Suppose That Two Dice Are Rolled Determine The Probability That The Sum Of The Numbers Showing On The

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When engaging with probability theory, one of the most intriguing and fundamental problems involves rolling dice. Whether in board games, statistical experiments, or mathematical puzzles, understanding the likelihood of various outcomes when rolling dice is essential. Among these, calculating the probability that the sum of the numbers showing on two rolled dice meets a specific criterion is a classic problem that exemplifies core concepts of probability, combinatorics, and probability distribution.

In this comprehensive article, we will explore the problem: Suppose That Two Dice Are Rolled Determine The Probability That The Sum Of The Numbers Showing On The. We will analyze different scenarios, develop formulas, and provide a step-by-step approach to calculating probabilities related to the sum of two dice. By the end, you will have a clear understanding of how to approach similar problems and the mathematical principles involved.

Understanding the Basics of Dice and Probabilities

The Nature of Dice

A standard die (singular of dice) is a cube with six faces, numbered 1 through 6. When rolled, each face has an equal probability of landing face-up, assuming the die is fair and unbiased.

Sample Space of Two Dice Rolls

When rolling two dice simultaneously, each die has 6 possible outcomes, leading to:
- Total sample space: 6 × 6 = 36 possible outcomes.

Each outcome can be represented as an ordered pair (a, b), where:
- a = number on the first die.
- b = number on the second die.

Since each die is independent, every pair has an equal probability of 1/36.

Possible Outcomes

The total sample space includes outcomes such as:
- (1, 1), (1, 2), ..., (1, 6),
- (2, 1), (2, 2), ..., (6, 6).

Calculating the Probability of a Sum in Two Dice Rolls

Defining the Desired Event

Suppose we want to find the probability that the sum of the two dice equals a specific number, say k. That is:
- Event A: "Sum of numbers on two dice = k".

Possible values of k range from 2 (minimum, 1+1) to 12 (maximum, 6+6).

Approach to Calculating the Probability

The probability of A is:

$$
P(A) = \frac{\text{Number of favorable outcomes}}{\text{Total outcomes in sample space}} = \frac{\text{Number of outcomes where sum = k}}{36}
$$

To find the numerator, we need to count how many pairs (a, b) satisfy:

$$
a + b = k
$$

with $a, b \in \{1, 2, 3, 4, 5, 6\}$.

Number of Outcomes for Each Possible Sum

Sum = 2

- Possible outcome: (1, 1)
- Count: 1

Sum = 3

- Possible outcomes: (1, 2), (2, 1)
- Count: 2

Sum = 4

- Possible outcomes: (1, 3), (2, 2), (3, 1)
- Count: 3

Sum = 5

- Possible outcomes: (1, 4), (2, 3), (3, 2), (4, 1)
- Count: 4

Sum = 6

- Possible outcomes: (1, 5), (2, 4), (3, 3), (4, 2), (5, 1)
- Count: 5

Sum = 7

- Possible outcomes: (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1)
- Count: 6

Sum = 8

- Possible outcomes: (2, 6), (3, 5), (4, 4), (5, 3), (6, 2)
- Count: 5

Sum = 9

- Possible outcomes: (3, 6), (4, 5), (5, 4), (6, 3)
- Count: 4

Sum = 10

- Possible outcomes: (4, 6), (5, 5), (6, 4)
- Count: 3

Sum = 11

- Possible outcomes: (5, 6), (6, 5)
- Count: 2

Sum = 12

- Possible outcome: (6, 6)
- Count: 1

Probability Calculations for Specific Sums

Using the counts above, the probability that the sum of two dice is equal to a specific number k is:

$$
P(\text{sum} = k) = \frac{\text{Number of outcomes with sum } k}{36}
$$

For example:
- $P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$

General Formula for the Probability of the Sum

The number of outcomes where the sum equals k can be expressed as:

$$
\text{Number of outcomes} =
\begin{cases}
k - 1, & \text{if } 2 \leq k \leq 7 \\
13 - k, & \text{if } 8 \leq k \leq 12
\end{cases}
$$

Thus, the probability is:

$$
P(\text{sum} = k) =
\begin{cases}
\frac{k - 1}{36}, & 2 \leq k \leq 7 \\
\frac{13 - k}{36}, & 8 \leq k \leq 12
\end{cases}
$$

Calculating Probabilities for Specific Sums: Examples

Example 1: Probability that the sum is 9

Using the count:

$$
\text{Number of outcomes} = 4
$$

So,

$$
P(\text{sum} = 9) = \frac{4}{36} = \frac{1}{9}
$$

Example 2: Probability that the sum is 4

Number of outcomes:

$$
3
$$

Thus,

$$
P(\text{sum} = 4) = \frac{3}{36} = \frac{1}{12}
$$

Extending to Multiple Events and Combined Probabilities

Probability of Sum Within a Range

Suppose you want to find the probability that the sum of two dice falls within a specific range, e.g., between 4 and 9 inclusive.

