

Solve Each Equation In The Interval From 0 To 2. Round Your Answer To The Nearest Hundredth. $\cos T = 1/4$

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Understanding how to solve trigonometric equations within specific intervals is a fundamental skill in mathematics, particularly in calculus and analytical geometry. In this article, we will explore the detailed process of solving the equation $\cos T = 1/4$ within the interval from 0 to 2 radians, including how to find approximate solutions rounded to the nearest hundredth. Whether you're a student preparing for exams or a math enthusiast seeking clarity, this guide will provide comprehensive insights into the problem-solving steps, relevant concepts, and application tips.

Introduction to the Problem

The given equation is:

```
```plaintext
cos T = 1/4
```
```

with the interval:

```
```plaintext
0 ≤ T ≤ 2
```
```

Here, T represents an angle in radians, and the goal is to find all solutions T within the specified interval that satisfy the equation. Since the cosine function is periodic, multiple solutions may exist within the interval, and understanding the properties of cosine is crucial to identifying all solutions.

Understanding the Cosine Function

Before solving the equation, it's essential to review the key properties of the cosine function:

Properties of Cosine Function

- **Periodicity:** Cosine is periodic with a period of 2π ; that is, $\cos(T + 2\pi) = \cos T$ for all T .
- **Range:** The output of cosine ranges from -1 to 1.

- **Symmetry:** Cosine is an even function, meaning $\cos(-T) = \cos T$.
- **Values at key points:** $\cos 0 = 1$, $\cos \pi/2 = 0$, $\cos \pi = -1$, etc.

Given that $\cos T = 1/4$, which lies between -1 and 1 , solutions exist within the interval $[0, 2\pi]$.

Approach to Solving $\cos T = 1/4$

The general approach involves:

1. Isolating the variable T .
2. Using inverse cosine ($\arccos$) to find principal solutions.
3. Considering the periodic nature of cosine to find all solutions within the interval.
4. Rounding the solutions to the nearest hundredth.

Let's proceed step-by-step.

Step 1: Applying the Inverse Cosine Function

The inverse cosine function, $\arccos$, helps find an angle T such that $\cos T$ equals a given value:

```
```plaintext
T = arccos(1/4)
```
```

Using a calculator set to radians, compute:

```
```plaintext
T1 = arccos(1/4)
```
```

Calculating this:

```
```plaintext
T1 ≈ arccos(0.25)
```
```

Using a calculator:

```
```plaintext
T1 ≈ 1.3181 radians
```
```

This is the principal value, which lies within $[0, \pi]$, as $\arccos$ always returns values in $[0, \pi]$.

Note: Since $\cos T$ is symmetric about the y-axis, the second solution within one period is related to the cosine's symmetry.

Step 2: Finding All Solutions in the Interval $[0, 2]$

The cosine function is symmetric, and solutions can be found using the property:

```
```plaintext
cos T = cos (2π - T)
```
```

However, our interval is from 0 to 2 radians, which is less than one full period ($2\pi \approx 6.2832$). Therefore, only solutions corresponding to T_1 and its supplementary angle within this interval need to be considered.

The other solution is:

```
```plaintext
T2 = 2π - T1
```
```

Calculating T_2 :

```
```plaintext
T2 ≈ 6.2832 - 1.3181 ≈ 4.9651 radians
```
```

Now, check whether these solutions are within the interval from 0 to 2:

- $T_1 \approx 1.3181 \rightarrow$ within $[0, 2]$
- $T_2 \approx 4.9651 \rightarrow$ exceeds 2, so outside the interval

Therefore, only $T \approx 1.3181$ radians is within the interval from 0 to 2.

Important: Since T_2 is outside $[0, 2]$, it is not considered a solution for this problem.

Final Solutions and Rounding

Based on the above steps, the only solution to $\cos T = 1/4$ in the interval $[0, 2]$ radians is approximately:

```
```plaintext
T ≈ 1.32 radians
```
```

rounded to the nearest hundredth.

Verification of the Solution

Verifying the solution is crucial to ensure accuracy:

- Calculate $\cos(1.32)$:

```
```plaintext
cos(1.32) ≈ 0.249
```
```

...

which is very close to 0.25, confirming the solution's correctness within typical calculator precision.

