function?

Yes, the inverse is $F^{-1}(x) = 5x + 3$, which is linear.

What is the domain and range of the inverse function for F(x) = (x - 3)/5?

The domain of the inverse is all real numbers, and the range is also all real numbers.

Can the inverse of $F(x) = (x - 3)/5$ be written as $y = 5x + 3$?

Yes, the inverse is $y = 5x + 3$.

What steps are involved in finding the inverse of $F(x) = (x - 3)/5$?

Replace $F(x)$ with y, swap x and y, then solve for y: $y = (x - 3)/5 \rightarrow x = (y - 3)/5 \rightarrow y = 5x + 3$.

Does the inverse function $F^{-1}(x) = 5x + 3$ satisfy the property that $F(F^{-1}(x)) = x$?

Yes, substituting $F^{-1}(x)$ into $F(x)$ confirms that $F(F^{-1}(x)) = x$.

Is the original function $F(x) = (x - 3)/5$ one-to-one and invertible?

Yes, it is linear and one-to-one, so it has an inverse.

What is the inverse function of $F(x) = (x - 3)/5$ expressed in terms of x?

$F^{-1}(x) = 5x + 3$.

If $F(x) = (x - 3)/5$, what is $F^{-1}(2)$?

$F^{-1}(2) = 5(2) + 3 = 10 + 3 = 13$.

Additional Resources

Inverse Functions: Unlocking the Secrets of Reversibility in Mathematics

Understanding inverse functions is a cornerstone of algebra and advanced mathematics, offering insight into how functions can be "reversed" to retrieve original inputs from outputs. In this article, we'll explore the process of finding the inverse of a specific function, $F(x) = (x - 3) / 5$, and how to select the correct inverse equation from multiple options. This deep dive will serve as both a detailed tutorial and a professional guide for students, educators, and math enthusiasts alike.

Deciphering the Given Function

Before diving into the inverse, it's essential to understand the structure and properties of the original function, $F(x) = (x - 3) / 5$.

Function Breakdown

- Type of Function: The given function is a linear function, characterized by its straight-line graph. Its general form can be expressed as f(x) = mx + b, where:
- m is the slope
- b is the y-intercept

- Given Function in Standard Form:
- F(x) = (x - 3) / 5
- This can be rewritten as:
- F(x) = (1/5) x - (3/5)

- Properties:
- Domain: All real numbers (ℝ), since dividing by 5 poses no restrictions.
- Range: All real numbers (ℝ), because the linear function spans the entire real line.
- Injectivity: The function is one-to-one (injective), since linear functions with non-zero slope are invertible over ℝ.
- Continuity and Differentiability: Both are continuous and differentiable everywhere on ℝ.

The Concept of Inverse Functions

An inverse function, denoted as F-1(x), essentially reverses the effect of the original function. If F(x) takes an input x and produces an output y, then F-1(x) takes y and returns x.

Key Point:
- The inverse "undoes" the original function.
- For a function F(x) to have an inverse, it must be bijective (both injective and surjective).

Mathematically:
- If y = F(x), then x = F-1(y).

Graphical Interpretation:
- The graph of an inverse function is the reflection of the original function across the line y = x.

Step-by-Step Derivation of the Inverse

Finding the inverse involves algebraic manipulation:

Step 1: Write the original function

- F(x) = (x - 3) / 5

Step 2: Replace F(x) with y

- y = (x - 3) / 5

Step 3: Swap x and y to reflect the inverse relationship

- x = (y - 3) / 5

Step 4: Solve for y

- Multiply both sides by 5:
- 5x = y - 3
- Add 3 to both sides:
- y = 5x + 3

Step 5: Express the inverse function

- F-1(x) = 5x + 3

This formula now allows us to find the original input x given an output of the original function.

Validating the Inverse Function

To ensure correctness, test the inverse with specific values:

- Suppose x = 4.
- Compute F(4):
- F(4) = (4 - 3) / 5 = 1/5 = 0.2
- Now, apply the inverse to this output:
- F-1(0.2) = 5(0.2) + 3 = 1 + 3 = 4
- The inverse correctly retrieves the original input, confirming accuracy.

Choosing the Correct Inverse Equation from Options

In many scenarios, multiple candidate equations are provided, and selecting the correct inverse requires understanding the derivation process and properties discussed. Here's a strategic approach:

1. Verify the Algebraic Structure

- The inverse should algebraically simplify to the form F-1(x) = 5x + 3 for the given function.

2. Confirm the Reflection Property

- Graphically, the inverse should be the original function reflected across y = x.

3. Test with Sample Values

- Pick test inputs and outputs to validate the candidate inverse.

4. Check Domain and Range

- The inverse should have domain ℝ (matching the original function's range) and range ℝ (matching the original's domain).

5. Discard Illogical or Non-Inverse Forms

- Any candidate that doesn't satisfy the inverse relationship or conflicts with the properties above should be eliminated.

Summary of Key Takeaways

- The original function F(x) = (x - 3) / 5 is linear, invertible, and straightforward to invert algebraically.
- The inverse function derived is F-1(x) = 5x + 3, which correctly reverses the original transformation.
- Validating the inverse through substitution confirms its correctness.
- Selection among multiple options involves algebraic verification, understanding reflection properties, and testing with specific values.

Final Thoughts

Understanding how to find and verify inverse functions is a fundamental skill in mathematics, providing insights into symmetry, invertibility, and function behavior. The specific case of F(x) = (x - 3) / 5 exemplifies how linear functions are easily inverted through algebraic manipulation. Whether you're tackling homework, preparing for exams, or exploring advanced topics like calculus and functional analysis, mastering inverse functions is essential for deep mathematical comprehension.

Remember: Always double-check your derived inverse, test with multiple values, and ensure the functional properties align. With practice, selecting the correct inverse from options becomes an intuitive process, reinforcing your overall grasp of function transformations and their inverses.

In conclusion, the inverse of F(x) = (x - 3) / 5 is F-1(x) = 5x + 3. Whether presented as a standalone equation or multiple-choice options, understanding the derivation process is crucial to making the correct selection and enhancing your mathematical maturity.

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