This involves summing individual probabilities:

$$
P(4 \leq \text{sum} \leq 9) = \sum_{k=4}^9 P(\text{sum} = k)
$$

Using the probabilities:

$$
P(4) = \frac{3}{36} = \frac{1}{12}
$$

$$
P(5) = \frac{4}{36} = \frac{1}{9}
$$
$$
P(6) = \frac{5}{36}
$$
$$
P(7) = \frac{6}{36} = \frac{1}{6}
$$
$$
P(8) = \frac{5}{36}
$$
$$
P(9) = \frac{4}{36} = \frac{1}{9}
$$

Adding these:
$$
P(4 \leq \text{sum} \leq 9) = \frac{3 + 4 + 5 + 6 + 5 + 4}{36} = \frac{27}{36} = \frac{3}{4}
$$

Special Cases and Symmetries

Symmetry in Probabilities

Notice that the probabilities are symmetric around the sum of 7:
- Sum = 2 has 1 outcome
- Sum = 12 has 1 outcome
- Sum = 3 has 2 outcomes
- Sum = 11 has 2 outcomes
- Sum = 4 has 3 outcomes
- Sum = 10 has 3 outcomes
- Sum = 5 has 4 outcomes
- Sum = 9 has 4 outcomes
- Sum = 6 has 5 outcomes
- Sum = 8 has 5 outcomes
- Sum = 7 has 6 outcomes

This symmetry simplifies calculations and supports intuitive understanding.

Applications

Frequently Asked Questions

What is the probability that the sum of two rolled dice is 7?

There are 6 favorable outcomes where the sum is 7 (1-6, 2-5, 3-4, 4-3, 5-2, 6-1) out of 36 possible outcomes, so the probability is 6/36 or 1/6.

How do you calculate the probability that the sum of two dice is greater than 8?

Count all outcomes where the sum exceeds 8 (sums of 9, 10, 11, 12). There are 4 (for 9), 3 (for 10), 2 (for 11), and 1 (for 12) outcomes, totaling 10. Probability = 10/36 = 5/18.

What is the probability that the sum of two dice is an even number?

Even sums are 2, 4, 6, 8, 10, 12. Counting outcomes for each, total favorable outcomes are 18. Therefore, probability = 18/36 = 1/2.

If two dice are rolled, what is the probability that the sum is 3?

The outcomes that sum to 3 are (1,2) and (2,1), totaling 2. Probability = 2/36 = 1/18.

What is the probability that the two dice show the same number (doubles)?

Doubles occur when both dice show the same number: (1,1), (2,2), (3,3), (4,4), (5,5), (6,6). Total of 6 outcomes, so probability = 6/36 = 1/6.

Determine the probability that the sum of the dice is less than 5.

Sum less than 5 includes sums of 2, 3, and 4. Outcomes are (1,1), (1,2), (2,1), (1,3), (3,1), (2,2). Total of 6 outcomes. Probability = 6/36 = 1/6.

What is the probability that the sum of two dice is 10 or more?

Sums of 10, 11, and 12. Outcomes for 10: (4,6), (5,5), (6,4); for 11: (5,6), (6,5); for 12: (6,6). Total outcomes = 3 + 2 + 1 = 6. Probability = 6/36 = 1/6.

If two dice are rolled, what is the probability that the sum is an odd number?

Odd sums are 3, 5, 7, 9, 11. Counting outcomes, total favorable outcomes are 18. Therefore, probability = 18/36 = 1/2.

What is the probability that the sum of two rolled dice is exactly 8?

The outcomes that sum to 8 are (2,6), (3,5), (4,4), (5,3), (6,2). Total of 5 outcomes. Probability = 5/36.

Additional Resources

Suppose That Two Dice Are Rolled Determine The Probability That The Sum Of The Numbers Showing On The

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Introduction

In the realm of probability and statistics, few scenarios are as classic and universally

recognized as the roll of two dice. Whether in board games, gambling, or educational demonstrations, understanding the likelihood of various outcomes when rolling dice is fundamental. Today, we delve into a specific question: Suppose that two dice are rolled, what is the probability that the sum of the numbers showing on the two dice equals a particular value? This exploration not only offers insights into basic probability calculations but also provides a foundation for more complex statistical reasoning.

Understanding the Basics: What Are Two Dice and Their Possible Outcomes?

Before calculating probabilities, it is crucial to understand the nature of the objects involved—two standard six-sided dice.