Additional Considerations and Tips

When solving such equations, consider the following tips:

- **Always check the interval:** Since cosine is periodic, solutions repeat every 2π ; ensure solutions lie within the specified interval.
- **Use calculator carefully:** Make sure it is set to radians unless specified otherwise.
- **Round appropriately:** Round to the nearest hundredth as required, but retain enough decimal places during calculations to avoid rounding errors affecting the final answer.
- **Understand symmetry:** For $\cos T = c$, the solutions are $T = \arccos c$ and $T = 2\pi - \arccos c$ within one period.
- **Convert to degrees if needed:** Sometimes, it helps to visualize solutions in degrees, but always convert back to radians for final answers if working in radians.

Summary of Key Steps

1. Compute $T_1 = \arccos(1/4)$.
2. Determine if the symmetric solution $2\pi - T_1$ is within the interval.
3. Only include solutions within $[0, 2]$.
4. Round the solutions to the nearest hundredth.

Additional Practice Problems

To strengthen understanding, try solving similar equations:

- $\cos T = -1/2$ in $[0, 2]$
- $\cos T = \sqrt{3}/2$ in $[0, 2]$
- $\cos T = 0.6$ in $[0, 2]$

Apply the same steps: find the principal value using $\arccos$, then find symmetric solutions, and check whether they lie in the interval.

Conclusion

In summary, solving $\cos T = 1/4$ within the interval from 0 to 2 radians involves calculating the inverse cosine, understanding the periodic nature of cosine, and selecting solutions that fall within the interval. The primary solution, approximately 1.32 radians, is the only solution in the specified interval when rounded to the nearest hundredth. Mastery of these steps enhances problem-solving skills in trigonometry and prepares you for more complex equations involving periodic functions.

If you follow this detailed approach, you'll be well-equipped to solve similar trigonometric equations efficiently and accurately for various intervals and values.

Frequently Asked Questions

What are the solutions to $\cos T = 1/4$ in the interval from 0 to 2 radians?

$T \approx 1.32$ radians and $T \approx 4.96$ radians.

How do I find the solutions to $\cos T = 1/4$ between 0 and 2?

Calculate the principal value $T = \arccos(1/4) \approx 1.32$ radians, then find the second solution as $T \approx 2\pi - 1.32 \approx 4.96$ radians.

Is the solution $T = \arccos(1/4)$ sufficient to find all solutions in $[0, 2]$?

Yes, because cosine is positive in the first and fourth quadrants within 0 to 2 radians, so the two solutions are $\arccos(1/4)$ and $2\pi - \arccos(1/4)$.

What is the value of $\arccos(1/4)$ rounded to the nearest hundredth?

Approximately 1.32 radians.

How do I convert the solutions from radians to degrees for $\cos T = 1/4$?

Multiply each radian value by $180/\pi$: for example, 1.32 radians $\approx 75.57^\circ$, and 4.96 radians $\approx 284.43^\circ$.

Are there any solutions outside the interval $[0, 2]$ for $\cos T = 1/4$?

No, within $[0, 2]$ radians, the solutions are approximately 1.32 and 4.96 radians; outside this interval, solutions are not considered.

Why do we need to consider both $\arccos(1/4)$ and $2\pi - \arccos(1/4)$ when solving $\cos T = 1/4$?

Because cosine is positive in both the first and fourth quadrants, corresponding to these two solutions within the interval.

Can I use a calculator to find the solutions to $\cos T = 1/4$? If so, how?

Yes, use the inverse cosine function: $T = \arccos(0.25)$. Then, find the second solution as $2\pi - T$.

What is the approximate value of the second solution to $\cos T = 1/4$ in radians?

Approximately 4.96 radians.

Additional Resources

Solving Trigonometric Equations: An In-Depth Exploration of $\cos T = 1/4$ in the Interval $[0, 2\pi]$

Introduction

When it comes to solving trigonometric equations, the process can sometimes appear daunting, especially for students new to the subject. However, with a systematic approach, understanding the fundamental properties of the cosine function, and the use of a calculator, solving equations like $\cos T = 1/4$ within a specified interval becomes manageable and even rewarding. In this article, we will explore the problem $\cos T = 1/4$ in detail, focusing on solutions within the interval $[0, 2\pi]$ (which corresponds to 0 to approximately 6.2832 radians). We will also guide you through the entire process, from understanding the problem to applying the calculator for precise answers rounded to the nearest hundredth.

