Solve The Following Inequalities And Show Your Solutions On The Number Line:Q.2.1.1 |2x-1| -7 > -3

Solve The Following Inequalities And Show Your Solutions On The Number Line:Q.2.1.1 |2x-1| -7 > -3

Understanding how to solve inequalities involving absolute value expressions is fundamental in algebra. Absolute value inequalities typically require considering both the positive and negative scenarios of the expression inside the absolute value. In this article, we will explore the inequality |2x - 1| - 7 > -3, analyze its solutions step by step, and illustrate these solutions on a number line for clarity.

Step 1: Rewrite the Inequality in a Simpler Form

The given inequality is:

$|2x - 1| - 7 > -3$

To approach this problem, first isolate the absolute value term:

$|2x - 1| - 7 > -3$

Add 7 to both sides to isolate the absolute value:

|2x - 1| > -3 + 7

|2x - 1| > 4

This simplifies our task to solving the inequality:

|2x - 1| > 4

Step 2: Recall the Definition of Absolute Value Inequalities

Absolute value inequalities compare the magnitude of an expression to a number, which leads to two cases:

- When the absolute value is greater than a positive number, the expression inside can be either greater than that number or less than its negative.

- When the absolute value is less than a positive number, the expression inside is between the negative and positive values.

In our case, since the inequality is |2x - 1| > 4, it means the expression 2x - 1 is either greater than 4 or less than -4.

Key Points:

- For |A| > c, where c > 0, the solution is A > c or A < -c.

- For |A| < c, the solution is -c < A < c.

Our current inequality uses the "greater than" sign, so we will analyze two cases accordingly.

Step 3: Solve the Two Cases

Case 1: $2x - 1 > 4$

Solve for x:

1. Write the inequality:

$2x - 1 > 4$

2. Add 1 to both sides:

$2x > 5$

3. Divide both sides by 2:

$x > \frac{5}{2}$

Solution for case 1:

$x > \frac{5}{2}$

Case 2: $2x - 1 < -4$

Solve for x:

1. Write the inequality:

$2x - 1 < -4$

2. Add 1 to both sides:

$2x < -3$

3. Divide both sides by 2:

$x < -\frac{3}{2}$

Solution for case 2:

$x < -\frac{3}{2}$

Step 4: Combine the Solutions

Since the original inequality $|2x - 1| > 4$ is satisfied when either condition holds, the overall solution set is the union of the two:

- All x such that $x < -1.5$

- All x such that x > 2.5

Expressed in interval notation:

(-\(\infty\), -\(\frac{3}{2}\)) ∪ (\(\frac{5}{2}\), \(\infty\))

Note: The points x = -1.5 and x = 2.5 are not included because the inequality is strict (>), not ≥.

Step 5: Show the Solution on the Number Line

Visualizing solutions on the number line helps in understanding the range of values satisfying the inequality.

Steps to illustrate:

- Draw a horizontal line representing the number line.

- Mark the critical points: -1.5 and 2.5.

- Use open circles at these points to indicate they are not included in the solution set.

- Shade the region to the left of -1.5, extending towards negative infinity.

- Shade the region to the right of 2.5, extending towards positive infinity.

Graphical Representation:

```
```

```
<====□================□====>
-1.5 2.5
```

- The circles at -1.5 and 2.5 are open, indicating these points are not part of the solution.

- The shading extends beyond these points to show all x less than -1.5 and greater than 2.5 satisfy the inequality.

Additional Notes and Summary

- Absolute value inequalities are solved by considering the definition of absolute value and splitting the problem into two cases based on the sign.

- The key to solving |A| > c (where c > 0) is to recognize that either A > c or A < -c.

- When graphing solutions, open circles are used at boundary points since the inequality is strict (>). If the inequality were □, closed circles would be appropriate.

- Always verify the solutions by substituting test points from each region into the original inequality to confirm they satisfy the condition.

Conclusion

The inequality $|2x - 1| - 7 > -3$ simplifies to $|2x - 1| > 4$. Its solutions are all x less than -1.5 and all x greater than 2.5. These solutions are represented on the number line as two rays extending to infinity in both directions, with open circles at the boundary points to indicate they are not included. Mastery of absolute value inequalities hinges on understanding their definition and the logical steps to split and solve the resulting cases. Visualizing solutions on the number line enhances comprehension and provides a clear picture of the solution set's structure.

