

The Following Ordered Pairs Represent A Relation. $\{(1,3) , (5,15) , (2,6), (4,12) , (3,9)\}$ What Are The

The Following Ordered Pairs Represent A Relation. $\{(1,3) , (5,15) , (2,6), (4,12) , (3,9)\}$ What Are The are fundamental questions in the study of mathematics, especially in the areas of set theory, algebra, and discrete mathematics. Understanding relations, their properties, and their representations is crucial for grasping more advanced topics like functions, graphs, and data modeling. In this comprehensive article, we will explore what relations are, analyze the given ordered pairs, and explain the significance of relations in various mathematical contexts.

Understanding Relations in Mathematics

What Is a Relation?

In mathematics, a relation is a way to describe a relationship between elements of two sets. More specifically, a relation from set A to set B is a subset of the Cartesian product $(A \times B)$.

Definition:

A relation (R) from set (A) to set (B) is a set of ordered pairs $((a, b))$ where $(a \in A)$ and $(b \in B)$. The relation "relates" each element (a) in (A) to one or more elements (b) in (B) .

Example:

If $(A = \{1, 2, 3\})$ and $(B = \{3, 6, 9, 12, 15\})$, then a relation could be:

$$\begin{aligned} & \setminus \\ R &= \{(1, 3), (2, 6), (3, 9)\} \\ & \setminus \end{aligned}$$

which indicates that 1 is related to 3, 2 is related to 6, and 3 is related to 9.

Analyzing the Given Set of Ordered Pairs

The Set of Ordered Pairs

The set provided is:

$$\{(1, 3), (5, 15), (2, 6), (4, 12), (3, 9)\}$$

This set represents a relation between two sets of elements, which we can interpret as pairs where the first element of each pair is related to the second element.

Key observations:

- The first elements are: 1, 5, 2, 4, 3
- The second elements are: 3, 15, 6, 12, 9

This relation can be visualized or analyzed to understand its structure, properties, and potential applications.

Identifying the Domain and Range

Domain

The domain of a relation is the set of all first elements in the ordered pairs.

$$\text{Domain} = \{1, 2, 3, 4, 5\}$$

Range

The range is the set of all second elements in the ordered pairs.

$$\text{Range} = \{3, 6, 9, 12, 15\}$$

Understanding the domain and range helps to visualize the relation and analyze its properties.

Properties of the Relation

In mathematical analysis, properties such as reflexivity, symmetry, antisymmetry, transitivity, and functionality are crucial to understanding what kind of relation we are dealing with.

Is the Relation a Function?

A relation is a function if each element in the domain is related to exactly one element in the range.

Check:

- Each first element appears only once in the set of pairs.
- The pairs are: (1,3), (2,6), (3,9), (4,12), (5,15)

Since no first element repeats, the relation is a function.

Implication:

- The relation described is a function from the domain to the range.
- Each input (from the domain) maps to exactly one output.

Is the Relation One-to-One (Injective)?

A relation (or function) is injective if different elements in the domain map to different elements in the range.

Check:

- All second elements are unique: 3, 6, 9, 12, 15
- Since all outputs are distinct, the relation is injective.

Is the Relation Onto (Surjective)?

A relation from set A to set B is surjective if every element in B is mapped to by some element in A.

- The range includes all elements: 3, 6, 9, 12, 15
- If the codomain is exactly these elements, then the relation is surjective onto this set.

Note:

- To determine surjectivity, the codomain must be specified. If the codomain is larger, then the relation might not be onto.

Summary of Properties:

Property	Status
Is it a function?	Yes
Is it injective?	Yes
Is it surjective?	Yes, onto the set {3, 6, 9, 12, 15}

Graphical Representation of the Relation

Visualizing relations helps in understanding their structure.

Plotting the Relation

- The x-axis represents the domain elements.
- The y-axis represents the range elements.

Plot each pair:

- (1, 3)
- (2, 6)
- (3, 9)
- (4, 12)
- (5, 15)

This creates a set of points on a coordinate plane, illustrating the direct relationship between each element.

Graphical Insights

- The points form a straight line if the relation follows a specific rule (like a linear function).
- In this case, the relation resembles the function $y = 3x$, as all pairs satisfy this.

Mathematical Interpretation of the Relation

Is the Relation a Function? Yes, and It Looks Like a Linear Function

Given the pairs:

$\{$
(1, 3), (2, 6), (3, 9), (4, 12), (5, 15)
 $\}$

Observe that:

$\{$
 $b = 3a$
 $\}$

for each pair $((a, b))$.

