The Factor (x + 2) Occurs In The Numerator And Denominator. There Will Be A Hole At X = 2.TrueO False

The Factor (x + 2) Occurs In The Numerator And Denominator. There Will Be A Hole At X = 2. True or False?

Understanding the behavior of rational functions around their factors is fundamental in algebra and calculus. When a factor like (x + 2) appears in both the numerator and denominator of a rational expression, it indicates a potential point of discontinuity known as a "hole." The question often arises: does the presence of the same factor in both parts automatically mean there is a hole at the corresponding value of x? This article explores this concept in depth, clarifying the conditions under which a hole exists and what it implies for the graph of the function.

What Is a Rational Function?

Definition

A rational function is a ratio of two polynomials, expressed as:

$$R(x) = \frac{P(x)}{Q(x)}$$

where $P(x)$ and $Q(x)$ are polynomials, and $Q(x) \neq 0$.

Graphical Behavior

The graph of a rational function can have various features:
- Vertical asymptotes where $Q(x) = 0$ and the numerator is non-zero.
- Holes where both numerator and denominator share common factors that are canceled out.
- Horizontal or oblique asymptotes depending on the degrees of the numerator and denominator.

Factors in Numerator and Denominator: What Do They Imply?

Common Factors and Simplification

When both numerator and denominator contain a common factor, such as $(x + 2)$, the first step is to factor both expressions completely and then simplify:

$$R(x) = \frac{\text{Factorized Numerator}}{\text{Factorized Denominator}}$$

If $(x + 2)$ appears as a factor in both, it can be canceled:

$$R(x) = \frac{\text{Remaining numerator}}{\text{Remaining denominator}}$$

However, this cancellation affects the domain and the nature of the function at specific points.

Implication of Cancellation

- The canceled factor indicates that at $x = -2$, the original function was undefined due to division by zero.
- After simplification, the function appears to be defined at $x = -2$ but with a "hole" or removable discontinuity in the graph.

Understanding Holes in the Graph

What Is a Hole?

A hole, also known as a removable discontinuity, occurs at a point $x = a$ where:
- The original function is undefined because the denominator equals zero.
- The numerator also equals zero at that point, indicating a common factor that can be canceled.
- After canceling, the function can be extended to be continuous at $x = a$, but the graph has a visible "hole."

How to Identify a Hole?

To determine if a point $x = a$ is a hole:
1. Factor the numerator and denominator completely.

2. Check if $(x - a)$ is a common factor.
3. If yes, cancel the factor.
4. The original function is undefined at $x = a$ (since the denominator was zero), but the simplified function is defined there.
5. The graph will show a hole at $x = a$.

Applying This to the Factor (x + 2)

Scenario: The Factor (x + 2) is in Both Numerator and Denominator

Suppose we have a rational function:

$$R(x) = \frac{(x + 2) \times \text{Other factors}}{(x + 2) \times \text{Other factors}}$$

- The factor (x + 2) appears in both numerator and denominator.
- When simplified, the (x + 2) cancels out:

$$R(x) = \frac{\text{Remaining numerator factors}}{\text{Remaining denominator factors}}$$

- The original function is undefined at $x = -2$, because plugging $x = -2$ into the original denominator yields zero.

Does this mean there is a hole at $x = -2$?

Yes, because:
- The factor (x + 2) cancels out, suggesting the discontinuity is removable.
- The original function has a division by zero at $x = -2$, but the simplified function is defined everywhere else.
- The graph will have a hole at $x = -2$.

Note: The statement in the prompt uses "x = 2," but the factor is (x + 2), which corresponds to $x = -2$. For clarity, the hole occurs at the root of the canceled factor, i.e., at $x = -2$.

False or True? Clarifying the Statement

Statement Analysis

> "The factor (x + 2) occurs in the numerator and denominator. There will be a hole at $x = 2$."

- Since $(x + 2) = 0$ when $x = -2$, the hole appears at $x = -2$, not at $x = 2$.
- Therefore, the statement is False if interpreted literally at $x = 2$.

Corrected Understanding

- The factor $(x + 2)$ has a zero at $x = -2$.
- When this factor appears in both numerator and denominator, there is a hole at $x = -2$, not at $x = 2$.

Conclusion: The statement in the prompt is False because the hole occurs at $x = -2$, not at $x = 2$.

Summary and Key Takeaways

Summary

- When a factor appears in both numerator and denominator, it indicates a removable discontinuity, or a hole.
- The hole occurs at the root of the canceled factor.
- The location of the hole depends on the zero of the common factor, not the factor itself.