Frequently Asked Questions

How do you start solving the inequality $|2x - 1| - 7 > -3$?

First, isolate the absolute value: $|2x - 1| > 4$, by adding 7 to both sides.

What is the next step after isolating the absolute value in $|2x - 1| > 4$?

Rewrite the inequality as two separate inequalities: $2x - 1 > 4$ and $2x - 1 < -4$.

How do you solve the inequality $2x - 1 > 4$?

Add 1 to both sides: $2x > 5$, then divide both sides by 2: $x > 2.5$.

What about solving $2x - 1 < -4$?

Add 1 to both sides: $2x < -3$, then divide both sides by 2: $x < -1.5$.

How can I represent the solution on the number line?

Plot open circles at $x = -1.5$ and $x = 2.5$, shading the regions $x < -1.5$ and $x > 2.5$.

What is the solution set for the inequality |2x - 1| - 7 > -3?

The solution set is x < -1.5 or x > 2.5.

Are there any restrictions or special cases to consider when solving this inequality?

No, since absolute value inequalities can be split into two linear inequalities without restrictions.

How do you verify the solutions to the inequality?

Choose test points from each region (e.g., x = -2 and x = 3), plug them into the original inequality, and check if it holds true.

What is the importance of showing solutions on the number line?

It provides a visual representation of the solution set, making it easier to understand the range of solutions clearly.

Additional Resources

Understanding and Solving the Inequality \(\left| 2x - 1 \right| - 7 > -3\): A Comprehensive Guide

---

Introduction

Inequalities involving absolute values are fundamental concepts in algebra, often encountered in various mathematical problems and real-world applications. They require a nuanced approach because the absolute value function introduces a piecewise component, reflecting the distance of a number from zero on the number line.

Today, we analyze the inequality:

$$
|2x - 1| - 7 > -3
$$

This problem not only involves solving for the variable x but also visualizing the solution set on the number line, which enhances understanding of how inequalities translate into geometric representations.

The Significance of Absolute Value in Inequalities

Before diving into solving the inequality, it is essential to understand what absolute value signifies:

- Absolute value of a real number a, denoted as $|a|$, measures the distance of a from zero on the real number line.
- The absolute value is always non-negative: $|a| \geq 0$.
- When solving inequalities involving absolute values, the main approach involves splitting the problem into cases based on the definition of the absolute value.

Understanding the properties of absolute value helps in:

- Simplifying inequalities.
- Recognizing when to split into multiple cases.
- Visualizing solutions on the number line.

Step-by-Step Solution of the Inequality

Let's analyze and solve the inequality:

$$
|2x - 1| - 7 > -3
$$

Step 1: Isolate the Absolute Value

Begin by isolating the absolute value term:

$$
|2x - 1| - 7 > -3
$$

Add 7 to both sides:

$$
|2x - 1| > 4
$$

This simple step transforms the inequality into a more manageable form.

Step 2: Understand the Inequality $|2x - 1| > 4$

The inequality states that the distance of $2x - 1$ from zero is greater than 4. Since the absolute value measures distance, the solutions are all values of x where $2x - 1$ is either:

- Greater than 4, or
- Less than -4

This leads us to the fundamental principle for solving inequalities involving absolute values:

$$
|A| > a \implies A > a \quad \text{or} \quad A < -a
$$

where A is an algebraic expression and $a > 0$.

Step 3: Split into Two Cases

Applying the above principle:

Case 1:

$$
2x - 1 > 4
$$

Case 2:

$$
2x - 1 < -4
$$

Solving Each Case Individually

Case 1: $2x - 1 > 4$

- Add 1 to both sides:

$$
2x > 5
$$

- Divide both sides by 2:

$$
x > \frac{5}{2}
$$

Solution to Case 1:

$$
x > 2.5
$$

Case 2: $2x - 1 < -4$

- Add 1 to both sides:

$$
2x < -3
$$

- Divide both sides by 2:

\[

$$
x < -\frac{3}{2}
$$

Solution to Case 2:

$$
x < -1.5
$$

Final Solution Set

Combining both cases, the set of all x satisfying the original inequality is:

$$
x < -1.5 \quad \text{or} \quad x > 2.5
$$

Expressed in interval notation:

$$
(-\infty, -\frac{3}{2}) \cup (2.5, \infty)
$$

Visualizing the Solution on the Number Line

Plotting the solutions on a number line helps to develop a geometric understanding of the inequality.

