The Length Of A Rectangular Garden Is 22 Feet Longer Than Its Width. If The Area Of The Garden Is 408

The Length Of A Rectangular Garden Is 22 Feet Longer Than Its Width. If The Area Of The Garden Is 408

Understanding the dimensions of a rectangular garden is essential for planning, designing, and maintaining a beautiful outdoor space. When given specific measurements, such as the relationship between the length and width and the total area, solving for the unknown dimensions becomes a valuable mathematical exercise. In this article, we will explore how to determine the length and width of a rectangular garden when its length exceeds its width by 22 feet, and its area is 408 square feet. We will also discuss related concepts and practical applications to help you confidently approach similar problems.

Understanding the Problem

Before diving into calculations, it's important to clearly understand what is being asked and the details provided:

- The length of the garden is 22 feet longer than its width.
- The area of the garden is 408 square feet.

From this information, our goal is to find the exact measurements of the length and width of the garden.

Setting Up the Mathematical Model

To solve this problem, we can translate the verbal information into algebraic expressions.

Step 1: Define Variables

Let:

- w = the width of the garden in feet
- l = the length of the garden in feet

Step 2: Express the Relationship

Since the length is 22 feet longer than the width:

- $l = w + 22$

Step 3: Write the Area Equation

The area of a rectangle is calculated as:

- Area = length × width

Given the area is 408 square feet:

- $l \times w = 408$

Substituting $l = w + 22$ into the area formula:

- $(w + 22) \times w = 408$

Solving for the Dimensions

Now, we solve the quadratic equation:

$(w + 22)w = 408$

which simplifies to:

$w^2 + 22w = 408$

Rearranged as:

$w^2 + 22w - 408 = 0$

This quadratic equation can be solved using factoring, completing the square, or the quadratic formula.

Step 1: Apply the Quadratic Formula

The quadratic formula:

$w = [-b \pm \sqrt{} (b^2 - 4ac)] / 2a$

where:

- $a = 1$
- $b = 22$
- $c = -408$

Step 2: Calculate the Discriminant

$D = b^2 - 4ac = (22)^2 - 4(1)(-408) = 484 + 1632 = 2116$

Step 3: Find the Square Root of the Discriminant

$\sqrt{} D = \sqrt{} 2116 = 46$

Step 4: Compute the Two Possible Solutions

$w = [-22 \pm 46] / 2$

- $w = (-22 + 46) / 2 = 24 / 2 = 12$
- $w = (-22 - 46) / 2 = -68 / 2 = -34$

Since width cannot be negative, we discard $w = -34$.

Step 5: Find the Length

Using $l = w + 22$:

- For $w = 12$ feet:

$l = 12 + 22 = 34$ feet

Final Dimensions of the Garden

Parameter	Measurement (feet)
Width	12
Length	34

The garden's width is 12 feet, and its length is 34 feet.

Verification of the Solution

Always verify your calculations:

- Calculate the area:

$12 \times 34 = 408$ square feet

which matches the given area, confirming our solution is correct.

Practical Applications of the Problem

Understanding how to set up and solve such algebraic problems has numerous practical applications:

How Garden Dimensions Affect Planning

- Design & Layout: Knowing the length and width helps in designing pathways, planting beds, and irrigation systems.
- Material Estimation: Calculating fencing, soil, or grass needed depends on accurate measurements.
- Cost Calculation: Budgeting for materials can be based on precise dimensions.

Using Algebra in Real-Life Scenarios

- Planning construction projects
- Calculating space for landscaping
- Estimating resources for outdoor renovations

Additional Related Problems

To deepen understanding, consider exploring similar problems:

1. A rectangular garden has a length that is 15 meters longer than its width, and its area is 600 square meters. Find its dimensions.
2. The width of a garden is 8 meters less than its length. If the area is 96 square meters, find the dimensions of the garden.
3. A rectangular park's length is twice its width. If the area is 2000 square feet, determine its length and width.

Practicing these variations enhances problem-solving skills and applies algebraic concepts to practical situations.

