The Function F(x) = X2 4x + 5 Is Shown On The Graph. On A Coordinate Plane, A Parabola Opens Down. It

The Function $F(x) = X^2 - 4x + 5$ Is Shown On The Graph. On A Coordinate Plane, A Parabola Opens Down. It

Understanding quadratic functions and their graphs is fundamental in algebra and calculus. The function $F(x) = x^2 - 4x + 5$ provides an excellent example to explore the properties of parabolas, especially those that open downward. In this comprehensive guide, we'll analyze this specific quadratic function, interpret its graph, and discuss its key features, including vertex, axis of symmetry, domain, range, and more. Whether you're a student seeking to master quadratic functions or a math enthusiast looking to deepen your understanding, this article will serve as an in-depth resource.

Analyzing the Quadratic Function $F(x) = x^2 - 4x + 5$

A quadratic function generally has the form $f(x) = ax^2 + bx + c$, where a, b, and c are constants. For the given function:

- $a = 1$
- $b = -4$
- $c = 5$

Since the coefficient a is positive ($a = 1$), the parabola opens upward. However, based on the initial description stating it "opens down," there may be a discrepancy. Let's clarify this as we analyze the function further.

Note: The function provided is $F(x) = x^2 - 4x + 5$, where the leading coefficient is positive, indicating an upward-opening parabola. If the intended function was intended to open downward, the quadratic might be $F(x) = -x^2 + 4x - 5$. For consistency, we will analyze the provided function with a positive leading coefficient, meaning the parabola opens upward.

Understanding the Graph of $F(x) = x^2 - 4x + 5$

Shape and Opening Direction

- Since $a = 1 > 0$, the parabola opens upward.
- The graph is symmetric about its axis of symmetry.
- The parabola has a U-shaped curve, with its vertex at the highest or lowest point depending on the direction of opening.

Key Features of the Parabola

To fully understand the graph, we need to identify several features:

- Vertex: The highest or lowest point on the parabola.
- Axis of Symmetry: The vertical line passing through the vertex.
- Y-intercept: The point where the graph crosses the y-axis.
- X-intercepts (Roots): The points where the graph crosses the x-axis.
- Domain and Range: The set of all possible x and y values for the function.

Calculating Key Features of $F(x) = x^2 - 4x + 5$

1. Vertex of the Parabola

The vertex form of a quadratic function is $f(x) = a(x - h)^2 + k$, where (h, k) is the vertex.

Method: Completing the Square

1. Start with the original function:

$F(x) = x^2 - 4x + 5$

2. Complete the square:

- Group the quadratic and linear terms:

$F(x) = (x^2 - 4x) + 5$

- Complete the square inside the parentheses:

- Take half of the coefficient of x (-4), which is -2
- Square it: $(-2)^2 = 4$

- Rewrite the expression:

$F(x) = (x^2 - 4x + 4) + 5 - 4$

$F(x) = (x - 2)^2 + 1$

3. Vertex Coordinates:

- $h = 2$
- $k = 1$

So, the vertex is at (2, 1).

Interpretation:

- Because the leading coefficient $a = 1 > 0$, the vertex is the minimum point of the parabola.
- The parabola opens upward, with its lowest point at (2,1).

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2. Axis of Symmetry

The axis of symmetry passes vertically through the vertex:

- $x = h = 2$

This line divides the parabola into two mirror-image halves.

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3. Y-Intercept

- Set $x = 0$:

$F(0) = 0^2 - 40 + 5 = 5$

- The y-intercept is at (0, 5).

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4. X-Intercepts (Roots)

Find x when $F(x) = 0$:

- Using the factored form or quadratic formula:

$F(x) = (x - 2)^2 + 1 = 0$

- Set equal to zero:

$(x - 2)^2 + 1 = 0$

- Subtract 1:

$(x - 2)^2 = -1$

- Since the square of a real number cannot be negative, there are no real roots.

Conclusion:

- The parabola does not cross the x-axis.
- The roots are complex conjugates.

Graphical Representation of $F(x) = x^2 - 4x + 5$

Based on the calculations:

- The parabola opens upward.
- Its vertex is at (2, 1).
- The y-intercept is at (0, 5).
- It has no real x-intercepts.
- The axis of symmetry is the vertical line $x = 2$.

Visual Summary:

- The graph is symmetric about $x = 2$.
- The lowest point is at (2, 1).
- The parabola rises on both sides from the vertex and crosses the y-axis at 5.

Real-World Applications of Quadratic Functions

Quadratic functions like $F(x) = x^2 - 4x + 5$ are prevalent in various fields:

- **Physics:** Modeling projectile motion, where the path of an object follows a parabola.

- **Engineering:** Analyzing structural elements subjected to forces that produce parabolic shapes.
- **Economics:** Calculating profit maximization or loss minimization, often represented by quadratic functions.
- **Biology:** Describing population growth or decay models where quadratic equations are involved.

Understanding the Significance of the Leading Coefficient

The coefficient 'a' in a quadratic function determines the parabola's opening direction and width.

Positive 'a' (a > 0):

- Parabola opens upward.
- Vertex is the minimum point.
- The parabola widens or narrows depending on the absolute value of 'a'.

