The Graph Of The Function F(x) = (x + 3)(x 1) Is Shown Below. On A Coordinate Plane, A Parabola Opens

The Graph Of The Function F(x) = (x + 3)(x 1) Is Shown Below. On A Coordinate Plane, A Parabola Opens. This statement sets the stage for exploring the fascinating world of quadratic functions and their graphical representations. Understanding how the algebraic form of a quadratic function relates to its graph is fundamental in algebra and calculus. In this article, we will analyze the quadratic function $F(x) = (x + 3)(x - 1)$, interpret its graph, and learn how to identify key features such as intercepts, vertex, axis of symmetry, and the direction of the parabola's opening.

Understanding the Quadratic Function $F(x) = (x + 3)(x - 1)$

Expanding the Function

To better analyze the graph, it's often helpful to write the quadratic in standard form. Expanding the factored form:

$$
F(x) = (x + 3)(x - 1) = x^2 - x + 3x - 3 = x^2 + 2x - 3
$$

This standard form, $y = x^2 + 2x - 3$, provides a straightforward way to identify key features of the parabola.

Recognizing the Quadratic Structure

Quadratic functions are polynomials of degree 2, and their graphs are parabolas. The general form:

$$
y = ax^2 + bx + c
$$

where $a \neq 0$, defines the parabola's shape and orientation. In our case:

- $a = 1$
- $b = 2$
- $c = -3$

Since $a > 0$, the parabola opens upward.

Key Features of the Graph of $F(x)$

Understanding the shape and position of the parabola involves identifying several key features:

1. X-Intercepts (Roots or Zeroes)

The points where the parabola crosses the x-axis.

- From the factored form, the roots are immediately visible:

$$
x + 3 = 0 \Rightarrow x = -3
$$
$$
x - 1 = 0 \Rightarrow x = 1
$$

- So, the parabola intersects the x-axis at (-3, 0) and (1, 0).

2. Y-Intercept

The point where the parabola crosses the y-axis, found by evaluating $F(0)$:

$$
F(0) = (0 + 3)(0 - 1) = 3 \times -1 = -3
$$

- The y-intercept is (0, -3).

3. Vertex of the Parabola

The vertex is the highest or lowest point on the parabola, depending on its orientation.

- For a quadratic $y = ax^2 + bx + c$, the x-coordinate of the vertex is:

$$
x_v = -\frac{b}{2a}
$$

- Substituting $a = 1$, $b = 2$:

$$
x_v = -\frac{2}{2 \times 1} = -\frac{2}{2} = -1
$$

- To find the y-coordinate, evaluate $F(-1)$:

$$
F(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4
$$

- The vertex is at (-1, -4).

4. Axis of Symmetry

The vertical line passing through the vertex, dividing the parabola into mirror images.

$$
x = x_v = -1
$$

5. Direction of Opening

Since the coefficient of x^2 (which is 1) is positive, the parabola opens upward.

Graphing the Function $F(x) = (x + 3)(x - 1)$

Step-by-Step Graphing Process

1. Plot the intercepts:

- X-intercepts at (-3, 0) and (1, 0)
- Y-intercept at (0, -3)

2. Plot the vertex:

- At (-1, -4)

3. Draw the axis of symmetry:

- The vertical line $x = -1$

4. Sketch the parabola:

- Use symmetry about the axis of symmetry
- Ensure the curve opens upward

Additional Points for Accuracy

To create a precise graph, select additional x-values and compute corresponding y-values:

x	$F(x) = x^2 + 2x - 3$	y-value
-2	$(-2)^2 + 2(-2) - 3 = 4 - 4 - 3 = -3$	(-2, -3)
0	$0 + 0 - 3 = -3$	(0, -3)
2	$4 + 4 - 3 = 5$	(2, 5)

Plotting these points ensures a smooth and accurate parabola.

Analyzing the Graph: Real-World Applications

Quadratic functions and their graphs are prevalent in various real-world scenarios, including physics, engineering, economics, and biology.

1. Projectile Motion

The path of an object thrown into the air follows a parabola; understanding the vertex and intercepts helps determine maximum height and time of flight.

2. Optimization Problems

Parabolas model maximum profit, minimum cost, or optimal resource allocation when represented as quadratic functions.

3. Design and Engineering

Architects and engineers use parabolic curves for bridges, arches, and satellite dishes for their structural and reflective properties.

Common Misconceptions and Clarifications

Misconception 1: The parabola always opens upward

Clarification: The opening direction depends on the coefficient a. If $a > 0$, it opens upward; if $a < 0$, downward.

Misconception 2: The roots are always at the vertex

Clarification: Roots are where the parabola crosses the x-axis; the vertex may be above or below the x-axis depending on the function.

Misconception 3: The vertex is always the highest or lowest point

Clarification: For upward-opening parabolas, the vertex is the minimum point; for downward, it is the maximum.

