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The function F, as given, appears to be a piecewise function, which means its definition varies depending on the value of the input variable x. Based on the information provided, the function is defined as $F(x) = x + 6$ when a certain condition involving x is satisfied, specifically when $-55x < 1$, and $F(x) = 9$ in some other case. To analyze and understand this function thoroughly, we will examine each part (a) to (d), exploring the domain, range, behavior, and other properties associated with it. This in-depth discussion will help clarify the structure and characteristics of the function.

Understanding the Function's Definition

Deciphering the Piecewise Structure

The given statement appears to have some formatting issues, but the core idea is that F(x) is defined differently based on the value of x. Typically, a piecewise function is written as:

```
F(x) = {
x + 6,  if -55x < 1
```

```
9,      otherwise
}
```

If this interpretation is correct, then the function is split into two parts:

- When the inequality -55x < 1 is true, F(x) = x + 6
- When the inequality is false, F(x) = 9

Rewriting the Domain Conditions

To understand where each part applies, we analyze the inequality:

```
-55x < 1
```

Dividing both sides by -55 (note that dividing by a negative reverses the inequality sign):

```
x > -\frac{1}{55}
```

This critical point divides the real number line into two regions:

1. For x > -1/55, the condition -55x < 1 holds, so F(x) = x + 6
2. For x □ -1/55, the condition -55x < 1 does not hold, so F(x) = 9

Part (a): Domain of the Function F

Defining the Domain Based on the Piecewise Conditions

The domain of a function is the set of all possible input values x for which the function is defined. Since F(x) is defined for all real numbers (either as x + 6 or as 9), the domain is:

- **Domain of F:** All real numbers, ▯

Because both parts of the function are defined over the entire real line, there are no restrictions on x. The piecewise definition simply specifies the output based on the value of x relative to -1/55.

Part (b): Range of the Function F

Finding the Range for Each Part

To determine the range, we analyze each piece separately.

When $x > -1/55$, $F(x) = x + 6$

- The domain for this part is (-1/55, □)
- As x approaches -1/55 from the right, F(x) approaches (-1/55) + 6 □ 6 - 1/55 □ 5.9818
- As x approaches □, F(x) □ □
- Therefore, the range for this part is:

```
(≈ 5.9818, ∞)
```

When x □ -1/55, F(x) = 9

- The output is constantly 9, regardless of x in this region

Combining the Ranges

The overall range of F(x) combines both parts:

- Values from approximately 5.9818 up to infinity (excluding the exact value at x = -1/55)
- The constant value 9

Since 9 is within the interval (□ 5.9818, □), the total range is:

```
[≈ 5.9818, ∞)
```

or, more precisely, the union of the interval (□ 5.9818, □) and the point 9, which is already included in that interval, so the range simplifies to:

```
(≈ 5.9818, ∞)
```

Part (c): Graphical Behavior of the Function F

Visualizing the Function

The graph of F(x) can be visualized as follows:

- A straight line segment for x > -1/55, with the equation y = x + 6
- A horizontal line at y = 9 for x □ -1/55

Plotting Key Points

1. At x = -1/55, the line y = x + 6 approaches approximately 5.9818

2. At x = -1/55, F(x) = 9 (the constant segment)

Discontinuity and Behavior

There is a jump discontinuity at x = -1/55, where the function jumps from approximately 5.9818 directly to 9. This discontinuity occurs because the definition switches from the linear segment to the constant value. For larger x, the function increases linearly without bound.

Part (d): Mathematical Properties of F

Continuity

- F(x) is continuous on the intervals ($-\infty$, -1/55) and (-1/55, ∞)
- There is a jump discontinuity at x = -1/55

Monotonicity

- For x > -1/55, F(x) = x + 6 is increasing
- For x $\leq$ -1/55, F(x) = 9 is constant (neither increasing nor decreasing)

Limits and Behavior at Critical Points

- Limit as x approaches -1/55 from the left: F(x) = 9 (constant)
- Limit as x approaches -1/55 from the right: F(x) □ 5.9818
- The function jumps from approximately 5.9818 to 9 at x = -1/55

Summary and Conclusion

In this analysis, we have dissected the piecewise function F(x), established its domain and range, visualized its graph, and examined its properties. The key takeaways are:

- The domain of F(x) is all real numbers, □.
- The range of F(x) is approximately (5.9818, □).
- The function exhibits a jump discontinuity at x = -1/55 but is otherwise continuous and increasing on its linear segment.
- The function's behavior is straightforward: a constant value for x less than or equal to -1/55, and a line with slope 1 for x greater than -1/55.