Negative 'a' (a < 0):

- Parabola opens downward.
- Vertex is the maximum point.

In our case: Since a = 1, the parabola opens upward, with the vertex as the lowest point.

Transformations and Graphing the Function

Understanding how to graph quadratic functions involves recognizing transformations:

- Vertical shifts: Changing c shifts the parabola up or down.
- Horizontal shifts: Replacing x with (x - h) shifts the parabola left or

right.
- Vertical stretches/compressions: Changing a affects the width.

For $F(x) = x^2 - 4x + 5$, the vertex form $F(x) = (x - 2)^2 + 1$ reveals:

- Horizontal shift: 2 units right.
- Vertical shift: 1 unit up.

This form simplifies graphing and understanding the function's behavior.

Summary of Key Graph Features

Feature	Description	Coordinates/Values
Vertex	Lowest point	(2, 1)
Axis of symmetry	Vertical line through vertex	$x = 2$
Y-intercept	Point where graph crosses y-axis	(0, 5)
X-intercepts	Roots of the function	None (complex roots)
Opening direction	Upward	Since $a = 1$

Additional Insights and Tips

- Completing the Square: A powerful method to find the vertex and transform the quadratic into vertex form.
- Quadratic Formula: Used to find roots when the parabola crosses the x-axis.
- Graphing: Draw the vertex first, then plot the y-intercept, and use symmetry to find the shape.
- Discriminant ($b^2 - 4ac$): For our function, the discriminant is $(-4)^2 - 415 = 16 - 20 = -4 < 0$, confirming no real roots.

Conclusion

The quadratic function $F(x) = x^2 - 4x + 5$ exemplifies the classic parabola with various key features that can be precisely calculated and visualized on a coordinate plane. Recognizing the vertex at (2, 1), the axis of symmetry at $x = 2$, and the y-intercept at (0, 5) allows for accurate graphing and analysis. Despite the

Frequently Asked Questions

What is the vertex of the parabola given by the function $F(x) = x^2 - 4x + 5$?

The vertex can be found by completing the square or using the vertex formula. The vertex is at (2, 1).

Does the parabola open upwards or downwards for the function $F(x) = x^2 - 4x + 5$?

Since the coefficient of x^2 is positive, the parabola opens upward. However, the question states it opens down, which suggests a different function or a typo.

What is the axis of symmetry for the given parabola?

The axis of symmetry is $x = 2$, which is the x-coordinate of the vertex.

How do you find the y-intercept of the parabola $F(x) = x^2 - 4x + 5$?

Substitute $x = 0$ into the function: $F(0) = 0 - 0 + 5 = 5$. So, the y-intercept is at (0, 5).

What is the significance of the parabola opening downward in relation to the quadratic function?

A downward-opening parabola indicates the quadratic coefficient is negative, meaning the parabola has a maximum point at the vertex. If the given function has a positive coefficient, then the parabola opens upward, so there may be a discrepancy in the description.

Based on the function $F(x) = x^2 - 4x + 5$, what is the minimum or maximum value of the parabola?

Since the parabola opens upward, it has a minimum at the vertex, which is at $y = 1$.

Additional Resources

The Function $F(x) = x^2 - 4x + 5$ Is Shown On The Graph: An In-Depth Analysis

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Introduction

Mathematics often presents itself as a language of patterns and relationships, with graphs serving as visual representations that bring these abstract concepts to life. Among these, quadratic functions hold a special place due to their distinctive parabolic shapes and their widespread application across scientific disciplines, engineering, economics, and beyond. In this context, the function $F(x) = x^2 - 4x + 5$ emerges as a classic example of a quadratic, with its graph revealing a parabola that opens downward under certain interpretations. However, as we explore this function in detail, it becomes apparent that its graph actually depicts a parabola opening upwards, with nuances that challenge initial impressions. This comprehensive review aims to dissect the function's properties, interpret its graph, and explore the mathematical implications behind its shape and position on the coordinate plane.

Understanding the Quadratic Function $F(x) = x^2 - 4x + 5$

General Form and Standard Characteristics

Quadratic functions are generally expressed as:

$$F(x) = ax^2 + bx + c$$

where:

- a determines the parabola's opening direction (upward if positive, downward if negative).
- b influences the position of the vertex horizontally.
- c affects the vertical shift.

For our specific function:

$$F(x) = x^2 - 4x + 5$$

- $a = 1$ (positive)
- $b = -4$
- $c = 5$

Given that $a = 1 > 0$, the parabola opens upward. This fundamental property is critical in interpreting the graph's orientation.

Vertex and Axis of Symmetry

The vertex of a parabola described by a quadratic function is a point of extremum—either a maximum or a minimum—depending on the parabola's opening direction.

The x-coordinate of the vertex is derived from:

$$ x_v = -\frac{b}{2a} $$

Substituting:

$$ x_v = -\frac{-4}{2 \times 1} = \frac{4}{2} = 2 $$

The y-coordinate is obtained by evaluating the function at $x = 2$:

$$ F(2) = (2)^2 - 4 \times 2 + 5 = 4 - 8 + 5 = 1 $$

Vertex: $(2, 1)$

Axis of symmetry: $x = 2$

Graphical Interpretation

- Since a = 1 > 0, the parabola opens upward.
- The vertex at $(2, 1)$ is the minimum point.
- The parabola is symmetric about the vertical line $x = 2$.

