The Function F Is Defined By F(x)= 2x^3-4x^2+1. The Application Of The Mean Value Theorem To F On The

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Understanding the behavior of functions and their properties is fundamental in calculus. Among the various theorems that facilitate this understanding, the Mean Value Theorem (MVT) stands out as a critical tool. In this article, we explore the application of the Mean Value Theorem to the function $F(x) = 2x^3 - 4x^2 + 1$. We will analyze how the theorem applies, interpret the implications of its results, and provide practical examples to illustrate its use.

Overview of the Function F(x)= 2x^3 - 4x^2 + 1

Before delving into the application of the Mean Value Theorem (MVT), it is essential to understand the nature of the function F(x).

Basic Properties of F(x)

- Polynomial Degree: F(x) is a cubic polynomial, which implies it is continuous and differentiable everywhere on the real line.
- Continuity and Differentiability: Since polynomial functions are continuous and differentiable for all real numbers, F(x) satisfies the prerequisites for applying the MVT.
- Behavior at Infinity: As x approaches positive or negative infinity, F(x) behaves like $2x^3$, tending towards infinity or negative infinity respectively.
- Critical Points: The function's critical points, where the derivative is zero, help identify local maxima and minima, which are also relevant when applying the MVT.

Calculating Derivatives of F(x)

- First Derivative:

$$
F'(x) = \frac{d}{dx}(2x^3 - 4x^2 + 1) = 6x^2 - 8x
$$

- Second Derivative:

$$

$$
F''(x) = \frac{d}{dx}(6x^2 - 8x) = 12x - 8
$$

Understanding these derivatives is critical since the MVT involves the function's slope over an interval.

The Mean Value Theorem (MVT): An Overview

The Mean Value Theorem is a fundamental theorem in calculus that connects the average rate of change of a function over an interval to its instantaneous rate of change at some point within that interval.

Statement of the Mean Value Theorem

Let f be a function that is:

- Continuous on a closed interval $[a, b]$,
- Differentiable on the open interval (a, b).

Then, there exists at least one point $c \in (a, b)$ such that:

$$
f'(c) = \frac{f(b) - f(a)}{b - a}
$$

This equation states that the instantaneous rate of change at c equals the average rate of change over $[a, b]$.

Interpretation of MVT

- The theorem guarantees the existence of at least one point where the tangent to the curve is parallel to the secant line connecting $(a, f(a))$ and $(b, f(b))$.
- It provides a bridge between average and instantaneous rates of change, which is vital for understanding function behavior.

Applying the Mean Value Theorem to F(x) on a Specific

Interval

To illustrate the application of the MVT to the function F(x), let’s consider a specific interval, for example, $[1, 3]$.

Step 1: Verify Conditions for MVT

- Continuity: F(x) is a polynomial, hence continuous on $[1, 3]$.
- Differentiability: F(x) is differentiable on $(1, 3)$.

Since both conditions are satisfied, the MVT applies.

Step 2: Calculate $F(1)$ and $F(3)$

$$
F(1) = 2(1)^3 - 4(1)^2 + 1 = 2 - 4 + 1 = -1
$$

$$
F(3) = 2(3)^3 - 4(3)^2 + 1 = 2(27) - 4(9) + 1 = 54 - 36 + 1 = 19
$$

Step 3: Compute the Average Rate of Change

$$
\frac{F(3) - F(1)}{3 - 1} = \frac{19 - (-1)}{2} = \frac{20}{2} = 10
$$

The average rate of change over $[1, 3]$ is 10.

Step 4: Find $c \in (1, 3)$ such that $F'(c) = 10$

Recall the derivative:

$$
F'(x) = 6x^2 - 8x
$$

Set $F'(c) = 10$:

$$
6c^2 - 8c = 10

$$

$$

Rearranged:

$$
6c^2 - 8c - 10 = 0
$$

Divide through by 2 for simplicity:

$$
3c^2 - 4c - 5 = 0
$$

Apply quadratic formula:

$$
c = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 3 \times (-5)}}{2 \times 3} = \frac{4 \pm \sqrt{16 + 60}}{6} = \frac{4 \pm \sqrt{76}}{6}
$$

Calculate $\sqrt{76} \approx 8.717$:

$$
c = \frac{4 \pm 8.717}{6}
$$

Two solutions:

1. $\displaystyle c = \frac{4 + 8.717}{6} \approx \frac{12.717}{6} \approx 2.119$

2. $\displaystyle c = \frac{4 - 8.717}{6} \approx \frac{-4.717}{6} \approx -0.786$

Since -0.786 is outside the interval $(1, 3)$, the relevant c is approximately 2.119.

Conclusion: There exists a point $c \approx 2.119$ in $(1, 3)$ where the instantaneous rate of change equals the average rate of change, confirming the MVT.