Key Points to Remember

- Identify common factors through full factoring of numerator and denominator.

- Cancel common factors to find the simplified form of the function.

- Determine the hole's location by solving $(x + 2) = 0$, which yields $x = -2$.

- The original function is undefined at the hole, but the simplified function is defined there, allowing the graph to be "patched" with a hole at that point.

Final Thoughts

Understanding the relationship between common factors and holes in rational functions is crucial for graphing and analyzing functions accurately. Recognizing that the hole occurs at the zero of the canceled factor, not at the factor itself, is key to correct interpretation. In the specific case of the factor \((x + 2) \), the hole appears at \(x = -2 \), making the original statement false if it claims a hole at \(x = 2 \).

In conclusion, the statement "The factor (x + 2) occurs in the numerator and denominator. There will be a hole at \(x = 2 \)." is False. The hole occurs at \(x = -2 \), the root of the canceled factor. Proper understanding of these principles allows students and mathematicians alike to accurately analyze and graph rational functions.

Frequently Asked Questions

Does the factor (x + 2) appear in both the numerator and denominator of a rational expression?

Yes, if $(x + 2)$ appears in both numerator and denominator, it is a common factor that can be canceled out.

What happens at x = 2 if (x + 2) is a factor in both numerator and denominator?

There will be a hole in the graph at $x = -2$, not at $x = 2$, because the factor is $(x + 2)$.

Is the statement 'The factor (x + 2) occurs in the numerator and denominator, creating a hole at x = 2' true or false?

False. The hole occurs at $x = -2$, not at $x = 2$.

How do you identify a hole in the graph of a rational function?

A hole appears at the x-value where a factor cancels out from numerator and denominator, which corresponds to the root of that factor.

If a factor (x + 2) appears in both numerator and denominator, what is the x-coordinate of the hole?

The x-coordinate of the hole is x = -2.

Is the statement 'There will be a hole at x=2' true if the common factor is (x + 2)?

No, the hole is at x = -2, not x = 2.

Can the factor (x + 2) be canceled out in the expression?

Yes, since it appears in both numerator and denominator, it can be canceled, creating a hole at x = -2.

What is the correct interpretation of the statement: 'The factor (x + 2) occurs in the numerator and denominator. There will be a hole at x=2'?

It is false because the hole occurs at x = -2, not at x = 2.

Additional Resources

Understanding the Role of Factors in Rational Expressions: When Does a Hole Occur at x = 2?

Introduction

Mathematics, especially algebra, often introduces students to the fascinating world of rational expressions—fractions where polynomials serve as numerators and denominators. Among the numerous concepts that come into play when simplifying these expressions, the idea of holes (also called removable discontinuities) is fundamental. When a factor appears in both the numerator and the denominator, it can lead to a situation where the function is undefined at a certain point, but that undefined point may be "removed" after simplification, resulting in a hole rather than a vertical asymptote.

One common question that arises in this context is: "If (x + 2) occurs in both the numerator and denominator, does that mean there will be a hole at x = 2?" The statement to evaluate is: True or False?

In this detailed review, we'll analyze this question thoroughly, exploring the underlying principles, illustrating with examples, and clarifying common misconceptions.

The Basics of Rational Functions and Factors

What Is a Rational Function?

A rational function is a ratio of two polynomials:

$$
f(x) = \frac{P(x)}{Q(x)}
$$

where both $P(x)$ and $Q(x)$ are polynomials.

Factors in Numerator and Denominator

- When a polynomial can be factored, its factors may appear in both the numerator and the denominator.
- For example:

$$
f(x) = \frac{(x + 2)(x - 3)}{(x + 2)(x + 5)}
$$

- In this case, the factor $(x + 2)$ appears in both numerator and denominator.

The Concept of Holes in Rational Functions

What Is a Hole?

- A hole in the graph of a rational function occurs at a specific x-value where the function is undefined (due to zero in the denominator), but the discontinuity can be "removed" by simplifying the function.
- When the common factor in numerator and denominator is canceled out, the point at which this factor equals zero becomes a hole.

How Do Holes Differ from Vertical Asymptotes?

- Vertical asymptotes occur where the function tends to infinity, typically at points where the denominator is zero and the factor isn't canceled out.

- Holes occur where the factor is canceled out, indicating the function is undefined there but can be redefined to make the function continuous.

Analyzing the Factor (x + 2)

Let's analyze the specific factor \((x + 2) \):

- The root of \((x + 2) = 0 \) is at \(x = -2 \).
- Notice that the question mentions x = 2, but the factor is \((x + 2) \), which zeros at \(x = -2 \), not at \(x = 2 \).