Clarifying the Misconception: The Parabola's Opening Direction

The initial statement suggests that "A parabola opens down" for the function in question. However, based on the algebraic analysis, this is inconsistent with the standard properties of quadratic functions.

Why the Misinterpretation Occurs

- The phrase might stem from a misreading or misstatement.
- Alternatively, it could be referencing a different function or a specific transformation.
- Or, perhaps, the description was referencing a different quadratic, and the function has been mischaracterized.

Confirming the Opening Direction

Given the positive value of a in the quadratic, the parabola must open upward. To visually confirm:

- Plot points above and below the vertex.
- Observe the parabola's shape in a coordinate plane.
- Use graphing tools or software for precise visualization.

Conclusion: The graph of $F(x) = x^2 - 4x + 5$ unquestionably opens upward, with a vertex at $(2,1)$.

Deep Dive into the Graph's Features

1. Vertex as the Minimum Point

The vertex $(2, 1)$ represents the lowest point on the parabola. This minimum value of the function:

$$F_{min} = 1$$

occurs at $x = 2$.

2. Y-Intercept

The y-intercept occurs when $x = 0$:

$$F(0) = 0^2 - 4 \times 0 + 5 = 5$$

Y-intercept: $(0, 5)$

3. X-Intercepts (Roots)

To find x-intercepts, solve:

$$x^2 - 4x + 5 = 0$$

Discriminant D:

$$D = b^2 - 4ac = (-4)^2 - 4 \times 1 \times 5 = 16 - 20 = -4$$

Since $D < 0$, there are no real roots, meaning the parabola does not cross the x-axis. The graph is entirely above the x-axis.

4. Symmetry and Width

The parabola's axis of symmetry:

$$x = 2$$

symmetrically divides the parabola.

The parabola's "width" is influenced by the coefficient a:

- Larger $|a|$ values produce narrower parabolas.
- Since $a=1$, the parabola has a standard width.

5. Range of the Function

Given the vertex is a minimum:

$$\text{Range}: [1, \infty)$$

The function values are always greater than or equal to 1.

Visualizing and Analyzing the Graph

Step-by-Step Graph Construction

1. Plot the vertex at $(2, 1)$.
2. Mark the y-intercept at $(0, 5)$ and compute additional points:
- $x = 1$:

$$F(1) = 1 - 4 + 5 = 2$$

- $x = 3$:

$$F(3) = 9 - 12 + 5 = 2$$

- $x = 4$:

$$F(4) = 16 - 16 + 5 = 5$$

3. Connect these points smoothly, noting the symmetry about $x=2$.

4. Confirm the parabola opens upward, with the lowest point at the vertex.

Implications of the Graph

- The parabola is entirely above the x-axis.
- The minimum value of the function is 1, achieved at $x=2$.
- The function does not intercept the x-axis, indicating complex roots.

Mathematical Significance and Applications

1. Vertex Form Conversion

Transforming the quadratic into vertex form:

$$F(x) = (x - h)^2 + k$$

where (h, k) is the vertex.

Given:

$$F(x) = x^2 - 4x + 5$$

Complete the square:

$$
\begin{aligned}
F(x) &= x^2 - 4x + 4 + 1 \\

&= (x - 2)^2 + 1
\end{aligned}
\]

This form highlights the parabola's vertex at $(2, 1)$, and shows the parabola's minimum value is 1, attained when $x=2$.

2. Applications in Real-World Contexts

Quadratic functions like $F(x) = x^2 - 4x + 5$ model various phenomena:

- Projectile motion (parabolic trajectories)
- Economics (profit maximization/minimization)
- Engineering design (parabolic reflectors)

Understanding the vertex and intercepts aids in optimizing systems and predicting behavior.

Summary of Key Findings

Feature	Details
Opening Direction	Upward (since $a=1 > 0$)
Vertex	$(2, 1)$
Axis of Symmetry	$x = 2$
Y-Intercept	$(0, 5)$
X-Intercepts	None (discriminant negative)
Range	$[1, \infty)$
Standard Form	$(x - 2)^2 + 1$

Addressing the Initial Misstatement

The assertion that "The parabola opens down" for this function is incorrect based on algebraic and graphical evidence. The function's positive leading coefficient definitively indicates an upward opening parabola. This clarification is vital for accurate interpretation, especially in educational and analytical contexts.

Conclusion

The quadratic function $F(x) = x^2 - 4x + 5$ exemplifies a classic upward-opening parabola with its vertex at $(2, 1)$. Its properties—such as the absence of real roots, the minimum value, and symmetry—are emblematic of standard quadratic behavior. This detailed investigation underscores the importance of rigorous algebraic analysis in correctly interpreting graph features and dispelling misconceptions. Whether used as a teaching tool or in applied sciences, understanding the precise shape and position of such functions enhances our capacity to model and analyze complex phenomena effectively.

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