The Area Of A Rectangle Is 108 M2 And Its Diagonal Is 15 M. Findthe Perimeter Of The Rectangle.Please

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Understanding the properties of rectangles is fundamental in geometry, especially when solving real-world problems involving area, diagonals, and perimeter. In this article, we will explore how to determine the perimeter of a rectangle when its area is 108 m^2 and its diagonal length is 15 meters. This problem combines concepts of area, the Pythagorean theorem, and perimeter calculation, providing a comprehensive example of geometric problem-solving.

Understanding the Problem

Before diving into calculations, it's important to understand what information is given and what is being asked.

Given Data:

- Area of the rectangle = 108 m^2
- Diagonal length = 15 meters

What is Being Asked:

- Find the perimeter of the rectangle.

The perimeter of a rectangle is the total length around the shape, calculated as:

Perimeter = 2 × (length + width)

However, to find the perimeter, we need to determine the length and width of the rectangle first, which are not directly given but can be deduced from the provided data.

Step 1: Define Variables and Set Up Equations

Let’s denote:

- Length of the rectangle = l meters
- Width of the rectangle = w meters

Based on the problem:

- Area: $l \times w = 108$ (Equation 1)
- Diagonal: $d = 15$ meters

Since the diagonal of a rectangle relates to its length and width through the Pythagorean theorem:

$$ l^2 + w^2 = d^2 $$

Substitute $d = 15$:

$$ l^2 + w^2 = 15^2 = 225 $$ (Equation 2)

Now, we have two equations:

1. $l \times w = 108$
2. $l^2 + w^2 = 225$

Our goal is to find l and w to compute the perimeter:

$$ P = 2(l + w) $$

Note: Since the equations are symmetric in l and w, we can solve for them using substitution or elimination methods.

Step 2: Express One Variable in Terms of the Other

From Equation 1:

$$ w = \frac{108}{l} $$

Substitute this into Equation 2:

$$ l^2 + \left(\frac{108}{l} \right)^2 = 225 $$

Simplify:

$$ l^2 + \frac{11664}{l^2} = 225 $$

Multiply through by l^2 to clear the denominator:

$$ l^4 + 11664 = 225 l^2 $$

Rearranged:

$$ l^4 - 225 l^2 + 11664 = 0 $$

Let’s set $x = l^2$, simplifying the quartic to a quadratic:

$$ x^2 - 225 x + 11664 = 0 $$

Now, solve this quadratic equation for x.

Step 3: Solve the Quadratic Equation for x

Using the quadratic formula:

$$ x = \frac{225 \pm \sqrt{(225)^2 - 4 \times 1 \times 11664}}{2} $$

Calculate discriminant:

$$ \Delta = 225^2 - 4 \times 11664 $$

$$ 225^2 = 50625 $$

$$ 4 \times 11664 = 46656 $$

$$ \Delta = 50625 - 46656 = 3969 $$

Find the square root:

$$ \sqrt{3969} = 63 $$

Now, compute x:

$$ x = \frac{225 \pm 63}{2} $$

Calculate both solutions:

1. $x = \frac{225 + 63}{2} = \frac{288}{2} = 144$
2. $x = \frac{225 - 63}{2} = \frac{162}{2} = 81$

Recall that $x = l^2$, so:

- If $l^2 = 144$, then $l = \pm 12$. Since length is positive, $l = 12$ meters.
- If $l^2 = 81$, then $l = \pm 9$. So, $l = 9$ meters.

Correspondingly, compute w:

- When $l = 12$:

$$ w = \frac{108}{l} = \frac{108}{12} = 9 \text{ meters} $$

- When $l = 9$:

$$ w = \frac{108}{9} = 12 \text{ meters} $$

Thus, the rectangle's dimensions are:

- Length = 12 meters and Width = 9 meters, or vice versa.

Note: The rectangle's dimensions are interchangeable; what matters is the pair (9, 12).

Step 4: Calculate the Perimeter

Now that we know the dimensions:

$$ P = 2(l + w) $$

Choose $l = 12$ m and $w = 9$ m:

$$ P = 2(12 + 9) = 2 \times 21 = 42 \text{ meters} $$

Alternatively, if $l = 9$ m and $w = 12$ m, the perimeter remains the same:

$$ P = 2(9 + 12) = 42 \text{ meters} $$

Therefore, the perimeter of the rectangle is 42 meters.

Summary of the Solution

To find the perimeter of a rectangle with an area of 108 m^2 and a diagonal of 15 meters:

- Set variables for length and width.
- Use the area to relate length and width.
- Apply the Pythagorean theorem for the diagonal.
- Form a quadratic in terms of l^2, solve for l.
- Find corresponding w, ensuring the dimensions satisfy both equations.
- Calculate the perimeter using $P = 2(l + w)$.

The final answer: The perimeter of the rectangle is 42 meters.

