Suppose F(x) = X^2/(x-12)^2Find The Intervals On Which F Is Increasing Or Decreasing. F Is Increasing

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Understanding the behavior of functions, particularly where they are increasing or decreasing, is fundamental in calculus. In this article, we will analyze the function $F(x) = x^2 / (x - 12)^2$ to determine the intervals where it is increasing or decreasing. This process involves calculating the derivative, analyzing its sign, and interpreting the results to understand the function's overall behavior.

Understanding the Function $F(x) = x^2 / (x - 12)^2$

Before diving into the calculus, it’s essential to understand the structure and properties of the function:

Function Composition and Domain

- The function is a rational function, with numerator x^2 and denominator $(x - 12)^2$.
- The denominator $(x - 12)^2$ is always positive except at x=12, where it is zero, causing the function to be undefined.
- Therefore, the domain of F(x) is all real numbers except x □ 12.

Behavior at Critical Points and Asymptotes

- $X = 12$ is a vertical asymptote because the function is undefined there.
- As x approaches 12 from the left or right, the function tends toward infinity or negative infinity depending on the direction.

Calculating the Derivative of F(x)

To find where F(x) is increasing or decreasing, we need to analyze its derivative, F'(x). The derivative provides the rate of change of the function at any point, indicating increasing behavior when $F'(x) > 0$ and decreasing behavior when $F'(x) < 0$.

Using the Quotient Rule

The quotient rule states that for a function $F(x) = u(x)/v(x)$,

$F'(x) = [u'(x) v(x) - u(x) v'(x)] / [v(x)]^2$

Let:

- $u(x) = x^2$

- $v(x) = (x - 12)^2$

Calculate derivatives:

- $u'(x) = 2x$

- $v'(x) = 2(x - 12)$

Applying the quotient rule:

$F'(x) = [2x (x - 12)^2 - x^2 2(x - 12)] / (x - 12)^4$

Simplify the Derivative

Numerator:

$= 2x(x - 12)^2 - 2x^2(x - 12)$

$= 2x(x - 12)[(x - 12) - x]$

$= 2x(x - 12)(-12)$

So:

$F'(x) = [2x(x - 12)(-12)] / (x - 12)^4$

Simplify numerator:

$= -24x(x - 12)$

Rewrite the derivative:

$F'(x) = -24x(x - 12) / (x - 12)^4$

Note that (x - 12) appears in both numerator and denominator. We can express the derivative as:

$F'(x) = -24x / (x - 12)^3$, for $x \neq 12$

Determining Intervals of Increase and Decrease

Having simplified F'(x) to:

$F'(x) = -24x / (x - 12)^3$

we can analyze the sign of F'(x) to determine increasing and decreasing intervals.

Critical Points

Critical points occur where $F'(x) = 0$ or is undefined:

- $F'(x) = 0$ when numerator = 0 □ $-24x = 0$ □ $x = 0$
- $F'(x)$ is undefined at $x = 12$ (vertical asymptote)

Note: $x=0$ is a critical point because the derivative equals zero there, and $x=12$ is a point of discontinuity.

Sign Analysis of F'(x)

Construct a sign chart for $F'(x)$ considering the critical points:

1. For $x < 0$:

- x is negative
- numerator: $-24x$ □ positive (since x is negative, $-24x > 0$)
- denominator: (x - 12)^3
- For $x < 12$, $(x - 12) < 0$, so (x - 12)^3 < 0
- Therefore:
- numerator > 0
- denominator < 0
- $F'(x)$ = positive / negative = negative
- $F(x)$ is decreasing on (-□, 0)

2. For $0 < x < 12$:

- $x > 0$
- numerator: $-24x$ □ negative
- denominator: (x - 12)^3
- Since $x < 12$, $(x - 12) < 0$, so denominator < 0
- Therefore:
- numerator < 0

- denominator < 0

- F'(x) = negative / negative = positive

- F(x) is increasing on (0, 12)

3. For x > 12:

- x > 0

- numerator: -24x → negative

- denominator: (x - 12)^3 > 0 (since x > 12)

- Thus:

- numerator < 0

- denominator > 0

- F'(x) = negative / positive = negative

- F(x) is decreasing on (12, ∞)

Summary of Increasing and Decreasing Intervals

Based on the sign analysis:

- F(x) is decreasing on

  - (-∞, 0)
  - (12, ∞)

- F(x) is increasing on

  - (0, 12)

Note that at x=0, the derivative is zero, indicating a potential local extremum, and at x=12, the function is not defined, with a vertical asymptote.

Interpreting the Behavior of F(x)

Understanding where a function increases or decreases provides insights into its overall shape:

Behavior Near Critical Points

- At x=0, the function reaches a critical point where the slope changes from negative to positive, indicating a local minimum.
- As x approaches 12 from the left, F(x) increases without bound, shifting to decreasing after 12.

