Solve The Equation. (x+7)(x-3)=(x+1)^{2} Select The Correct Choice Below And Fill In Any Answer Boxes

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Are you struggling with solving quadratic equations? Do you want to master the process of solving equations like $(x+7)(x-3) = (x+1)^2$? This comprehensive guide will walk you through the step-by-step process of solving this specific equation, help you understand the underlying concepts, and provide tips to select the correct answer choice. Whether you're a student preparing for exams or someone interested in enhancing your algebra skills, this article is designed to be clear, detailed, and easy to follow.

Understanding the Equation

Before diving into the solution, let's analyze the given equation:

$(x+7)(x-3) = (x+1)^2$

This is a quadratic equation involving binomials on both sides. The goal is to find all real values of x that satisfy this equality.

Key Points:

- The left side is a product of two binomials: $(x+7)(x-3)$.
- The right side is a perfect square: $(x+1)^2$.
- The equation is set equal to zero when rearranged, which helps in solving.

Step-by-Step Solution Process

Let's break the solving process into clear, manageable steps.

Step 1: Expand Both Sides

To compare and solve the equation, expand both sides into polynomial form.

Expand $(x+7)(x-3)$:

- Use the distributive property (FOIL method):

$(x)(x) = x^2$

$(x)(-3) = -3x$

$(7)(x) = 7x$

$(7)(-3) = -21$

- Combine like terms:

$x^2 + (-3x + 7x) - 21 = x^2 + 4x - 21$

Expand $(x+1)^2$:

- Use the binomial square formula:

$(x+1)^2 = x^2 + 2x + 1$

Now, the equation becomes:

$x^2 + 4x - 21 = x^2 + 2x + 1$

Step 2: Bring All Terms to One Side

Subtract $x^2 + 2x + 1$ from both sides:

$(x^2 + 4x - 21) - (x^2 + 2x + 1) = 0$

Simplify:

$x^2 + 4x - 21 - x^2 - 2x - 1 = 0$

Combine like terms:

$(x^2 - x^2) + (4x - 2x) + (-21 - 1) = 0$

$0 + 2x - 22 = 0$

Resulting in:

$2x - 22 = 0$

Step 3: Solve for x

Isolate x:

$2x = 22$

$x = 22 / 2$

$x = 11$

Checking for Additional Solutions

At this point, the simplified process yielded one solution, $x = 11$. However, since the original equation involves quadratic expressions, it's prudent to verify if any other solutions exist, especially in cases where quadratic equations have multiple roots.

Note: The previous steps simplified the problem to a linear equation, indicating potentially only one real solution. But for completeness, let's verify whether any extraneous solutions or additional roots exist.

Step 4: Verify the Solution

Substitute $x = 11$ into the original equation:

Left side:

$(x+7)(x-3) = (11+7)(11-3) = (18)(8) = 144$

Right side:

$(x+1)^2 = (11+1)^2 = (12)^2 = 144$

Since both sides equal 144, $x = 11$ is a valid solution.

Conclusion: The only real solution to the equation is $x = 11$.

Additional Considerations

While the problem simplifies to a linear solution, in more complex quadratic equations, you might encounter two solutions. Here's how to handle such situations:

- Use the quadratic formula when necessary:

For equations in the form $ax^2 + bx + c = 0$, solutions are:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$

- Always check solutions in the original equation to avoid extraneous roots introduced during algebraic manipulations.

Choosing the Correct Answer and Filling in Answer Boxes

Suppose the question provides multiple-choice options for the solution, like:

a) x = 11

b) x = -11

c) x = 0

d) x = 7

In this case, based on our calculations, the correct choice is:

a) x = 11

If the answer box is provided for you to fill in, simply input:

11

Always double-check your answer by substituting back into the original equation to confirm the solution's validity.

Summary of Key Steps for Solving Similar Equations

- Expand binomials and products thoroughly.
- Simplify both sides into polynomial form.
- Bring all terms to one side to set the equation to zero.
- Use appropriate methods (factoring, quadratic formula, completing the square) to solve.
- Verify solutions in the original equation.
- Be cautious of extraneous solutions, especially when squaring both sides.

Tips for Success in Solving Quadratic Equations

- Always perform algebraic operations carefully to avoid errors.
- Remember that squaring both sides can introduce extraneous solutions; verify solutions afterwards.

- Use the quadratic formula when factoring is difficult.
- Keep track of your steps and double-check the calculations.
- Practice solving different types of quadratic equations to become proficient.

Conclusion

Solving equations like $(x+7)(x-3) = (x+1)^2$ may seem challenging at first glance, but with a systematic approach, it becomes manageable. By expanding, simplifying, and verifying your solutions, you can confidently find the correct answer. In this particular problem, the solution is $x = 11$, which satisfies the original equation. Remember, understanding each step and verifying your results are key to mastering algebraic equations.

Whether you're preparing for tests, homework, or just want to improve your algebra skills, practicing such problems will enhance your problem-solving abilities. Keep practicing, stay attentive to details, and you'll become adept at solving quadratic equations in no time!

Frequently Asked Questions

What is the first step to solve the equation $(x+7)(x-3) = (x+1)^2$?

Expand both sides of the equation to eliminate the parentheses and simplify each side.

After expanding, what form should the equation take?

The equation should be in quadratic form: $ax^2 + bx + c = 0$.

How do you expand $(x+7)(x-3)$?

Use the distributive property: $xx + x(-3) + 7x + 7(-3) = x^2 - 3x + 7x - 21 = x^2 + 4x - 21$.

How do you expand $(x+1)^2$?

Use the formula for a perfect square: $(x+1)^2 = x^2 + 2x + 1$.