Understanding the Equation: $\cos T = 1/4$

Before diving into the solution process, it's essential to understand what the equation $\cos T = 1/4$ signifies:

- **Cosine Function Overview:** The cosine function, $\cos T$, measures the horizontal coordinate of a point on the unit circle corresponding to an angle T (measured in radians). Its range is $[-1, 1]$, meaning it can take any value between -1 and 1 inclusive.

- **Equation Context:** The equation asks us to find all angles T within the interval $[0, 2\pi]$ where the cosine value equals $1/4$, a positive value less than 1, indicating the angle is in the first or fourth quadrants.

Step 1: Recognize the Nature of the Solution

Since $\cos T = 1/4$:

- The solutions will be angles T where the cosine function attains 0.25.
- Because $\cos T$ is positive, solutions are in quadrants I (0 to $\pi/2$ radians) and IV ($3\pi/2$ to 2π radians).

This insight reduces the solution search to specific regions of the unit circle.

Step 2: Isolate the Variable and Use Inverse Cosine (Arccos)

To find the solutions:

1. Apply the inverse cosine function (arccos):

```
\[
T = \arccos(1/4)
\]
```

2. Compute the principal value:

Using a calculator set to radians (since the interval is in radians),

```
\[
T_1 = \arccos(0.25)
\]
```

3. Find the second solution based on cosine symmetry:

Cosine is symmetric about the vertical axis, meaning if T_1 is a solution, then $2\pi - T_1$ is also a solution within $[0, 2\pi]$.

Step 3: Calculating the Numerical Values

Now, it's time to perform the actual calculations:

- Calculate T_1 :

```
\[
T_1 = \arccos(0.25)
\]
```

- Calculate T_2 :

```
\[
T_2 = 2\pi - T_1
\]
```

Using a calculator:

- $\arccos(0.25)$:

```
\[
T_1 \approx \boxed{1.32 \text{ radians}}
\]
```

(rounded to the nearest hundredth)

- Second solution:

```
\[
```

$$T_2 = 2\pi - 1.32 \approx 6.2832 - 1.32 = \boxed{4.96 \text{ radians}}$$

Step 4: Confirm the Solutions Fall Within the Interval

Both solutions, $T \approx 1.32$ and $T \approx 4.96$, are within $[0, 2\pi]$:

- $0 \leq 1.32 \leq 6.2832$
- $0 \leq 4.96 \leq 6.2832$

Thus, both are valid solutions in the given interval.

Step 5: Final Answer with Rounded Values

Rounding to the nearest hundredth:

$$\begin{aligned} T_1 &\approx 1.32 \text{ radians} \\ T_2 &\approx 4.96 \text{ radians} \end{aligned}$$

These are the solutions to $\cos T = 1/4$ in $[0, 2\pi]$.

Additional Tips for Solving Similar Equations

- Always verify the range of the cosine function: Since $\cos T$ ranges from -1 to 1 , check if the given value is within this domain before attempting to solve.
- Use a calculator with inverse cosine capability and ensure it's set to radians if the interval is in radians.
- Remember the symmetry of cosine: If $\cos T = k$, solutions are $T = \arccos(k)$ and $T = 2\pi - \arccos(k)$ within $[0, 2\pi]$.
- Adjust solutions for other intervals as necessary, especially when dealing with solutions outside the primary interval.

Conclusion

Solving $\cos T = 1/4$ within the interval $[0, 2\pi]$ is a straightforward process that hinges on understanding the properties of the cosine function and effectively utilizing inverse trigonometric functions. The key steps involve:

- Recognizing the symmetry of cosine.
- Computing the primary inverse cosine value.
- Determining the second solution using the cosine symmetry property.
- Rounding the solutions to the nearest hundredth for precision and clarity.

Final solutions:

- $T \approx 1.32$ radians
- $T \approx 4.96$ radians

This process exemplifies how fundamental trigonometric principles combined with calculator tools can efficiently solve equations that at first glance might seem complex. Whether you're preparing for exams or applying these concepts in real-world modeling, mastering these steps will enhance your confidence and problem-solving skills in trigonometry.

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