Implications of the Mean Value Theorem for F(x)

Applying the MVT to the function F(x) reveals critical insights into its behavior:

1. Existence of Critical Points

- Since $F'(c) = 10$ at $c \approx 2.119$, this point indicates where the tangent slope matches the

average change over the interval.
- The specific value of the derivative at c helps identify potential local extrema, which can be further investigated via the second derivative.

2. Monotonicity and Increasing/Decreasing Intervals

- The derivative $F'(x) = 6x^2 - 8x$ can be analyzed to determine where the function increases or decreases.
- Factoring:

$$
F'(x) = 2x(3x - 4)
$$

- Critical points at $x=0$ and $x=\frac{4}{3}$.

- Sign analysis:

x interval	Sign of $F'(x)$	Behavior of F(x)
$(-\infty, 0)$	Negative	Decreasing
$(0, \frac{4}{3})$	Positive	Increasing
$(\frac{4}{3}, \infty)$	Positive	Increasing

- Therefore, F(x) decreases on $(-\infty,0)$, increases on $(0, \infty)$.

3. Confirmation of the Function's Shape

- The critical point at $x=0$ is a local minimum because $F''(x) = 12x - 8$:

$$
F''(0) = -8 < 0
$$

indicating concave down at $x=0$.

- The point at $x=\frac{4}{3}$:

$$
F''\left(\frac{4}{3}\right) = 12 \times \frac{4}{3} - 8 = 16 - 8 = 8 > 0
$$

indicating concave up, a potential local maximum or inflection point.

Further Applications of the Mean Value Theorem to F(x)

Beyond specific intervals, the MVT can have broader implications for the function F(x):

1. Verifying the Function's Behavior Over Larger Intervals

- By selecting different intervals, one can determine where the function's slope matches certain values.
- For example, analyzing \([0, 2]\) or \

Frequently Asked Questions

What is the main condition for applying the Mean Value Theorem to the function F(x)= 2x^3 - 4x^2 + 1 on an interval?

The main condition is that the function F(x) must be continuous on the closed interval and differentiable on the open interval, which is satisfied by the given polynomial on any interval.

How do you find the value c guaranteed by the Mean Value Theorem for F(x)= 2x^3 - 4x^2 + 1 on a specific interval?

You compute F'(x) and then set F'(c) equal to the average rate of change over the interval, i.e., (F(b) - F(a)) / (b - a), and solve for c within (a, b).

What does the Mean Value Theorem tell us about the behavior of the function F(x) on an interval?

It guarantees that there exists at least one point c in the interval where the instantaneous rate of change (F'(c)) equals the average rate of change over the entire interval.

Can the Mean Value Theorem be used to determine whether F(x) is increasing or decreasing on an interval?

While the MVT itself doesn't directly determine increasing or decreasing behavior, analyzing F'(x) helps, and MVT guarantees points where the function's slope matches average change, indicating potential local extrema.

How can the application of the Mean Value Theorem help in understanding the function's critical points for F(x)= 2x^3 -

$4x^2 + 1$?

By finding points where $F'(x)$ equals the average rate of change or zero, the MVT can help identify potential critical points where the function may have local maxima or minima.

Additional Resources

The Function F Is Defined By $F(x) = 2x^3 - 4x^2 + 1$. The Application Of The Mean Value Theorem To F On The

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Introduction

In the realm of calculus, the Mean Value Theorem (MVT) stands as a fundamental pillar, bridging the behavior of functions with their derivatives. Its applications extend across various fields—from physics to engineering, economics to computer science—making it a vital tool for analysis and problem-solving. This article delves deeply into the function $F(x) = 2x^3 - 4x^2 + 1$, exploring its properties, and rigorously applying the Mean Value Theorem (MVT) to uncover insights about its behavior over specified intervals. Through this comprehensive examination, we aim to illustrate not just the theoretical underpinnings but also practical implications of MVT in analyzing polynomial functions.

Understanding the Function $F(x) = 2x^3 - 4x^2 + 1$

Before applying the Mean Value Theorem, it is essential to thoroughly understand the properties of the function $F(x)$. Being a cubic polynomial, it exhibits characteristics typical of such functions, including potential inflection points, local maxima and minima, and specific end behavior.

Basic Properties and Domain

- Domain: The function $F(x)$ is a polynomial, hence defined for all real numbers ($\mathbb{R}$).
- Range: To determine the range, analyze the critical points and the end behavior.

End Behavior

- As $x \to +\infty$, $F(x) \to +\infty$.
- As $x \to -\infty$, $F(x) \to -\infty$.

This indicates the function is unbounded both above and below.

Critical Points and Extrema

To locate critical points, compute the first derivative:

$$
F'(x) = \frac{d}{dx}(2x^3 - 4x^2 + 1) = 6x^2 - 8x
$$

Set $F'(x) = 0$:

$$
6x^2 - 8x = 0 \implies 2x(3x - 4) = 0
$$

Solutions:

$$
x = 0 \quad \text{or} \quad x = \frac{4}{3}
$$

These are potential critical points where F may have local maxima or minima.