Summary and Key Takeaways

- The quadratic function $F(x) = (x + 3)(x - 1)$ expands to $x^2 + 2x - 3$, revealing its structure.
- The parabola opens upward because the leading coefficient $a = 1$ is positive.
- It intersects the x-axis at $x = -3$ and $x=1$.
- The y-intercept occurs at $(0, -3)$.
- The vertex is at $(-1, -4)$, found using $x_v = -b/(2a)$.
- The axis of symmetry is the vertical line $x = -1$.
- Additional points help in sketching the parabola accurately.
- Understanding these features aids in interpreting the graph in real-world contexts.

Final Thoughts

Graphing quadratic functions like \(F(x) = (x + 3)(x - 1) \) is an essential skill in mathematics. Recognizing the relationship between factored, expanded, and vertex forms allows for a comprehensive understanding of the parabola's shape and position. Whether analyzing projectile trajectories or optimizing business models, mastery of quadratic graphs provides valuable insights across numerous fields. Remember to identify key features, plot critical points, and leverage symmetry for accurate and meaningful graph interpretations.

Frequently Asked Questions

What is the general shape of the graph of the function F(x) = (x + 3)(x - 1)?

The graph is a parabola that opens upward because the quadratic coefficient is positive.

What are the x-intercepts of the parabola F(x) = (x + 3)(x - 1)?

The x-intercepts are at x = -3 and x = 1.

What is the vertex of the parabola F(x) = (x + 3)(x - 1)?

The vertex can be found by calculating the axis of symmetry at x = -1, and plugging into the function to find the y-coordinate, which is F(-1) = (-1 + 3)(-1 - 1) = 2 (-2) = -4. So, the vertex is at (-1, -4).

How can you determine whether the parabola opens upward or downward?

By examining the leading coefficient of the quadratic form. Since the expanded form of F(x) = x^2 + 2x - 3 has a positive x^2 coefficient, the parabola opens upward.

What is the axis of symmetry for the parabola F(x) = (x + 3)(x - 1)?

The axis of symmetry is the vertical line x = -1.

How does the factored form of the quadratic help in graphing?

The factored form reveals the x-intercepts directly, making it easier to sketch the parabola accurately.

What is the range of the function F(x) = (x + 3)(x - 1)?

Since the parabola opens upward and has a minimum at the vertex, the range is $y \geq -4$.

How can you verify the shape of the parabola from its equation?

By expanding the factored form to standard quadratic form and noting the positive coefficient of x^2, confirming it opens upward.

What are some key features to label when graphing this parabola?

Key features include the x-intercepts at -3 and 1, the vertex at (-1, -4), the axis of symmetry at x = -1, and the parabola opening upward.

If you wanted to find the y-intercept of the parabola, what would it be?

The y-intercept occurs when x = 0. Substituting x=0 into F(x): F(0) = (0 + 3)(0 - 1) = 3 (-1) = -3. So, the y-intercept is at (0, -3).

Additional Resources

The Graph Of The Function F(x) = (x + 3)(x - 1) Is Shown Below. On A Coordinate Plane, A Parabola Opens

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Introduction

Mathematics, at its core, is a visual language. The graphs of functions serve as a bridge between abstract algebraic expressions and tangible geometric representations. Among these, quadratic functions hold a special place due to their fundamental properties and wide-ranging applications. In particular, the function $F(x) = (x + 3)(x - 1)$ offers an intriguing case study in understanding parabola behavior, vertex form, roots, and the overall shape of quadratic graphs. This article delves deeply into the characteristics of this specific quadratic function, exploring its graphical features, algebraic properties, and implications for mathematical analysis.

Understanding the Function $F(x) = (x + 3)(x - 1)$

At first glance, the function appears as a product of two linear factors. Such factorizations are instrumental in interpreting the graph's roots and symmetry.

Algebraic Expansion and Standard Form

Expanding $F(x)$:

$$
F(x) = (x + 3)(x - 1) = x^2 - x + 3x - 3 = x^2 + 2x - 3
$$

The quadratic is now expressed in standard form:

$$
F(x) = x^2 + 2x - 3
$$

This form is crucial for analyzing the parabola's vertex, axis of symmetry, and direction of opening.

Key Features of the Parabola

1. Roots (Zeros) of the Function

The roots are the points where $F(x) = 0$:

$$
(x + 3)(x - 1) = 0 \Rightarrow x + 3 = 0 \Rightarrow x = -3
$$
$$
x - 1 = 0 \Rightarrow x = 1
$$

Interpretation:

- The parabola intersects the x-axis at $x = -3$ and $x = 1$.

- The roots are real and distinct, indicating the parabola crosses the x-axis at two points.

