

The Perimeter Of A Triangle Is 39 Inches. If The Length Of The Shortest Side Is 1/2 The Length Of The

The Perimeter Of A Triangle Is 39 Inches. If The Length Of The Shortest Side Is 1/2 The Length Of The

Understanding the properties of triangles is fundamental in geometry, especially when it comes to calculating side lengths and perimeters. In this article, we explore a specific problem: given that the perimeter of a triangle is 39 inches and the shortest side is half the length of another side, how can we determine the lengths of all sides? We will delve into the concepts involved, develop formulas, and work through detailed examples to provide clear insights into solving such problems.

Understanding the Basic Concepts

Before diving into the specifics of the problem, it's essential to review some fundamental concepts related to triangles.

Perimeter of a Triangle

The perimeter of a triangle is the total length around the triangle, calculated by summing the lengths of all three sides:

- $P = a + b + c$

where a , b , and c are the lengths of the sides.

Types of Triangles

Triangles can be classified based on their side lengths:

- Equilateral: all sides equal
- Isosceles: two sides equal
- Scalene: all sides different

Additionally, triangles are classified based on angles:

- Acute: all angles less than 90°
- Right: one angle exactly 90°
- Obtuse: one angle greater than 90°

For our problem, the key focus is on the relationship between side lengths, especially the shortest side.

Key Relationships and Constraints

In any triangle, the following properties hold:

- The sum of the lengths of any two sides must be greater than the third side (triangle inequality theorem).
- The shortest side must be less than the other sides, but all sides are positive real numbers.

Given these principles, we are tasked with determining the individual side lengths based on the perimeter and the relationship between the shortest side and another side.

Formulating the Problem

Given:

- The perimeter $P = 39$ inches
- The shortest side, let's denote it as s
- The shortest side is half the length of another side, which we will specify as side b (assuming side b is the one related to s)

The problem states:

- > The length of the shortest side is $1/2$ the length of the other side.

Assuming:

- Shortest side = s
- The other side (say b) = $2s$

Now, we need to find the lengths of all three sides, which include s , b , and the third side c .

Variables and Assumptions

- Let's denote:
- Shortest side = s
- Second side = $2s$ (since it's twice the shortest)
- Third side = c (unknown)

Because the perimeter is known:

$$P = s + 2s + c = 39 \text{ inches}$$

which simplifies to:

$$3s + c = 39$$

From here, c can be expressed in terms of s :

$$c = 39 - 3s$$

Our goal:

- Find values of s and c that satisfy the triangle inequality conditions.

Applying the Triangle Inequality

The triangle inequality states:

- $a + b > c$
- $a + c > b$
- $b + c > a$

Applying these to our sides:

- $s + 2s > c \rightarrow 3s > c$
- $s + c > 2s \rightarrow c > 2s - s \rightarrow c > s$
- $2s + c > s \rightarrow c > s - 2s \rightarrow c > -s$ (which is always true since $c > 0$)

Given $c = 39 - 3s$, the inequalities become:

- $3s > 39 - 3s \rightarrow 6s > 39 \rightarrow s > 6.5$
- $c > s \rightarrow 39 - 3s > s \rightarrow 39 > 4s \rightarrow s < 9.75$
- $c > 0 \rightarrow 39 - 3s > 0 \rightarrow 3s < 39 \rightarrow s < 13$

Combining these constraints:

$s > 6.5$ and $s < 9.75$

Thus, the shortest side s is between 6.5 and 9.75 inches.

Calculating Possible Side Lengths

Now, we can analyze specific values within the interval $s \in (6.5, 9.75)$. Let's pick some representative values and compute the corresponding third side c .

Example Calculations

- **For $s = 7$ inches:**

$$c = 39 - 3(7) = 39 - 21 = 18 \text{ inches}$$

Check inequalities:

$$- 3s = 21 > c = 18 \quad \square$$

$$- c = 18 > s = 7 \quad \square$$

$$- c = 18 > 0 \quad \square$$

All conditions satisfied. Triangle sides: 7, 14, 18 inches (since $b = 2s = 14$ inches).

- **For $s = 8$ inches:**

$$c = 39 - 3(8) = 39 - 24 = 15 \text{ inches}$$

Check inequalities:

- $3s = 24 > c = 15$ ✓
- $c = 15 > s = 8$ ✓
- $c = 15 > 0$ ✓
Sides: 8, 16, 15 inches.

• **For $s = 9$ inches:**

$c = 39 - 3(9) = 39 - 27 = 12$ inches
Check inequalities:
- $3s = 27 > c = 12$ ✓
- $c = 12 > s = 9$ ✓
- $c = 12 > 0$ ✓
Sides: 9, 18, 12 inches.

These examples illustrate the range of possible side lengths that satisfy the given conditions.

Summary of Findings

- The shortest side s must be greater than 6.5 inches and less than 9.75 inches.
- The third side c is determined by $c = 39 - 3s$.
- The second side b is double the shortest side, i.e., $b = 2s$.
- The triangle inequality conditions are satisfied within this interval.

Additional Considerations and Variations

While the above calculations assume that the shortest side is half of side b , similar problems can be approached with different relationships:

- If the shortest side relates to side c instead of side b
- If the sides are in different ratios
- When the triangle is equilateral or isosceles with specific constraints

Understanding the relationships allows for flexible problem solving.

Practical Applications

These types of problems are common in:

- Architectural design
- Engineering
- Construction
- Educational contexts to develop problem-solving skills

They help reinforce understanding of geometric properties and algebraic manipulation.