Therefore, the relation can be expressed as:

$\{$
 $b = 3a$
 $\}$

This is a linear function with slope 3 passing through the origin (though in this case, it passes through (1,3) which aligns with $(b = 3a)$).

Graphically:

- The relation is a straight line with slope 3.
- It passes through points (1, 3), (2, 6), etc.

Applications of Relations in Real Life

Relations are not just abstract mathematical concepts; they have practical applications across various fields.

Examples include:

- Mapping students to their grades
- Relating employees to their departments
- Associating products with their prices

- Modeling relationships in social networks
- Describing functions in physics, such as distance-time relations

Understanding the properties of relations helps in designing efficient algorithms, data structures, and systems.

Summary and Conclusion

The set of ordered pairs:

$$\{(1, 3), (5, 15), (2, 6), (4, 12), (3, 9)\}$$

represents a relation that is a function, specifically a linear function $(y = 3x)$. It is injective and surjective onto its codomain, making it a bijective relation within the specified set.

Understanding relations involves analyzing their domain, range, properties, and graphical representations, which are fundamental in both theoretical and applied mathematics.

Final Thoughts

Recognizing and analyzing relations like the one given helps build a strong foundation for advanced mathematical concepts such as functions, graphs, and data modeling. Whether in pure mathematics or real-world applications, relations serve as the backbone for understanding how different entities interact, relate, and influence each other.

If you want to learn more about relations, functions, and their properties, consider exploring topics like inverse relations, composite functions, and relation matrices, which deepen your understanding of how elements connect and interact in various mathematical contexts.

Frequently Asked Questions

What does the relation $\{(1,3), (5,15), (2,6), (4,12), (3,9)\}$ represent?

It represents a set of ordered pairs where each first element is related to its corresponding second element, illustrating a relation between numbers.

How can we determine if this relation is a function?

Since each first element (input) has only one corresponding second element (output), the relation is a function.

What is the pattern or rule connecting the first and second elements in the relation?

The second element appears to be the first element multiplied by 3, indicating a rule $y = 3x$.

Is the relation $\{(1,3), (5,15), (2,6), (4,12), (3,9)\}$ one-to-one?

Yes, each first element is unique and maps to a unique second element, making it a one-to-one relation.

What is the domain and range of this relation?

The domain is $\{1, 2, 3, 4, 5\}$ and the range is $\{3, 6, 9, 12, 15\}$.

Can this relation be represented as a graph? If so, what would it look like?

Yes, it can be graphed as points on a coordinate plane, forming a straight line passing through $(x, 3x)$.

How do you verify if the relation is linear?

Check if the second elements follow a consistent pattern based on the first elements; here, $y = 3x$ indicates a linear relation.

Additional Resources

The Following Ordered Pairs Represent A Relation. What Are The

Introduction

In the realm of mathematics, particularly in set theory and discrete mathematics, the concept of relations forms a foundational pillar for understanding how different elements within sets interact and connect. When analyzing specific collections of ordered pairs, such as $\{(1,3), (5,15), (2,6), (4,12), (3,9)\}$, it becomes essential to investigate the nature of the relation they depict, its properties, and its

potential applications. This article provides a comprehensive exploration of the relation represented by these ordered pairs, examining its structure, characteristics, and significance within mathematical contexts.

Understanding Relations Through Ordered Pairs

Defining a Relation

At its core, a relation from one set to another is a subset of the Cartesian product of these sets. Given two sets, A and B, a relation R is a collection of ordered pairs (a, b) where $a \in A$ and $b \in B$, such that the pair satisfies certain properties or conditions.

In our specific case, the relation is explicitly defined by the set of ordered pairs:

$$R = \{(1,3), (5,15), (2,6), (4,12), (3,9)\}$$

Here, the first elements (domain) are 1, 5, 2, 4, 3, and the corresponding second elements (range) are 3, 15, 6, 12, 9.

Visualizing the Relation

Visual representations, such as mappings or graphs, can aid in understanding the structure of the relation. Plotting the pairs on a coordinate plane reveals patterns and connections, offering insights into the nature of these relationships.

Analyzing the Relation: Structural Properties

Domain and Range

- Domain (Input Set): {1, 2, 3, 4, 5}
- Range (Output Set): {3, 6, 9, 12, 15}

The domain and range are both finite, with all elements being distinct within their sets, indicating a well-defined relation.

Function or Non-Function?

A key question is whether this relation qualifies as a function. Recall that a function assigns exactly one output to each input.