Steps to sketch the solution:

1. Draw a horizontal line representing real numbers.
2. Mark the critical points:
- $x = -1.5$ (or $-\frac{3}{2}$)
- $x = 2.5$
3. Indicate the solution regions:
- Shade to the left of -1.5, extending infinitely.
- Shade to the right of 2.5, extending infinitely.
4. Use open circles at -1.5 and 2.5 because the inequality is strict ($>$, $<$). No inclusion of the boundary points.

This visual approach confirms that solutions are all x less than -1.5 or greater than 2.5.

Additional Considerations and Variations

1. Equality Case:

If the inequality were $|2x - 1| \geq 4$, the solution set would include the boundary points:

$$
x = -1.5 \quad \text{or} \quad x = 2.5
$$

which would be represented on the number line with closed circles.

---

2. Graphical Interpretation of Absolute Value Inequalities:

- The graph of $|2x - 1|$ is a V-shaped graph opening upwards with the vertex at $x = \frac{1}{2}$.
- The inequality $|2x - 1| > 4$ corresponds to the parts of the graph above the lines $y=4$ and below $y=-4$, but since absolute value cannot be negative, only the parts where the graph exceeds 4 are relevant.

Real-World Applications of Such Inequalities

Understanding and solving inequalities like $|2x - 1| > 4$ has practical implications in various fields:

- Engineering: Determining tolerances where a measurement deviates beyond acceptable limits.
- Physics: Calculating regions where a quantity exceeds certain thresholds.
- Finance: Analyzing ranges where deviations from expected values are significant.
- Statistics: Establishing confidence intervals based on deviations from a mean.

Practice Problems and Further Exploration

To deepen understanding, consider practicing similar inequalities:

1. $|3x + 2| < 7$
2. $|x - 4| \leq 5$
3. $|5x - 3| > 10$
4. $|x/2 + 1| < 3$

For each, follow the same step-by-step process:

- Isolate the absolute value.
- Split into cases based on the inequality.

- Solve each case separately.
- Interpret solutions and visualize on the number line.

Summary and Key Takeaways

- Absolute value inequalities involve splitting into two cases based on the definition $|A| > a$ or $|A| < a$.
- For $|A| > a$, solutions are where $A > a$ or $A < -a$.
- For $|A| < a$, solutions are where $-a < A < a$.
- Always consider whether the inequality is strict or inclusive to decide between open or closed intervals.
- Visualizing solutions on the number line enhances understanding and communication of the solution set.

Conclusion

The inequality $|2x - 1| - 7 > -3$ simplifies elegantly into $|2x - 1| > 4$, with solutions $x < -1.5$ or $x > 2.5$. This showcases the power of absolute value properties and the importance of systematic reasoning in algebra. Mastery of such problems fosters a deeper understanding of inequalities, which are foundational in mathematics and its applications.

Understanding the geometric interpretation complements algebraic solutions, making the concepts more tangible. Whether for academic purposes or real-world problem-solving, mastering inequalities involving absolute values is essential, and visual tools like number line diagrams play a crucial role in this mastery.

Final Remarks

Always approach absolute value inequalities with a clear plan:

- Isolate the absolute value.
- Determine the appropriate case splitting.
- Solve each case carefully.
- Interpret and visualize the solutions.

Through practice and visualization, solving inequalities involving absolute values becomes an intuitive process, bridging the gap between algebraic manipulation and geometric understanding.

Solve The Following Inequalities And Show Your Solutions On The Number Line Q 2 1 1 2x 1 7 Gt 3

Solve The Following Inequalities And Show Your Solutions On The Number Line Q 2 1 1 2x 1 7 Gt 3

Related Articles

- The Increase-domestic-employment Argument For Tariff Protection Is Based On The Belief That Select An
- The Earth's Ionosphere Has An Index Of Refraction Less Than 1 (due To It Being A Plasma). Draw What A
- The Only Person With The Legal Power To Accept An Offer And Create A Contract Is The: Multiple Choice

Back to Home