Graphical Representation

Visualizing the function:

- The graph declines from negative infinity up to x=0.

- It then rises from x=0 to x=12.

- After x=12, the graph declines towards negative infinity again.

- Vertical asymptote at x=12 indicates an unbounded behavior.

Conclusion

By analyzing the derivative of $F(x) = x^2 / (x - 12)^2$, we determined that:

- The function is decreasing on the intervals $(-\infty, 0)$ and $(12, \infty)$.
- The function is increasing on the interval (0, 12).
- Critical points at $x=0$ and the vertical asymptote at $x=12$ are key to understanding the function's behavior.

This comprehensive analysis aids in graphing the function accurately and understanding its increasing/decreasing behavior, which is crucial in calculus applications such as optimization and curve sketching.

For further insights into calculus and function analysis, consider exploring topics like critical point classification, concavity, and limits at infinity to deepen your understanding of function behavior.

Frequently Asked Questions

How do I determine the intervals where the function $F(x) = x^2 / (x - 12)^2$ is increasing?

To find where F(x) is increasing, first compute its derivative F'(x), then identify the intervals where $F'(x) > 0$. These intervals indicate where the function is increasing.

What is the derivative of $F(x) = x^2 / (x - 12)^2$?

Using the quotient rule, the derivative is $F'(x) = [2x(x - 12)^2 - x^2 2(x - 12)] / (x - 12)^4$, which simplifies to $F'(x) = 2x(x - 12) / (x - 12)^4$.

How do I find critical points for $F(x) = x^2 / (x - 12)^2$?

Set $F'(x) = 0$ and solve for x. Since $F'(x) = 2x(x - 12) / (x - 12)^4$, the numerator equals zero when $x = 0$ or $x = 12$, but $x = 12$ is undefined in the original function, so only $x = 0$ is a critical point.

On which intervals is F(x) decreasing?

F(x) is decreasing where $F'(x) < 0$. Since $F'(x) = 2x(x - 12) / (x - 12)^4$, the sign depends on the numerator $2x(x - 12)$, which is negative when x □ (0, 12) and $x < 0$ or $x > 12$, considering the domain restrictions.

What is the overall increasing interval for $F(x) = x^2 / (x - 12)^2$?

F(x) is increasing on the intervals where $F'(x) > 0$. From the derivative, $F'(x) > 0$ when x □ (-□, 0) and (12, □), considering the function's domain and the sign of the numerator.

Are there any asymptotes or domain restrictions I should consider when analyzing F(x)?

Yes, the function has a vertical asymptote at $x = 12$, where the denominator is zero. The domain is all real numbers except $x = 12$, and this affects the intervals where the function is increasing or decreasing.

Additional Resources

Suppose F(x) = \(\frac{x^2}{(x-12)^2}\). Find the intervals on which F is increasing or decreasing. F is increasing.

Understanding the behavior of functions, especially in terms of increasing and decreasing intervals, is a fundamental aspect of calculus. The function \(F(x) = \frac{x^2}{(x-12)^2} \) presents an interesting case due to its rational form and the potential for vertical asymptotes and critical points. Analyzing where this function increases or decreases requires a detailed examination of its derivatives, critical points, and the domain. This article provides a comprehensive exploration of this function, breaking down the steps involved in determining its increasing and decreasing intervals and discussing the implications of these behaviors.

Introduction to the Function $F(x) = \frac{x^2}{(x-12)^2}$

The function $F(x) = \frac{x^2}{(x-12)^2}$ is a rational function characterized by a numerator x^2 and denominator $(x-12)^2$. Since the denominator involves a squared term, the function is undefined at $x = 12$, which introduces a vertical asymptote at this point. The numerator, being a perfect square, is always non-negative, and the entire function is defined for all real x except $x = 12$.

This function exhibits symmetry because it is even; that is, $F(-x) = F(x)$, owing to the squares in both numerator and denominator. The behavior of $F(x)$ near the asymptote and at large $|x|$ values reveals critical insights into its increasing or decreasing nature.

Domain and Asymptotic Behavior

Domain of $F(x)$

The function $F(x)$ is defined for all real numbers except at $x = 12$, where the denominator becomes zero:

$$
\text{Domain}: \quad (-\infty, 12) \cup (12, \infty)
$$

This discontinuity splits the domain into two intervals, which need separate analysis.

Vertical Asymptote at $x = 12$

As $x \to 12^-$, $(x - 12)^2 \to 0^+$, making $F(x) \to +\infty$. Similarly, as $x \to 12^+$, $F(x) \to +\infty$. Thus, $x=12$ is a vertical asymptote where the function tends to infinity from both sides.