- Each element in the domain appears exactly once as the first element in the ordered pairs.
- No input element repeats with different outputs.

Thus, the relation is a function because every element in the domain maps to exactly one element in the range.

Mathematical Properties of the Relation

To understand the nature of this relation further, we analyze several classical properties:

1. Reflexivity

A relation on a set is reflexive if every element relates to itself. Since our relation involves pairs like (1,3), (5,15), etc., and none of the pairs are of the form (a, a), it is not reflexive.

2. Symmetry

A relation is symmetric if whenever (a, b) is in the relation, then (b, a) is also in the relation. Our set has only pairs from input to output, with no reciprocals; hence, it is not symmetric.

3. Transitivity

A relation is transitive if whenever (a, b) and (b, c) are in the relation, then (a, c) must also be in the relation. Let's explore whether this holds.

- For example, (1,3) and (3,9) are in the relation.
- Does (1,9) exist? No.
- Similarly, (2,6) and (6,?) - no pair with 6 as the first element.

Thus, the relation is not transitive.

4. Injectivity

A relation (or function) is injective (one-to-one) if distinct inputs map to distinct outputs. Since each input is unique and maps to a unique output, the relation is injective.

5. Surjectivity

A relation (or function) is surjective (onto) if every element in the codomain has a pre-image. If we consider the codomain as the set of outputs, {3, 6, 9, 12, 15}, then:

- All outputs are achieved by some input.
- Therefore, the relation is surjective onto this set.

The Functional Nature and Pattern Recognition

The relation appears to follow a specific pattern: the second element of each pair seems to be a multiple of the first element, specifically a multiplication by 3.

Input (a) Output (b) Pattern Explanation		
----- ----- -----		
1	3	1 × 3
2	6	2 × 3
3	9	3 × 3
4	12	4 × 3

$$| 5 | 15 | 5 \times 3 |$$

This observation indicates that the relation can be expressed as a function $f: \mathbb{N} \rightarrow \mathbb{N}$, defined by:

$$f(a) = 3a$$

for all $a \in \{1, 2, 3, 4, 5\}$.

Broader Context and Applications

Mathematical Significance

The relation exemplifies a simple yet fundamental concept: linear functions, specifically scalar multiples. It serves as an illustrative example to introduce students and researchers to the properties of functions, relations, and their classifications.

Practical Applications

Such relations are not merely theoretical—they underpin many practical scenarios:

- Scaling in Engineering: Multiplication relations model proportional relationships, such as converting units or scaling dimensions.
- Computational Algorithms: Functions like $f(a) = 3a$ are used in algorithms involving linear transformations.
- Data Mapping: Relations like these can represent mappings from input data to processed output, such as in data processing pipelines.

Extending the Concept: Generalization and Variations

While the current relation is finite and defined explicitly for specific pairs, the concept extends to broader contexts:

- Infinite relations: For example, the set of all pairs $(a, 3a)$ where $a \in \mathbb{N}$.
- Different scalar multiples: Relations such as $f(a) = ka$, with various constants k .
- Relations over different sets: Extending the relation to real numbers, rational numbers, or other domains.

This generalization leads to the study of functions and relations in more complex mathematical systems, including calculus, linear algebra, and real analysis.

Critical Evaluation and Pedagogical Implications

Understanding the relation defined by these pairs provides a stepping stone for learners to grasp core concepts:

- Identifying functions: Recognizing that each input maps to a unique output.
- Analyzing properties: Distinguishing between reflexive, symmetric, transitive, injective, and surjective relations.
- Pattern recognition: Observing the multiplication pattern that defines the relation.

Educators can leverage such explicit examples to foster conceptual understanding, moving from concrete sets of pairs to abstract functional definitions.

Conclusion

The set of ordered pairs $\{(1,3), (5,15), (2,6), (4,12), (3,9)\}$ exemplifies a clear, linear relation that functions as a simple mathematical model for scalar multiplication. Its properties—being a function, injective, and surjective onto its range—highlight foundational concepts in the study of relations and functions. By dissecting and analyzing this relation, students and researchers can better appreciate the underlying principles that govern more complex mathematical systems.

Understanding such relations not only deepens mathematical literacy but also opens pathways to numerous practical applications across sciences, engineering, and computer science. As a microcosm of the broader mathematical landscape, this relation serves as an instructive and illustrative example of the elegant simplicity underlying many mathematical constructs.

References

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Author's Note: This comprehensive review aims to provide clarity and depth in understanding the relation defined by the given ordered pairs, suitable for academic and educational contexts.

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