What is the resulting quadratic equation after expansion?

Set the expanded forms equal: $x^2 + 4x - 21 = x^2 + 2x + 1$, then simplify to find the solutions.

How do you simplify the equation x^2 + 4x - 21 = x^2 + 2x + 1?

Subtract x^2 + 2x + 1 from both sides to get: (x^2 + 4x - 21) - (x^2 + 2x + 1) = 0.

What is the simplified form after subtraction?

Simplify to: (x^2 - x^2) + (4x - 2x) + (-21 - 1) = 0, which simplifies further to 2x - 22 = 0.

How do you solve for x in the equation 2x - 22 = 0?

Add 22 to both sides: 2x = 22, then divide both sides by 2: x = 11.

Are there any other solutions to the original equation?

No, solving the quadratic gives x = 11. Verify by substituting back into the original equation to check.

What is the final answer to the equation (x+7)(x-3) = (x+1)^2?

The solution is x = 11.

Additional Resources

Mathematical Problem Solving

Introduction to the Equation

Mathematics often presents us with equations that challenge our problem-solving skills and deepen our understanding of algebraic principles. The equation at hand:

$$
(x + 7)(x - 3) = (x + 1)^2
$$

is a classic example of a quadratic equation presented in factored form, equated to a squared binomial. Its structure offers multiple pathways for solution, including expansion, simplification, and the application of quadratic formulas. As an expert in algebra, I will guide you through an in-depth analysis of this equation, explore the methods to solve it, and help you select the correct solution.

Understanding the Equation

Before diving into solving, it’s essential to understand the components of the equation:

- Left side: $(x + 7)(x - 3)$ is a product of two binomials.
- Right side: $(x + 1)^2$ is a perfect square binomial.

This setup suggests that expanding both sides and bringing everything to one side to set the equation to zero is a straightforward approach. Alternatively, recognizing the structure can hint at solutions via factorization or substitution.

Method 1: Expansion and Simplification

The most systematic approach involves expanding both sides to form a quadratic equation, then solving for x.

Step 1: Expand the Left Side

Apply the distributive property (FOIL method):

$$
(x + 7)(x - 3) = x \times x + x \times (-3) + 7 \times x + 7 \times (-3)
$$

which simplifies to:

$$
x^2 - 3x + 7x - 21 = x^2 + 4x - 21
$$

Step 2: Expand the Right Side

$$
(x + 1)^2 = x^2 + 2x + 1
$$

Step 3: Set the Equation

Now, equate both sides:

$$
x^2 + 4x - 21 = x^2 + 2x + 1
$$

Step 4: Simplify and Solve

Subtract x^2 from both sides:

$$
4x - 21 = 2x + 1
$$

Bring like terms together:

$$
4x - 2x = 1 + 21
$$

$$
2x = 22
$$

Divide both sides by 2:

$$
x = 11
$$

This is our first potential solution.

Method 2: Alternative Approach — Re-arrangement

and Factoring

Sometimes, directly expanding may seem cumbersome, especially for more complex equations. Alternatively, observe the original equation:

$$
(x + 7)(x - 3) = (x + 1)^2
$$

One can consider setting the equation as:

$$
(x + 7)(x - 3) - (x + 1)^2 = 0
$$

which is equivalent to:

$$
x^2 + 4x - 21 - (x^2 + 2x + 1) = 0
$$

Simplify:

$$
x^2 + 4x - 21 - x^2 - 2x - 1 = 0
$$

$$
(x^2 - x^2) + (4x - 2x) + (-21 - 1) = 0
$$

$$
0 + 2x - 22 = 0
$$

which simplifies to:

$$
2x = 22
$$

and again yields:

$$
x = 11
$$

Therefore, the only solution, in this case, is $x = 11$.

Verification of the Solution

It’s always prudent to verify solutions by substituting back into the original equation.

Substitute $x = 11$:

Left side:

$$
(11 + 7)(11 - 3) = (18)(8) = 144
$$

Right side:

$$
(11 + 1)^2 = (12)^2 = 144
$$

Since both sides are equal, $x=11$ is confirmed as a valid solution.

Discussion of Possible Multiple Solutions

Given the algebraic steps, the solution appears to be unique. However, to be thorough, one might consider whether the original equation could have additional solutions, especially if it involved quadratic factors or extraneous roots.

In this case, the simplification reveals a linear equation after reduction, indicating only one solution. This aligns with the nature of the original problem, which simplifies neatly to $x=11$.

Choosing the Correct Solution and Filling the Answer Box

Based on the comprehensive analysis above, the correct solution to the equation:

\[

(x + 7)(x - 3) = (x + 1)^2
\]

is \(x = 11\).

If the problem provides multiple-choice options, the correct choice is the one that corresponds to \(x=11\).

Answer Box:

\[
\boxed{11}
\]

Summary and Key Takeaways

- The process involved expanding both sides, simplifying, and solving for \(x\).
- The solution was verified by substitution.
- The problem demonstrated that sometimes seemingly complex equations reduce to simple linear solutions.
- Always verify solutions in the original equation to avoid extraneous roots.

Additional Tips for Solving Similar Equations

1. Always expand and simplify before solving.
2. Check for extraneous roots, especially when dealing with squares or roots.
3. Use substitution or rearranged forms when direct expansion seems cumbersome.
4. Verify solutions by substitution to confirm correctness.
5. Be mindful of the nature of the equation — linear, quadratic, or higher degree — to choose an appropriate solving method.

In conclusion, mastering the art of solving equations like \((x+7)(x-3) = (x+1)^2\) enhances algebraic intuition and problem-solving efficiency, skills essential for progressing in mathematics.

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Below And Fill In Any Answer Boxes

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