The Structure of a Die

Each die has six faces numbered from 1 to 6. When rolling a single die, each face has an equal chance of landing face-up, giving a probability of 1/6 for each outcome.

The Sample Space for Two Dice

When rolling two dice simultaneously, the total number of possible outcomes—referred to as the sample space—is the product of the individual possibilities:

- Each die has 6 outcomes.
- Therefore, total outcomes = 6 (for the first die) × 6 (for the second die) = 36.

Each outcome can be represented as an ordered pair (a, b), where 'a' is the number on the first die, and 'b' is the number on the second die. For example, (3, 5) indicates the first die shows a 3, and the second die shows a 5.

Calculating the Probability of Specific Sums

The core question involves determining the probability that the sum of the numbers on the two dice equals a particular number, say, 'k'. To do this systematically, we analyze all possible outcomes and identify those that meet the sum criterion.

Possible Sums and Their Ranges

- The smallest sum occurs when both dice show 1:

sum = 2.
- The largest sum occurs when both dice show 6: sum = 12.
- Therefore, the sums can range from 2 to 12.

Step-by-Step Methodology for Calculating Probabilities

1. Identify All Outcomes That Yield the Desired Sum

For each sum 'k' in 2 through 12, list all pairs (a, b) such that:

a + b = k, with 1 ≤ a ≤ 6 and 1 ≤ b ≤ 6.

For example, for sum = 7:

Possible pairs are (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1).

Total outcomes for sum = 7: 6.

2. Count the Number of Favorable Outcomes

Count how many pairs satisfy the sum condition for each value of 'k'.

Sum (k)	Favorable Outcomes	Count
2	(1,1)	1
3	(1,2), (2,1)	2
4	(1,3), (2,2), (3,1)	3
5	(1,4), (2,3), (3,2), (4,1)	4
6	(1,5), (2,4), (3,3), (4,2), (5,1)	5
7	(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)	6
8	(2,6), (3,5), (4,4), (5,3), (6,2)	5
9	(3,6), (4,5), (5,4), (6,3)	4
10	(4,6), (5,5), (6,4)	3
11	(5,6), (6,5)	2
12	(6,6)	1

3. Calculate the Probability

Since all 36 outcomes are equally likely, the probability of any specific sum 'k' is:

P(sum = k) = (Number of outcomes with sum k) / 36

For example, the probability that the sum of the two dice equals 7:

P(sum = 7) = 6 / 36 = 1/6 ≈ 16.67%.

Visual Representation: Probability Distribution of the Sum

Plotting the probabilities for each possible sum provides a clear visual understanding of how likely each outcome is. The distribution resembles a bell-shaped curve centered around 7, which is the most probable sum.

Probability Distribution Table:

Sum (k)	Number of Outcomes	Probability
2	1	1/36
3	2	2/36 = 1/18
4	3	3/36 = 1/12
5	4	4/36 = 1/9
6	5	5/36
7	6	6/36 = 1/6
8	5	5/36
9	4	4/36 = 1/9
10	3	3/36 = 1/12
11	2	2/36 = 1/18
12	1	1/36

This table highlights the symmetry of outcomes around the middle sum, 7, which has the highest likelihood.

Special Cases and Variations

While the above calculations assume fair six-sided dice, variations can occur:

- **Loaded or biased dice: Probabilities shift based on design.**
- **Dice with different numbers of sides: For example, an 8-sided die combined with a 6-sided die alters the total outcomes and sum probabilities.**
- **Multiple rolls: Extending the concept to multiple rolls involves convolution of distributions.**

Practical Applications of Probability Calculations

Understanding these probabilities has tangible applications:

- **Gaming strategies: Players can assess the likelihood of winning or losing based on dice outcomes.**
- **Educational tools: Demonstrating fundamental principles of probability and combinatorics.**
- **Statistical modeling: Simulating scenarios where outcomes depend on repeated random events.**

Conclusion

The calculation of probabilities when rolling two dice offers a fascinating glimpse into the principles of combinatorics and probability theory. By systematically analyzing all possible outcomes and identifying those that meet specific conditions, such as a particular sum, we gain a clear understanding of the likelihood of various events. The symmetry and structure of these outcomes not only reinforce foundational concepts in probability but also serve as a gateway to more advanced statistical reasoning. Whether for academic purposes, gaming, or real-world decision-making, mastering these calculations enhances our ability to interpret and predict uncertainty in diverse contexts.

In essence, the probability that the sum of two rolled dice equals a particular number is determined by counting the favorable outcomes and dividing by the total possible outcomes. This straightforward yet profound approach underscores the beauty and utility of probability theory in understanding randomness and chance.

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