Behavior at Infinity

- As $x \to +\infty$:

$$
F(x) \sim \frac{x^2}{x^2} = 1
$$

- As $x \to -\infty$:

$$
F(x) \sim \frac{x^2}{x^2} = 1
$$

In both cases, the function approaches 1, which suggests a horizontal asymptote at $y=1$.

Calculating the Derivative $F'(x)$

To find the increasing or decreasing intervals, we need to analyze the first derivative $F'(x)$.

Given:

$$
F(x) = \frac{x^2}{(x-12)^2}
$$

Applying the quotient rule:

$$

F'(x) = \frac{(2x)(x-12)^2 - x^2 \cdot 2(x-12)}{(x-12)^4}

\]

Simplify numerator:

\[

N(x) = 2x(x-12)^2 - 2x^2(x-12)

\]

Factor numerator:

\[

N(x) = 2x (x-12)^2 - 2x^2 (x-12)

\]

\[

= 2x (x-12) [(x-12) - x]

\]

\[

= 2x (x-12) [x - 12 - x]

\]

\[

= 2x (x-12) (-12)

\]

\[

= -24 x (x-12)

\]

Thus:

\[

F'(x) = \frac{-24 x (x-12)}{(x-12)^4}

\]

Further simplification:

$$
F'(x) = \frac{-24 x}{(x-12)^3}
$$

Critical Points and Sign Analysis

Critical points occur where $F'(x) = 0$ or $F'(x)$ is undefined.

- Set numerator zero:

$$
-24 x = 0 \implies x=0
$$

- The derivative is undefined where the denominator is zero:

$$
(x-12)^3 = 0 \implies x=12
$$

But $x=12$ is outside the domain, so it does not serve as a critical point but remains a vertical asymptote.

Thus, the only critical point in the domain is at $x=0$.

Determining the Sign of $F'(x)$

- For $x \in (-\infty, 0)$:

$$
F'(x) = \frac{-24 x}{(x-12)^3}
$$

Since $x<0$, numerator $-24 x > 0$ (because $x<0 \Rightarrow -24 x > 0$), and denominator $(x-12)^3$ is negative (since $x<12 \Rightarrow x-12<0 \Rightarrow (x-12)^3<0$).

Therefore:

$$
F'(x) = \frac{\text{positive}}{\text{negative}} = \text{negative}
$$

This indicates F is decreasing on $(-\infty, 0)$.

- For $x \in (0, 12)$:

Numerator $-24 x < 0$ (since $x>0$), denominator $(x-12)^3<0$ (since $x<12$).

Thus:

$$
F'(x) = \frac{\text{negative}}{\text{negative}} = \text{positive}
$$

So, F is increasing on $(0, 12)$.

- For $x \in (12, \infty)$:

Numerator $-24 x < 0$ (since $x>0$), denominator $(x-12)^3 > 0$, so:

$$
F'(x) = \frac{\text{negative}}{\text{positive}} = \text{negative}

\]

Therefore, F is decreasing on $(12, \infty)$.

Summary of Increasing and Decreasing Intervals

Interval	Sign of $F'(x)$	Behavior of $F(x)$
$(- \infty, 0)$	Negative	Decreasing
$(0, 12)$	Positive	Increasing
$(12, \infty)$	Negative	Decreasing

Note: The function has a vertical asymptote at $x=12$ and a critical point at $x=0$.

Graphical Interpretation and Features

Understanding the graph of $F(x) = \frac{x^2}{(x-12)^2}$ aids in visualizing the intervals of increase and decrease.

Features:

- Symmetry: The function is even, i.e., symmetric with respect to the y-axis.
- Vertical asymptote: At $x=12$, with the function tending to $+\infty$ from both sides.
- Horizontal asymptote: At $y=1$, since $F(x) \to 1$ as $x \to \pm \infty$.
- Critical point at $x=0$: This point is a minimum, as the function transitions from decreasing to increasing.

Behavior:

- The function decreases from $+\infty$ as $x \to -\infty$ toward the minimum at $x=0$.

- From $x=0$ up to $x=12$, the function increases from 1 to $+\infty$.

- After $x=12$, the function decreases from $+\infty$ back toward 1 as $x \to \infty$.

Conclusion

Analyzing $F(x) = \frac{x^2}{(x-12)^2}$ reveals that the function is decreasing on $(-\infty, 0)$ and $(12, \infty)$, and increasing on $(0, 12)$. The critical point at $x=0$ marks the minimum value of the function, occurring at $F(0) = 0$. The vertical asymptote at $x=12$ and the

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Suppose F X X 2 X 12 2find The Intervals On Which F Is Increasing Or Decreasing F Is Increasing

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