Suppose A Normal Distribution Has A Mean Of 48 And A Standard Deviation Of 5. What Is The Probability

Suppose A Normal Distribution Has A Mean Of 48 And A Standard Deviation Of 5. What Is The Probability of a particular event occurring within this distribution? Understanding how to determine probabilities associated with normal distributions is fundamental in statistics, especially in fields like psychology, finance, engineering, and social sciences. This article will explore the process of calculating probabilities for a normal distribution characterized by a mean of 48 and a standard deviation of 5. We will delve into the concepts of standard normal distribution, z-scores, and how to interpret the probability in various contexts.

Understanding the Normal Distribution

What Is a Normal Distribution?

A normal distribution, also known as a Gaussian distribution, is a symmetric bell-shaped curve that describes how the values of a continuous variable are distributed. Most of the data points cluster around the mean, with fewer observations appearing as you move further away in either direction. This distribution model is pervasive because many natural phenomena tend to follow it, such as heights, test scores, measurement errors, and more.

Key Characteristics of a Normal Distribution

- Symmetry: The distribution is perfectly symmetrical around the mean.
- Mean = Median = Mode: All three measures of central tendency coincide.

- Bell-shaped curve: The highest point is at the mean, tapering off equally on both sides.
- Empirical Rule: Approximately 68% of data falls within one standard deviation, 95% within two, and 99.7% within three.

Parameters of Our Distribution

In our case, the normal distribution has:
- Mean (μ): 48
- Standard Deviation (σ): 5

This means most data points will fall between approximately 33 and 63 (within three standard deviations).

Calculating Probabilities in a Normal Distribution

Understanding the Problem

The core question is: given the distribution parameters, what is the probability that a randomly selected value falls within a certain range? For example:
- What is the probability that a value is less than 50?
- What is the probability that a value is greater than 55?
- What is the probability that a value falls between 45 and 55?

To answer these, we convert raw scores to z-scores and use the standard normal distribution.

What Is a Z-Score?

A z-score indicates how many standard deviations a particular value (X) is from the mean:

$$ z = \frac{X - \mu}{\sigma} $$

This transformation allows us to look up probabilities in standard normal distribution tables or use calculator functions.

Step-by-Step: Calculating Probabilities

1. Identify the value or range for which you want the probability

Suppose we want to find:

- The probability that a value is less than 50.
- The probability that a value is between 45 and 55.
- The probability that a value exceeds 55.

2. Convert raw scores to z-scores

Using the formula:

$$ z = \frac{X - 48}{5} $$

- For X = 50:

$$ z = \frac{50 - 48}{5} = \frac{2}{5} = 0.4 $$

- For X = 45:

$$ z = \frac{45 - 48}{5} = \frac{-3}{5} = -0.6 $$

- For X = 55:

$$ z = \frac{55 - 48}{5} = \frac{7}{5} = 1.4 $$

3. Use standard normal distribution tables or calculator functions to

find probabilities

Standard normal tables or software functions provide the cumulative probability $P(Z \leq z)$.

For example:

- $P(Z \leq 0.4) \approx 0.6554$ (65.54%)
- $P(Z \leq -0.6) \approx 0.2743$ (27.43%)
- $P(Z \leq 1.4) \approx 0.9192$ (91.92%)

4. Interpret the probabilities

- Probability that a value is less than 50:

$$P(X < 50) = P(Z < 0.4) \approx 0.6554$$

- Probability that a value is between 45 and 55:

$$P(45 < X < 55) = P(Z < 1.4) - P(Z < -0.6)$$

$$\approx 0.9192 - 0.2743 = 0.6449$$

- Probability that a value exceeds 55:

$$P(X > 55) = 1 - P(Z < 1.4) \approx 1 - 0.9192 = 0.0808$$

Applying the Concepts: Examples of Probability Calculations

Example 1: Probability of a Value Less Than a Specific Number

Suppose you want to find the probability that a randomly selected value from this distribution is less than 45.

- Convert to z-score:

$$z = \frac{45 - 48}{5} = -0.6$$

- Find cumulative probability:

$$ P(Z < -0.6) \approx 0.2743 $$

- Interpretation: Approximately 27.43% of the values are less than 45.

Example 2: Probability of a Value Falling Within a Range

Find the probability that a value lies between 45 and 55.