This detailed understanding of the piecewise function F provides a comprehensive foundation for further applications, such as solving equations involving F or analyzing similar piecewise functions in advanced mathematics.

Frequently Asked Questions

What is the piecewise definition of the function F(x)?

The function $F(x)$ is defined as $F(x) = x + 6$ if $-55x < 1$, and $F(x) = 9$ otherwise.

How do you determine the interval where F(x) equals x + 6?

$F(x)$ equals $x + 6$ when the inequality $-55x < 1$ holds true, which simplifies to $x > -1/55$.

What is the value of F(x) when x is less than or equal to -1/55?

When x □ -1/55, then -55x □ 1, so $F(x) = 9$.

Calculate F(0). What does it evaluate to?

Since $0 > -1/55$, we use $F(x) = 9$, so $F(0) = 9$.

For which x-values does F(x) = x + 6?

$F(x) = x + 6$ for all $x > -1/55$.

Additional Resources

In-Depth Analysis of the Function F: Definition, Properties, and Applications

Understanding the behavior and properties of mathematical functions is fundamental in various fields such as calculus, algebra, and applied mathematics. In this detailed review, we examine a specific piecewise function, denoted as F, which is defined with particular conditions. This exploration will include a comprehensive breakdown of its structure, domain, range, and various features, along with solutions to the parts (a) to (d) as specified.

Introduction to the Function F

The function F under consideration is described as follows:

$$
F(x) =
\begin{cases}
x + 6, & \text{if } -55x < 1 \\
9, & \text{otherwise}
\end{cases}
$$

At first glance, this piecewise function appears straightforward, yet it encompasses several nuanced features that warrant detailed analysis. Our goal is to dissect the function thoroughly, addressing each of the parts (a) to (d), which likely involve domain determination, range identification, continuity analysis, and possibly other properties.

Part (a): Determining the Domain of F

Definition of the Domain

The domain of a function is the set of all input values (x-values) for which the function is defined. In the case of this piecewise function, the definition hinges on the condition $-55x < 1$.

Step-by-Step Domain Analysis

1. Analyze the inequality $-55x < 1$:

- Divide both sides by -55, noting that dividing by a negative number reverses the inequality:

$$
-55x < 1 \quad \Rightarrow \quad x > -\frac{1}{55}
$$

- Important: The inequality flips because we're dividing by a negative number.

2. Interpreting the piecewise definition:

- For all x satisfying $x > -\frac{1}{55}$, the function is defined as $F(x) = x + 6$.

- For x such that $-55x \geq 1$, i.e., $x \leq -\frac{1}{55}$, the function takes the value $F(x) = 9$.

3. Domain conclusion:

- The entire real line is covered by these two cases; the function is defined for all real x because the conditions partition the real line completely:

- When $x > -\frac{1}{55}$, $F(x) = x + 6$.

- When $x \leq -\frac{1}{55}$, $F(x) = 9$.

Final Domain:

$$
\boxed{
\text{Domain of } F: \quad (-\infty, -\frac{1}{55}] \cup (-\frac{1}{55}, \infty) = \mathbb{R}
}
$$

In essence, the domain is all real numbers, since the function is defined for every real x, but with different expressions depending on the region.

Part (b): Finding the Range of F

Understanding the Range

The range is the set of all possible output values of F as x varies over its domain. Given the piecewise nature, we analyze each case separately.

Case 1: $x > -\frac{1}{55}$

- Here, $F(x) = x + 6$.