Conclusion

Determining the dimensions of a rectangular garden when given the relationship between length and width and the total area is a fundamental algebraic problem with real-world relevance. By translating the problem into an algebraic equation, applying the quadratic formula, and verifying the results, you can confidently find the measurements needed for your outdoor projects. Whether designing a new garden, planning landscaping, or just honing your math skills, understanding these principles is invaluable. Remember, the key steps involve defining variables, setting up equations, solving the quadratic, and verifying your answer, ensuring accurate and reliable results every time.

Frequently Asked Questions

What is the width of the rectangular garden if its length is 22 feet longer than its width and the area is 408 square feet?

Let the width be x feet. Then, the length is x + 22 feet. The area equation is x(x + 22) = 408. Solving for x: x^2 + 22x = 408, or x^2 + 22x - 408 = 0. Using the quadratic formula, x = [-22 ± √(22^2 - 4 1 (-408))]/2. Simplifying, x = [-22 ± √(484 + 1632)]/2 = [-22 ± √2116]/2 = [-22 ± 46]/2. Taking the positive root, x = (-22 + 46)/2 = 24/2 = 12. So, the width is 12 feet.

What is the length of the garden based on the given information?

Since the length is 22 feet longer than the width and the width is 12 feet, the length is 12 + 22 = 34 feet.

How do you set up the equation for the area of a rectangular garden with a given length and width?

The area of a rectangle is calculated as length multiplied by width. If the width is x and the length is x + 22, then the area equation is x(x + 22) = 408.

Is the quadratic equation derived from the problem solvable using the quadratic formula?

Yes, the quadratic equation x^2 + 22x - 408 = 0 can be solved using the quadratic formula, as it is a standard quadratic equation.

What is the discriminant in the quadratic equation for this problem?

The discriminant is Δ = b^2 - 4ac. Here, a=1, b=22, c=-408. So, Δ = 22^2 - 41(-408) = 484 + 1632 = 2116.

Are both roots of the quadratic equation valid for the width of the garden?

Only the positive root makes sense physically, since width cannot be negative. In this case, the positive root is 12, so the width is 12 feet.

What is the significance of the length-to-width relationship in this problem?

The relationship that the length is 22 feet longer than the width allows us to express the length in terms of the width, enabling us to set up the quadratic equation for the area.

Can this problem be solved graphically, and how?

Yes, by graphing the equations y = x and y = x + 22, then plotting the area condition, you can visually identify the dimensions where the area equals 408 square feet.

What is the purpose of using the quadratic formula in this problem?

The quadratic formula helps find the possible values of the width that satisfy the area condition when the relationship between length and width is linear, as in this problem.

Additional Resources

The Length Of A Rectangular Garden Is 22 Feet Longer Than Its Width: An In-Depth Exploration

When designing a garden, understanding the relationship between its length, width, and area is fundamental for efficient planning and aesthetic appeal. This scenario, where the length of a rectangular garden exceeds its width by 22 feet and the total area is known to be 408 square feet, provides a practical example of how algebraic methods can help in real-world landscaping projects. By analyzing this problem thoroughly, homeowners, landscapers, and students alike can gain insight into how dimensions interplay to produce desired garden sizes and shapes.

Understanding the Problem: The Key Variables

Before delving into calculations, it's crucial to comprehend the variables involved:

- Width (w): The shorter side of the rectangle, which is unknown initially.
- Length (l): The longer side, which is 22 feet longer than the width; thus, l = w + 22.
- Area (A): Given as 408 square feet.

These variables form the foundation for setting up algebraic equations that help determine the exact dimensions of the garden.

Formulating the Mathematical Model

The problem involves translating the real-world scenario into an algebraic model:

- Since the length exceeds the width by 22 feet, the relationship is:
$l = w + 22$

- The area of a rectangle is calculated as:
Area = Length × Width

- Given the area is 408 square feet, the equation becomes:
$(w + 22) \times w = 408$

This quadratic equation serves as the starting point for finding the dimensions of the garden.

Solving the Quadratic Equation

The algebraic expression expands as:

$(w + 22)w = 408$
$w^2 + 22w = 408$

Rearranged into standard quadratic form:

$w^2 + 22w - 408 = 0$

To solve this quadratic, the quadratic formula is most effective:

$w = [-b \pm \sqrt{} (b^2 - 4ac)] / 2a$

where $a = 1$, $b = 22$, $c = -408$.