- Convert to z-scores:

$$ z_1 = -0.6, \quad z_2 = 1.4 $$

- Use standard normal table:

$$ P(Z < 1.4) \approx 0.9192, \quad P(Z < -0.6) \approx 0.2743 $$

- Calculate the probability:

$$ P(45 < X < 55) = 0.9192 - 0.2743 = 0.6449 $$

- Interpretation: About 64.49% of the observations fall within this range.

Example 3: Probability of a Value Greater Than a Certain Point

What is the probability that a value exceeds 55?

- Convert to z-score:

$$ z = 1.4 $$

- Find the complement:

$$ P(X > 55) = 1 - P(Z < 1.4) \approx 1 - 0.9192 = 0.0808 $$

- Interpretation: Approximately 8.08% of values are greater than 55.

Advanced Topics: Using Technology to Calculate Probabilities

While tables are useful, modern tools such as calculator functions, statistical software, or online calculators make these calculations quicker and more precise.

Using Excel or Google Sheets

- Use the `NORM.DIST` function:

$$ \text{NORM.DIST}(X, \text{mean}, \text{standard deviation}, \text{cumulative}) $$

- Example:

$$ \text{NORM.DIST}(50, 48, 5, TRUE) \approx 0.6554 $$

- To find the probability of being between two values:

$$ P = \text{NORM.DIST}(55, 48, 5, TRUE) - \text{NORM.DIST}(45, 48, 5, TRUE) $$

Using Python or R

- Python (scipy library):

```python
from scipy.stats import norm
mean = 48
std_dev = 5
prob_less_than_50 = norm.cdf(50, loc=mean, scale=std_dev)
prob_between = norm.cdf(55, loc=mean, scale=std_dev) - norm.cdf(45, loc=mean, scale=std_dev)
```