- As x approaches $-\frac{1}{55}$ from above:

$$
\lim_{x \to -\frac{1}{55}^+} F(x) = -\frac{1}{55} + 6
$$

Calculate:

$$
-\frac{1}{55} + 6 = 6 - \frac{1}{55} = \frac{330}{55} - \frac{1}{55} = \frac{329}{55}
$$

- As x approaches infinity:

$$
\lim_{x \to \infty} F(x) = \infty
$$

- Therefore, on this interval:

$$
F(x) \in \left(\frac{329}{55}, \infty \right)
$$

Case 2: $x \leq -\frac{1}{55}$

- Here, $F(x) = 9$.

- Since this is a constant value, the range contribution is simply:

$$
\{9\}
$$

Combining the ranges:

- The outputs from the first case are all real numbers greater than $\frac{329}{55}$.

- The second case contributes a single value, 9.

- Important Observation:

- Is 9 included in the first case?

- To check, see if $x > -\frac{1}{55}$ can produce $F(x) = 9$:

$$
x + 6 = 9 \quad \Rightarrow \quad x = 3
$$

- But $x=3$ is greater than $-\frac{1}{55}$, so in the domain where $F(x) = x + 6$, $F(3) = 9$.

- Therefore, the value 9 is achieved at $x=3$ in the first case, meaning the range is continuous from $\frac{329}{55}$ up to infinity, including 9.

Final Range:

$$
\boxed{
\text{Range of } F: \left[\frac{329}{55}, \infty \right)
}
$$

- Because x can be chosen to yield any value above $\frac{329}{55}$, and $F(x)$ can exactly be 9 (by selecting $x=3$), the range is continuous from $\frac{329}{55}$ to infinity, including 9.

Part (c): Investigating Continuity and Discontinuity

Continuity Analysis

The key points to analyze are where the piecewise definitions meet, particularly at the boundary $x = -\frac{1}{55}$.

1. Continuity at $x = -\frac{1}{55}$

- Left-hand limit (approach from below):

For $x \leq -\frac{1}{55}$, $F(x) = 9$.

$$
\lim_{x \to -\frac{1}{55}^-} F(x) = 9
$$

- Right-hand limit (approach from above):

For $x > -\frac{1}{55}$, $F(x) = x + 6$.

$$
\lim_{x \to -\frac{1}{55}^+} F(x) = -\frac{1}{55} + 6 = \frac{329}{55}
$$

- Value at $x = -\frac{1}{55}$:

Since the piecewise definition specifies $F(x) = 9$ for $x \leq -\frac{1}{55}$, then:

$$
F\left(-\frac{1}{55}\right) = 9
$$

\]

- Comparison:

- Left-hand limit = 9
- Right-hand limit = $\frac{329}{55} \approx 5.98$
- Function value at the point = 9

- Conclusion:

Since the left-hand limit equals the function value (both 9), but the right-hand limit ≈ 5.98 does not equal 9, the function is discontinuous at $x = -\frac{1}{55}$.

Type of discontinuity: Jump discontinuity.

2. Continuity elsewhere

- For $x > -\frac{1}{55}$, $F(x) = x + 6$, which is continuous everywhere.

- For $x < -\frac{1}{55}$, $F(x) = 9$, which is constant and continuous.

- The only discontinuity is at the boundary point $x = -\frac{1}{55}$.

Part (d): Additional Properties and Applications

1. Behavior at Infinity

- As $x \to \infty$:

\[
F(x) = x + 6 \to \infty
\]

- As \(x \to -\infty\):

\[
F(x) = 9
\]

Since for \(x \leq -\frac{1}{55}\), \(F(x) = 9\), the function approaches 9 from the left, remaining constant.

2. Derivative and Monotonicity

- For \(x > -\frac{1}{55}\):

\[
F'(x) = 1
\]

indicating that \(F\) is strictly increasing on this interval.

- For \(x \leq -\frac{1}{55}\):

\[
F(x) = 9 \quad

The Function F Is Defined As Follows Complete Parts A To D

Below X 6 If 55x Lt 1 F X 9

The Function F Is Defined As Follows Complete Parts A To D Below X 6 If 55x Lt 1 F X 9

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