Additional Tips for Solving Similar Problems

- Always verify your solutions by substituting back into original equations.

- Remember the Pythagorean theorem applies to right-angled triangles, which is the case for rectangle diagonals.
- When solving quadratic equations, check both solutions to identify physically meaningful dimensions.
- Recognize that dimensions of rectangles can often be interchanged, so the order doesn't affect the perimeter calculation.

Conclusion

Solving for the perimeter of a rectangle given its area and diagonal involves applying algebra, the Pythagorean theorem, and quadratic equations. This process enhances understanding of how different geometric properties are interconnected. By mastering these techniques, you can confidently approach a variety of rectangle-related problems in mathematics and real-world applications.

Keywords: rectangle perimeter, area of rectangle, diagonal of rectangle, geometry problems, Pythagorean theorem, solving quadratic equations, rectangle dimensions, perimeter calculation

Frequently Asked Questions

Given the area of a rectangle is 108 m² and its diagonal is 15 m, how can we find its length and width?

Let the length be L and the width be W. From the area, we have $L \times W = 108$. From the diagonal, by Pythagoras theorem, $L^2 + W^2 = 15^2 = 225$. Solving these two equations simultaneously allows us to find L and W.

What are the steps to determine the perimeter of the rectangle in this problem?

First, find the length and width using the area and diagonal equations. Once you have L and W, calculate the perimeter using $P = 2(L + W)$.

How do you set up equations to solve for length and width given area and diagonal length?

Use the area equation: $L \times W = 108$, and the Pythagorean theorem: $L^2 + W^2 = 225$. These form a system of equations to solve for L and W.

Can you provide the calculations to find the rectangle’s

dimensions?

Yes. From $L \times W = 108$, $W = 108 / L$. Substitute into $L^2 + W^2 = 225$: $L^2 + (108 / L)^2 = 225$. Solve this quadratic to find L, then W.

What is the approximate perimeter of the rectangle after solving for dimensions?

After solving, the dimensions are approximately $L \approx 12$ and $W \approx 9$. The perimeter is then $P = 2(12 + 9) = 42$ meters.

Why is the Pythagorean theorem applicable in this problem?

Because the diagonal of a rectangle forms a right triangle with its length and width, so the Pythagorean theorem relates these sides.

Are there multiple solutions for the dimensions of the rectangle in this problem?

No, since length and width are positive and based on the given area and diagonal, the dimensions are uniquely determined.

What formula do you use to find the perimeter once dimensions are known?

Perimeter = 2 × (length + width). After calculating length and width, simply substitute them into this formula.

Is it necessary to use quadratic equations to solve for the rectangle's dimensions?

Yes, because substituting $W = 108 / L$ into the diagonal equation results in a quadratic in L, which can be solved to find the dimensions.

Additional Resources

Understanding the Problem:

Let's begin by examining the problem statement carefully:

- The area of the rectangle is 108 m^2.
- The diagonal length of the rectangle is 15 meters.
- The goal is to find the perimeter of the rectangle.

This problem combines fundamental concepts of geometry involving areas, diagonals, and perimeters of rectangles. To thoroughly understand and solve this, we need to explore each component systematically.

Fundamentals of Rectangles

Before diving into calculations, it’s essential to recall some fundamental properties of rectangles:

- A rectangle has opposite sides equal.
- The area (A) of a rectangle is calculated as:

$$ A = length \times width $$

- The perimeter (P) of a rectangle is:

$$ P = 2 \times (length + width) $$

- The diagonal (d) of a rectangle relates to its length and width via the Pythagorean theorem:

$$ d = \sqrt{length^2 + width^2} $$

Understanding these formulas is crucial because they form the basis for the problem at hand.

Breaking Down the Given Data

Given:

- Area, $A = 108\, m^2$
- Diagonal, $d = 15\, m$

Unknown:

- Perimeter, P

To find the perimeter, we first need to determine the rectangle’s length and width.

Mathematical Approach to the Problem

The problem involves two key equations:

1. Area Equation:

$$ length \times width = 108 $$

2. Diagonal Equation (Pythagoras):

$$ length^2 + width^2 = 15^2 = 225 $$

Let’s denote:

- $l =$ length
- $w =$ width

From the area:

$$ l \times w = 108 \quad ...(1) $$

From the diagonal:

$$ l^2 + w^2 = 225 \quad ...(2) $$

Solving for Length and Width

To find l and w, we need to solve the system of equations.