This is a crucial point: the zero of the factor \((x + 2) \) is at \(x = -2 \), not \(x = 2 \).

Evaluating the Given Statement

The statement is:

> "The Factor (x + 2) Occurs In The Numerator And Denominator. There Will Be A Hole At X = 2."

- Is this true or false?

Answer: False.

Because:

- The factor \((x + 2) \) zeros out at \(x = -2 \), not at \(x = 2 \).
- If the factor \((x + 2) \) appears in both numerator and denominator, the potential hole is at \(x = -2 \), not at \(x = 2 \).

Deep Dive: Why the Location of the Zero Matters

The Zero of the Factor

- For \((x + 2) \), the zero occurs at \(x = -2 \).
- The hole's location depends on where the factor equals zero, not where the factor is written in the expression.

Visualizing the Graph

Suppose:

$$
f(x) = \frac{(x + 2)(x - 3)}{(x + 2)(x + 5)}
$$

- Simplify by canceling $(x + 2)$:

$$
f(x) = \frac{x - 3}{x + 5} \quad \text{for} \quad x \neq -2
$$

- At $x = -2$, the original function is undefined, creating a hole.
- The hole occurs at $x = -2$, and the value of the function at this point is not defined unless we define it by the simplified expression.

Contrasting with $x = 2$

- If we consider a factor $(x - 2)$, its zero occurs at $x = 2$.
- If the factor $(x + 2)$ appears in both numerator and denominator, the hole occurs at $x = -2$, not $x = 2$.

Practical Examples

Let's examine some concrete examples to clarify the concept:

Example 1: Correct Identification of the Hole

$$
f(x) = \frac{(x + 2)(x - 4)}{(x + 2)(x + 1)}
$$

- Cancel $(x + 2)$:

$$
f(x) = \frac{x - 4}{x + 1}
$$

- The original function is undefined at $x = -2$ (since denominator zero), but after simplification, the hole is at $x = -2$.

Key point: The hole is at $x = -2$, because of the zero of the canceled factor $(x + 2)$.

Example 2: Misconception about $x = 2$

Suppose someone writes:

$$
g(x) = \frac{(x + 2)(x - 3)}{(x + 2)(x - 5)}
$$

- The zero of $(x + 2)$ is at $x = -2$.
- The potential hole occurs at $x = -2$.

But if the statement claims there's a hole at $x = 2$ because of the factor $(x + 2)$, that's incorrect.

Clarifying the Confusion

The confusion often arises because:

- The factor is written as $(x + 2)$, which zeros at $x = -2$.
- The question references $x = 2$ instead of $x = -2$.

Therefore:

- The presence of $(x + 2)$ in numerator and denominator implies a potential hole at $x = -2$.
- It does not imply a hole at $x = 2$.

Summary of Key Points

- The factor $(x + 2)$ zeros at $x = -2$.
- If $(x + 2)$ appears in both numerator and denominator, the function has a hole at $x = -2$.
- The statement claims a hole at $x = 2$, which is incorrect because $(x + 2) = 0$ only when $x = -2$.

Additional Considerations

- If a different factor such as $(x - 2)$ appears in both numerator and denominator, then the hole would

be at \(x = 2 \).
- Always identify the zero of the factor to determine the location of the hole.
- When simplifying rational functions, cancel common factors to find holes, but remember to note the \(x \)-values where these factors equal zero.

Final Verdict

The statement is: False.

The occurrence of the factor \((x + 2) \) in both numerator and denominator results in a hole at \(x = -2 \), not at \(x = 2 \). Therefore, claiming there's a hole at \(x=2 \) based on the factor \((x + 2) \) is incorrect.

Concluding Remarks

Understanding the relationship between factors and holes in rational functions is essential for accurate graphing and analysis. Recognizing where the zeros of factors occur and how they influence the function's domain allows mathematicians and students to predict and interpret the behavior of complex functions effectively.

Always double-check the factors and their corresponding zeros before concluding about the presence and location of holes or asymptotes. This careful approach leads to a deeper grasp of algebraic concepts and enhances problem-solving accuracy.

In summary:

- The factor \((x + 2) \) zeros at \(x = -2 \).
- A common factor appearing in numerator and denominator indicates a potential hole at \(x = -2 \).
- The statement referencing a hole at \(x=2 \) when the factor is \((x + 2) \) is incorrect—it's a classic example of a misconception that can be clarified through understanding zeros and factors.

End of review.

[The Factor X 2 Occurs In The Numerator And Denominator There Will Be A Hole At X 2 Trueo False]

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