2. Vertex of the Parabola

The vertex is the maximum or minimum point of the parabola. For $F(x) = x^2 + 2x - 3$, the vertex can be found via:

$$
x_{v} = -\frac{b}{2a} = -\frac{2}{2} = -1
$$

Substituting $x = -1$:

$$
F(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4
$$

Vertex Coordinates:

$$
(-1, -4)
$$

This point is the parabola's lowest point since the coefficient $a = 1 > 0$ indicates the parabola opens upward.

3. Axis of Symmetry

The parabola's symmetry axis runs through its vertex:

$$
x = -1
$$

The axis divides the parabola into two mirror-image halves.

4. Direction of Opening

Since the coefficient of x^2 is positive ($a = 1$), the parabola opens upward.

5. Y-intercept

Set $x = 0$:

$$
F(0) = 0^2 + 2(0) - 3 = -3
$$

Y-intercept at $(0, -3)$.

Graphical Analysis and Visualization

Symmetry and Shape

The graph of $F(x)$ displays a classic upward-opening parabola with the vertex at $(-1, -4)$. The roots at $x = -3$ and $x = 1$ mark the points where the parabola intersects the x-axis, creating a symmetric shape about the line $x = -1$.

Behavior at Extremes

- As $x \to \infty$, $F(x) \to \infty$.
- As $x \to -\infty$, $F(x) \to \infty$.

This indicates the parabola extends infinitely upward on both ends.

Key Points Summary:

Point	Coordinates	Description
Roots	(-3, 0), (1, 0)	x-intercepts
Vertex	(-1, -4)	Minimum point
Y-intercept	(0, -3)	Crosses y-axis

Decomposition and Transformations

Understanding the function as a product of linear factors aids in visualizing transformations from the basic parabola $y = x^2$.

1. Factored form:

$$
F(x) = (x + 3)(x - 1)
$$

- The roots -3 and 1 shift the parabola horizontally to intersect the x-axis at these points.

2. Standard form:

$$
F(x) = x^2 + 2x - 3
$$

- Completing the square reveals transformations:

$$
F(x) = (x + 1)^2 - 4
$$

which is the vertex form.

Vertex Form and Its Significance

Expressing the quadratic as:

$$
F(x) = (x + 1)^2 - 4
$$

provides immediate insights:

- The parabola is a shift of $y = x^2$ by 1 unit left and 4 units down.
- The vertex at $(-1, -4)$ is clearly visible.
- The parabola opens upward, consistent with the positive coefficient.

Applications and Implications

Quadratic functions like $F(x) = (x + 3)(x - 1)$ are fundamental in various fields:

- Physics: Projectile motion analysis, where the parabola models paths.
- Economics: Cost and revenue functions often involve quadratics.
- Engineering: Structural analysis and design.

The explicit roots and vertex facilitate optimization problems, such as finding minimum costs or maximum efficiencies.

Investigative Observations

The Nature of Roots and Discriminant

The discriminant D of $ax^2 + bx + c$:

$$
D = b^2 - 4ac = (2)^2 - 4(1)(-3) = 4 + 12 = 16
$$

- Since $D > 0$, the parabola has two distinct real roots, confirming the x-intercepts at -3 and 1.

Symmetry and Reflection

The axis of symmetry at $x = -1$ divides the parabola into two mirror images. For instance:

- Distance from the vertex to root at -3:

$$
|-1 - (-3)| = 2
$$

- Similarly, the root at 1:

$$
|1 - (-1)| = 2
$$

reflects the symmetry.

Visual Representation and Graphing Considerations

While the above analysis provides a precise description, visualizing the function's graph offers deeper comprehension. Key considerations include:

- Plotting the roots, vertex, y-intercept, and additional points for accuracy.
- Observing the parabola's symmetry about $x = -1$.
- Noting the parabola's concavity (opening upward).

Modern graphing tools or software can recreate the curve, validating algebraic insights.

Conclusion

The quadratic function $F(x) = (x + 3)(x - 1)$ exemplifies the fundamental properties of parabolas—roots, vertex, axis of symmetry, and direction of opening. Its analysis underscores the importance of algebraic manipulation, geometric interpretation, and the interplay between factored and standard forms. Beyond its theoretical value, this function serves as a foundational element in applied mathematics, physics, engineering, and economics. Its graph, a symmetric upward-opening parabola, encapsulates the elegant harmony between algebra and geometry, demonstrating the power of mathematical visualization in understanding complex relationships.

References and Further Reading

- Stewart, J. (2015). Precalculus: Mathematics for Calculus. Cengage Learning.
- Larson, R., & Hostetler, R. (2013). Precalculus with Limits. Cengage Learning.
- Khan Academy. (n.d.). Quadratic functions and their graphs. Retrieved from https://www.khanacademy.org/math/algebra/quadratics

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Note: For precise graphical illustrations, utilize graphing calculators or
software such as Desmos, GeoGebra, or Wolfram Alpha to observe the behavior
of \( F(x) \) and verify the analytical observations provided herein.
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