Step 1: Express one variable in terms of the other

From equation (1):

$$ w = \frac{108}{l} $$

Step 2: Substitute into equation (2):

$$ l^2 + \left(\frac{108}{l} \right)^2 = 225 $$

This becomes:

$$ l^2 + \frac{108^2}{l^2} = 225 $$

Simplify numerator:

$$ 108^2 = 11664 $$

So:

$$ l^2 + \frac{11664}{l^2} = 225 $$

Step 3: Multiply through by l^2 to clear the denominator:

$$l^4 + 11664 = 225 l^2$$

Step 4: Rearrange to form a quadratic in l^2:

$$l^4 - 225 l^2 + 11664 = 0$$

Let’s set:

$$x = l^2$$

Then the equation becomes:

$$x^2 - 225 x + 11664 = 0$$

Solving the Quadratic Equation for x

Use quadratic formula:

$$x = \frac{225 \pm \sqrt{(225)^2 - 4 \times 1 \times 11664}}{2}$$

Calculate discriminant:

$$\Delta = 225^2 - 4 \times 11664$$

$$225^2 = 50625$$

$$4 \times 11664 = 46656$$

Therefore:

$$\Delta = 50625 - 46656 = 3979$$

Calculate the square root:

$$\sqrt{3979} \approx 63.12$$

Now, find x:

$$x = \frac{225 \pm 63.12}{2}$$

Two solutions:

1. $x_1 = \frac{225 + 63.12}{2} = \frac{288.12}{2} = 144.06$

2. $x_2 = \frac{225 - 63.12}{2} = \frac{161.88}{2} = 80.94$

Recall $x = l^2$, so:

- $l^2 = 144.06 \Rightarrow l = \sqrt{144.06} \approx 12.00\, m$
- $l^2 = 80.94 \Rightarrow l = \sqrt{80.94} \approx 8.99\, m$

Corresponding widths:

- When $l \approx 12.00\, m$:

$$w = \frac{108}{12} = 9\, m$$

- When $l \approx 8.99\, m$:

$$w = \frac{108}{8.99} \approx 12\, m$$

Note: Both pairs $(l, w) = (12, 9)$ and $(8.99, 12)$ satisfy the conditions because the rectangle's dimensions are interchangeable – length and width are not fixed in orientation.

Choosing the Correct Dimensions

Since the rectangle's dimensions can be labeled as length or width without loss of generality, both pairs are valid:

- Pair 1: $l = 12\, m$, $w = 9\, m$
- Pair 2: $l \approx 9\, m$, $w \approx 12\, m$

Either set will give the same perimeter because:

$$P = 2 (l + w)$$

Calculating the perimeter:

$$P = 2 (12 + 9) = 2 \times 21 = 42\, m$$

or

$$P = 2 (9 + 12) = 2 \times 21 = 42\, m$$

Thus, the perimeter of the rectangle is 42 meters.

Final Summary and Interpretation

- The key to solving this problem was recognizing the relationship between the dimensions via the Pythagorean theorem and the area.
- The quadratic equation derived from the substitution process yielded two solutions for the squares of the sides, which are symmetric and valid.
- Both solutions lead to the same perimeter due to the symmetry of the rectangle's dimensions.

Additional Considerations and Practical Applications

Understanding such problems is crucial not just in academic contexts but also in real-world applications:

- Construction and Architecture: When designing rooms or plots, knowing how to derive dimensions from area and diagonal helps optimize space and materials.
- Landscaping and Plot Design: Ensuring that a plot fits specific area and diagonal constraints is common in land planning.
- Manufacturing: Cutting materials in specific shapes often involves calculating dimensions from given constraints.

Moreover, mastering these calculations enhances spatial reasoning and problem-solving skills, which are valuable in various STEM fields.

Summary of the Key Steps in the Solution

1. Identify the given data: area and diagonal length.
2. Write down the relevant formulas: area, Pythagorean theorem for the diagonal.
3. Express one dimension in terms of the other: \(w = \frac{108}{l}\).
4. Substitute into the diagonal equation: leading to a quadratic in \(l^2\).
5. Solve the quadratic: find possible values of \(l\) and \(w\).
6. Calculate the perimeter: sum of sides multiplied by 2.

This systematic approach not only solves the problem but also reinforces understanding of the geometric principles involved.

Conclusion

The problem of finding the perimeter of a rectangle given its area and diagonal length exemplifies the harmonious interplay of algebra and geometry. By leveraging fundamental formulas and solving quadratic equations, we determined that the rectangle's perimeter is 42 meters. This comprehensive analysis underscores the importance of a structured problem-solving approach and deepens one's appreciation of geometric relationships in real-world contexts.

In summary:

- The rectangle's dimensions are approximately 12 meters and 9 meters.
- The perimeter corresponding to these dimensions is 42 meters.
- The problem elegantly combines algebraic manipulation with geometric principles, serving as an excellent example for learners and professionals alike.

If you're interested in further exploring related problems, consider variations such as:

- What if the diagonal length was different?
- How would changes in area affect the dimensions?
- Can similar methods be applied to other quadrilaterals?

With a solid understanding of the fundamentals, tackling such problems becomes more intuitive and rewarding.

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