Calculating the discriminant:

$\Delta = b^2 - 4ac = (22)^2 - 4(1)(-408) = 484 + 1632 = 2116$

Since the discriminant is positive, there are two real solutions:

$w = [-22 \pm \sqrt{} 2116] / 2$

$\sqrt{} 2116 = 46$

Thus,

$w = [-22 + 46] / 2 = 24 / 2 = 12$
$w = [-22 - 46] / 2 = -68 / 2 = -34$

In the context of a physical garden, the width cannot be negative, so:

$w = 12$ feet

Using the relationship for length:

$l = w + 22 = 12 + 22 = 34$ feet

Final dimensions:

- Width = 12 feet
- Length = 34 feet

Area verification:

$12 \times 34 = 408$ square feet, which matches the given area.

Features and Pros of these Dimensions

Understanding the calculated dimensions allows for multiple benefits:

- Efficient Space Utilization: The dimensions provide a proportional layout suitable for various garden features.
- Design Flexibility: The longer length allows for planting along the sides or creating pathways.
- Ease of Maintenance: Smaller widths and the specific length facilitate easier watering and mowing.
- Cost-Effective Planning: Knowing exact measurements helps budget for fencing, soil, and plants.

Potential Challenges and Considerations

While the dimensions are mathematically precise, practical considerations include:

- Limited Width: At only 12 feet, the garden might be narrow for some features like wide flower beds or vegetable patches.
- Space Constraints: The length of 34 feet might be insufficient for larger garden structures or features.
- Sunlight Exposure: Narrower gardens may receive less sunlight in certain areas, affecting plant growth.
- Design Limitations: The fixed proportions may restrict certain landscaping styles.

Real-World Applications and Tips

Knowing how to derive the dimensions from area and length-width relationships is valuable in several scenarios:

- Landscape Planning: Use these calculations to plan plant arrangements, walkways, and features.
- Fencing and Boundary Design: Accurate measurements ensure cost-effective fencing.
- Garden Bed Design: Decide on the size and shape of garden beds based on available space.
- Compliance and Regulations: Ensure the garden fits within property boundaries and local regulations.

Tips for Effective Garden Design Based on Calculated Dimensions:

- Consider creating multi-layered planting beds to maximize space.
- Incorporate vertical gardening to compensate for limited width.
- Use the longer side for pathways or seating areas.
- Plan irrigation systems based on the precise measurements to avoid wastage.

Alternative Scenarios and Variations

If the area or the difference in length and width changes, the problem can be recalculated similarly:

- If the area increases: For example, area of 600 sq ft with the same difference.
- If the difference in length and width is different: For example, 15 feet instead of 22.
- Incorporating other constraints: such as maximum width for space reasons or specific design features.

By mastering the algebraic approach demonstrated here, you can adapt to various design constraints and preferences.

Conclusion

The problem of determining the dimensions of a rectangular garden where the length exceeds the width by a fixed amount and the total area is known offers a clear example of applying algebra to real-world situations. Through setting up and solving a quadratic equation, we find that the garden measures 12 feet in width and 34 feet in length, totaling an area of 408 square feet. These calculations not only serve as a useful planning tool but also highlight the importance of mathematical literacy in practical design projects. Whether you're a homeowner planning your backyard, a student learning algebra, or a professional landscaper, understanding these relationships enhances your ability to create functional and beautiful outdoor spaces efficiently.

The Length Of A Rectangular Garden Is 22 Feet Longer Than Its Width If The Area Of The Garden Is 408

The Length Of A Rectangular Garden Is 22 Feet Longer Than Its Width If The Area Of The Garden Is 408

Related Articles

- The Framers Of The Constitution Gave Specific Powers To Both The Nation As Well As Individual States.
- This Is Just So Dum Why Do They Make The App If Nobody Is Going To Help Me And Other Some People Have
- This Is A Graphing Problem And I Am Trying To Find The X-intercepts And The Y-intercepts. Please Show

Back to Home