- R:

```r
mean <- 48
sd <- 5
prob_less_than_50 <- pnorm(50, mean=mean, sd=sd)
prob_between <- pnorm(55, mean=mean, sd=sd) - pnorm(45, mean=mean, sd=sd)
```

```

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## Conclusion: The Significance of Probability in Normal Distributions

Calculating probabilities within a normal distribution is a fundamental skill in statistics. Knowing how to convert raw scores to z-scores, interpret cumulative probabilities, and apply these concepts to real-world data enables informed decision-making and analysis. Whether you're assessing the likelihood of a measurement falling within certain bounds or evaluating risks and outcomes, understanding the properties of the normal distribution and how to work with it is invaluable.

Remember, the key steps involve identifying the relevant values, transforming them into z-scores, and utilizing standard normal distribution tables or software tools to find the probabilities. This process applies broadly to various scenarios, making it a versatile and essential aspect of statistical analysis.

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In summary:

- The distribution has a mean of 48 and a standard deviation of 5.
- To find the probability of a value falling within a certain range, convert the bounds to z-s

## Frequently Asked Questions

### Suppose a normal distribution has a mean of 48 and a standard
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deviation of 5. What is the probability that a randomly selected value is less than 45?

First, calculate the z-score: (45 - 48) / 5 = -0.6. Using standard normal distribution tables or calculator, P(Z < -0.6) □ 0.2743. Therefore, the probability is approximately 27.43%.

What is the probability that a value from this distribution exceeds 53?

Calculate z-score: (53 - 48) / 5 = 1.0. Using standard normal tables, P(Z > 1.0) □ 0.1587. So, the probability is approximately 15.87%.

What percentage of data falls between 43 and 53 in this distribution?

Calculate z-scores: (43 - 48)/5 = -1.0 and (53 - 48)/5 = 1.0. P(-1.0 < Z < 1.0) □ 0.6826. Thus, about 68.26% of data falls between 43 and 53.

If a value is at the 90th percentile, what is its approximate value in this distribution?

The z-score for the 90th percentile is approximately 1.28. Calculate value: 48 + (1.28)(5) □ 48 + 6.4 = 54.4. So, the 90th percentile is approximately 54.4.

What is the probability that a value is more than 55?

Calculate z-score: (55 - 48)/5 = 1.4. P(Z > 1.4) □ 0.0808. Therefore, the probability is approximately 8.08%.

What is the probability that a value is exactly 48 in this continuous distribution?

In a continuous distribution like the normal distribution, the probability of exactly any specific value is zero.

If a value is 3 standard deviations above the mean, what is its corresponding value in this distribution?

Calculate: 48 + (3)(5) = 48 + 15 = 63. The value is 63.

What is the probability that a value lies between 45 and 55?

Calculate z-scores: (45 - 48)/5 = -0.6 and (55 - 48)/5 = 1.4. P(-0.6 < Z < 1.4) □ P(Z < 1.4) - P(Z < -0.6) □ 0.9192 - 0.2743 = 0.6449. So, approximately 64.49%.

What is the cutoff value for the top 5% of this distribution?

Find z-score for the 95th percentile, approximately 1.645. Calculate value: 48 + (1.645)(5) □ 48 + 8.225 = 56.225. The cutoff is approximately 56.23.

How do you compute the probability that a value from this distribution is between 46 and 50?

Calculate z-scores: (46 - 48)/5 = -0.4 and (50 - 48)/5 = 0.4. P(-0.4 < Z < 0.4) □ P(Z < 0.4) - P(Z < -0.4) □ 0.6554 - 0.3446 = 0.3108. So, about 31.08%.

Additional Resources

Normal Distribution Analysis: Understanding Probabilities with a Mean of 48 and Standard Deviation of 5

In the realm of statistical analysis, the normal distribution—often called the bell curve—is a fundamental concept that underpins many fields, from economics and engineering to social sciences and health sciences. Its importance stems from the fact that many natural phenomena tend to follow this distribution, especially when dealing with large datasets. In this article, we'll explore an illustrative example: a normal distribution with a mean (μ) of 48 and a standard deviation (□) of 5. Our goal is to understand the probability of certain events within this distribution, providing a comprehensive guide that can serve both novices and seasoned statisticians.

Understanding the Normal Distribution: The Foundation

Before delving into specific probability calculations, it's essential to grasp the core concepts of the normal distribution.

What Is a Normal Distribution?

A normal distribution is a symmetric, bell-shaped curve characterized by its mean and standard deviation:

- Mean (μ): The central point around which data is distributed. In our case, μ = 48.
- Standard Deviation (σ): Measures the spread or dispersion of the data. Here, σ = 5.

The probability density function (PDF) describes how data points are distributed around the mean. The higher the standard deviation, the wider and flatter the bell curve; the smaller the standard deviation, the narrower and taller it appears.

Properties of the Normal Distribution

- Symmetry: The curve is symmetric about the mean.
- 68-95-99.7 Rule: Approximately 68% of data falls within one standard deviation, 95% within two, and 99.7% within three.

- Empirical Rule: Provides quick estimates of probabilities for data within certain ranges.

Setting the Scenario: Distribution with $\mu=48$ and $\sigma=5$

Our specific distribution follows:

- $\mu = 48$
- $\sigma = 5$

This means that:

- About 68% of the data falls between 43 and 53 ($\mu \pm \sigma$).
- About 95% falls between 38 and 58 ($\mu \pm 2\sigma$).
- About 99.7% falls between 33 and 63 ($\mu \pm 3\sigma$).

Suppose we are interested in calculating the probability that a randomly selected value from this distribution falls within a certain range, exceeds a particular threshold, or is below a certain point. To do this systematically, we leverage the standard normal distribution and z-scores.

Standardizing Data: The Role of Z-Scores

What Is a Z-Score?

A z-score indicates how many standard deviations a particular value (X) is from the mean:

$$Z = \frac{X - \mu}{\sigma}$$

This transformation converts any normal distribution to the standard normal distribution, which has a mean of 0 and a standard deviation of 1. Using z-scores simplifies probability calculations because standard normal distribution tables and software functions are widely available.

Calculating Z-Scores

Given a value, X, the z-score is computed as:

$$Z = \frac{X - 48}{5}$$

For example, if X = 55:

$$Z = \frac{55 - 48}{5} = 1.4$$

This indicates that 55 is 1.4 standard deviations above the mean.

Calculating Probabilities: Step-by-Step Approach

Suppose we want to find the probability that a randomly selected value from our distribution is less than a specific value, greater than a certain cutoff, or within a range.

1. Probability that X is less than a specific value ($P(X < x)$)

Example: What is the probability that a value is less than 50?

- Calculate the z-score:

$$Z = \frac{50 - 48}{5} = 0.4$$

- Use standard normal distribution tables or software to find $P(Z < 0.4)$.

From standard normal tables:

$$P(Z < 0.4) \approx 0.6554$$

Interpretation: About 65.54% of the values are less than 50.

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2. Probability that X exceeds a certain value ($P(X > x)$)

Example: Probability that X is greater than 55.

- Compute z-score:

$$Z = \frac{55 - 48}{5} = 1.4$$

- Find $P(Z > 1.4)$:

Since tables give $P(Z < 1.4) \approx 0.9192$,

$$ P(Z > 1.4) = 1 - 0.9192 = 0.0808 $$

Interpretation: There's roughly an 8.08% chance that a value exceeds 55.

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3. Probability that X falls between two values ($P(a < X < b)$)

Example: Probability X is between 45 and 55.

- Calculate z-scores:

$$ Z_a = \frac{45 - 48}{5} = -0.6 $$

$$ Z_b = \frac{55 - 48}{5} = 1.4 $$

- Find $P(-0.6 < Z < 1.4)$:

From tables:

$$ P(Z < 1.4) \approx 0.9192 $$

$$ P(Z < -0.6) \approx 0.2743 $$

Thus,

$$ P(-0.6 < Z < 1.4) = 0.9192 - 0.2743 = 0.6449 $$

Interpretation: Approximately 64.49% of data falls between 45 and 55.

Applying the Method to Specific Probabilities

Let's explore some common probability questions within this context.

Question 1: What is the probability that a value is less than 45?

- Compute z-score:

$$ Z = \frac{45 - 48}{5} = -0.6 $$

- Find P(Z < -0.6):

$$ \approx 0.2743 $$

Answer: There is roughly a 27.43% chance that a randomly selected value from the distribution is less than 45.

Question 2: What is the probability that a value exceeds 53?

- Z-score:

$$ Z = \frac{53 - 48}{5} = 1.0 $$

- Find $P(Z > 1.0)$:

$$1 - P(Z < 1.0) \approx 1 - 0.8413 = 0.1587$$

Answer: Approximately 15.87% chance that the value exceeds 53.

Question 3: What is the probability that a value lies between 45 and 53?

- Z-scores:

$$Z_{45} = -0.6$$

$$Z_{53} = 1.0$$

- Probabilities:

$$P(Z < 1.0) \approx 0.8413$$

$$P(Z < -0.6) \approx 0.2743$$

- Difference:

$$0.8413 - 0.2743 = 0.5670$$

Answer: About 56.70% of the values lie between 45 and 53.

Advanced Applications: Using Software and Tables

While tables and manual calculations are instructive, modern statistical tools like software packages (Excel, R, Python) significantly streamline probability calculations.

Excel Example:

- To find $P(X < x)$:

```excel
=NORM.DIST(x, 48, 5, TRUE)
```

- To find $P(X > x)$:

```excel
=1 - NORM.DIST(x, 48, 5, TRUE)
```

Python Example:

```python
import scipy.stats as stats

Probability X < x
prob_less = stats.norm.cdf(x, loc=48, scale=5)

Probability X > x
prob_greater = 1 - stats.norm.cdf(x, loc=48, scale=5)

Probability between a and b
```

```
prob_between = stats.norm.cdf(b, loc=48, scale=5) - stats.norm.cdf(a, loc=48, scale=5)
```

Interpreting Probabilities in Real-World Contexts

Understanding these probabilities enables decision-making across various domains:

- Quality Control: Estimating the probability of products exceeding certain defect thresholds.
- Finance: Calculating the likelihood of returns falling below or above specific levels.
- Education: Determining the proportion of students scoring within certain ranges.
- Healthcare: Assessing the chances of a measurement (e.g., blood pressure) exceeding health thresholds.

For our distribution with $\mu = 48$ and $\sigma = 5$, these calculations help tailor policies, expectations, or interventions based on the likelihood of specific outcomes.

Conclusion: Mastering Normal Distribution Probabilities

Analyzing a normal distribution with given parameters involves understanding its core properties, converting raw data to z-scores, and employing standard normal distribution tables or software to compute probabilities. The example of a distribution with a

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