Calculate the second derivative to classify these points:

$$
F''(x) = \frac{d}{dx} (6x^2 - 8x) = 12x - 8
$$

- At $x=0$:

$$
F''(0) = -8 < 0
$$

Indicating a local maximum at $x=0$.

- At $x= \frac{4}{3}$:

$$
F''\left(\frac{4}{3}\right) = 12 \times \frac{4}{3} - 8 = 16 - 8 = 8 > 0
$$

Indicating a local minimum at $x=\frac{4}{3}$.

Corresponding function values:

- $F(0) = 2(0)^3 - 4(0)^2 + 1 = 1$
- $F\left(\frac{4}{3}\right) = 2 \left(\frac{4}{3}\right)^3 - 4 \left(\frac{4}{3}\right)^2 + 1$

Calculations:

$$
\left(\frac{4}{3}\right)^3 = \frac{64}{27}
$$

$$
\left(\frac{4}{3}\right)^2 = \frac{16}{9}
$$

Thus,

$$
F\left(\frac{4}{3}\right) = 2 \times \frac{64}{27} - 4 \times \frac{16}{9} + 1 = \frac{128}{27} - \frac{64}{9} + 1
$$

Express all terms with denominator 27:

$$
\frac{128}{27} - \frac{64}{9} + 1 = \frac{128}{27} - \frac{64 \times 3}{27} + \frac{27}{27} = \frac{128 - 192 + 27}{27} = \frac{-37}{27}
$$

This is approximately -1.370.

Summary of critical points:

Critical Point	Type	$F(x)$ value
$x=0$	Local maximum	1
$x=\frac{4}{3}$	Local minimum	$-\frac{37}{27}$ (~ -1.370)

Applying the Mean Value Theorem to F on Specific Intervals

The Mean Value Theorem (MVT) states that for a function f continuous on $[a, b]$ and differentiable on (a, b), there exists at least one point $c \in (a, b)$ such that:

$$
f'(c) = \frac{f(b) - f(a)}{b - a}
$$

This theorem essentially guarantees the existence of a point where the instantaneous rate of change (derivative) matches the average rate of change over the interval.

Conditions for MVT Application

- Continuity: Polynomial functions are continuous everywhere; $F(x)$ meets this criterion on all intervals.
- Differentiability: Polynomials are also differentiable everywhere; $F(x)$ satisfies this condition globally.

Therefore, MVT applies to any interval $[a, b]$.

Case Studies of F on Selected Intervals

Interval 1: $[0, 1]$

Calculate $F(0)$ and $F(1)$:

$$
F(0) = 1
$$
$$
F(1) = 2(1)^3 - 4(1)^2 + 1 = 2 - 4 + 1 = -1
$$

Average rate of change:

$$
\frac{F(1) - F(0)}{1 - 0} = \frac{-1 - 1}{1} = -2
$$

By MVT, there exists $c \in (0, 1)$ such that:

$$
F'(c) = -2
$$

Recall:

$$
F'(x) = 6x^2 - 8x
$$

Set equal to -2:

$$
6x^2 - 8x = -2
$$
\[

$$
6x^2 - 8x + 2 = 0
$$

Divide through by 2:

$$
3x^2 - 4x + 1 = 0
$$

Solve using quadratic formula:

$$
x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 3 \times 1}}{2 \times 3} = \frac{4 \pm \sqrt{16 - 12}}{6} = \frac{4 \pm \sqrt{4}}{6}
$$

$$
x = \frac{4 \pm 2}{6}
$$

Solutions:

- $x = \frac{4 + 2}{6} = 1$, which is at the boundary of the interval (not strictly inside).
- $x = \frac{4 - 2}{6} = \frac{2}{6} = \frac{1}{3}$

Since $c \in (0, 1)$, the valid point is $c = \frac{1}{3}$.

Interpretation: At $x = \frac{1}{3}$, the instantaneous rate of change equals the average rate over $[0, 1]$.

Interval 2: $[1, 2]$

Calculate $F(1)$ and $F(2)$:

$$
F(1) = -1
$$

$$
F(2) = 2(8) - 4(4) + 1 = 16 - 16 + 1 = 1
$$

Average rate:

$$
\frac{1 - (-1)}{2 - 1} = \frac{2}{1} = 2
$$

Set $F'(c) = 2$:

\[
6c^2 - 8c = 2
\]
\[
6c^2 - 8c - 2 = 0
\]

Divide through by 2:

\[
3c^2 - 4c - 1 = 0
\]

Quadratic solution:

\[
c = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 3 \times (-1)}}{2 \times 3} = \frac{4 \pm \sqrt{16 + 12}}{

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