Conclusion

Calculating the side lengths of a triangle given a perimeter and a relationship between sides involves a systematic approach:

1. Establish variables and relationships.
2. Use the perimeter to create an equation.
3. Apply the triangle inequality to find valid ranges.
4. Calculate specific examples within the constraints.

In our case, with a perimeter of 39 inches and the shortest side being half of another side, the shortest side must be between 6.5 and 9.75 inches, leading to corresponding side lengths that satisfy all triangle properties.

By mastering these methods, students and professionals can approach complex geometric problems with confidence, ensuring accurate and meaningful solutions.

Frequently Asked Questions

What is the total perimeter of a triangle if the sum of all sides equals 39 inches?

The total perimeter of the triangle is 39 inches, meaning the sum of the lengths of its three sides is 39 inches.

If the shortest side of a triangle is half the length of another side, how can we determine the side lengths?

By setting variable expressions for the sides and using the perimeter condition, we can solve for the side lengths algebraically.

Given the perimeter is 39 inches and the shortest side is half the length of the middle side, how do you find each side?

Let the shortest side be x , then the middle side is $2x$, and the longest side can be found by subtracting the sum of these from 39 inches, then solving for x .

Can the shortest side of the triangle be determined if its length is half of the second side and the perimeter is known?

Yes, by expressing the other sides in terms of the shortest side and using the perimeter, you can solve for the shortest side's length.

What are the possible side lengths of the triangle if the shortest side is half the length of the second side and the perimeter is 39 inches?

Possible side lengths can be found by solving the system: $x + 2x + \text{the third side} = 39$ inches, with the triangle inequality constraints.

Does the triangle inequality theorem apply to the sides with the given conditions?

Yes, the triangle inequality theorem must be satisfied, meaning the sum of any two sides must be greater than the third side.

How do you verify that the found side lengths form a valid triangle?

By checking that the sum of any two sides exceeds the remaining side for the calculated side lengths.

What algebraic steps are involved in solving for the side lengths given the perimeter and the ratio of sides?

Set variables for the sides, express the known ratio, write an equation for the perimeter, and solve the resulting algebraic equation for the variable.

Is it possible to have multiple solutions for the side lengths under these conditions?

Typically, such problems yield a unique solution, but depending on the ratios and constraints, multiple solutions might be possible if the triangle inequalities are satisfied.

Additional Resources

Perimeter

When exploring the fascinating world of triangles, one of the most fundamental concepts is understanding their perimeter — the total length around the shape. In this detailed analysis, we'll delve into a specific problem: The perimeter of a triangle is 39 inches. If the length of the shortest side is half the length of the longest side, what are the possible side lengths? This scenario not only challenges our understanding of basic geometry but also provides insight into algebraic problem-solving and the properties of triangles.

Understanding the Problem: Perimeter and Side Ratios

Before diving into calculations, it's vital to comprehend the core components of the problem:

- Perimeter: The sum of all three side lengths of a triangle. Given as 39 inches.
- Shortest Side: Let's denote this as x . According to the problem, this is half the length of the longest side.
- Longest Side: We'll denote this as $2x$ because it's twice the shortest side.
- Remaining Side: This is the unknown side, which we'll call y .

The problem asks us to determine the possible lengths of all three sides, given the perimeter and the ratio between the shortest and longest sides.

Formulating the Mathematical Model

To approach this problem systematically, let's translate the given information into algebraic expressions.

Defining the Sides

- Shortest side: x
- Longest side: $2x$
- Remaining side: y , which is unknown

Perimeter Equation

Since the perimeter is the sum of all three sides:

$$x + y + 2x = 39$$

Simplify:

$$3x + y = 39$$

From this, we can express y as:

$$y = 39 - 3x$$

Constraints Based on Triangle Inequality

A key aspect in triangle problems is the triangle inequality theorem, which states:

- The sum of any two sides must be greater than the third side.

Applying this to our sides:

$$1. \ (x + y > 2x \)$$

$$2. \ (x + 2x > y \)$$

$$3. \ (y + 2x > x \)$$

Let's analyze each:

$$1. \ (x + y > 2x \)$$

$$\begin{aligned} & \ [\\ & x + y > 2x \implies y > x \\ & \] \end{aligned}$$

$$2. \ (x + 2x > y \)$$

$$\begin{aligned} & \ [\\ & 3x > y \\ & \] \end{aligned}$$

$$3. \ (y + 2x > x \)$$

$$\begin{aligned} & \ [\\ & y + 2x > x \implies y + x > 0 \\ & \] \end{aligned}$$

Since side lengths are positive, this is always true and doesn't restrict us further.

Now, substituting $(y = 39 - 3x \)$, we get:

$$- \ (y > x \implies 39 - 3x > x \implies 39 > 4x \implies x < \frac{39}{4} = 9.75 \)$$

$$- \ (3x > y \implies 3x > 39 - 3x \implies 6x > 39 \implies x > 6.5 \)$$

Additionally, $(y \)$ must be positive:

$$\begin{aligned} & \ [\\ & y = 39 - 3x > 0 \implies 39 > 3x \implies x < 13 \\ & \] \end{aligned}$$

Combining all inequalities:

$$\begin{aligned} & \ [\\ & 6.5 < x < 9.75 \\ & \] \end{aligned}$$

Exploring the Possible Side Lengths

Given the bounds for x , the shortest side, we now analyze specific values within this range to determine the corresponding side lengths and verify the triangle inequality.

Step 1: Calculating y

Recall:

$$y = 39 - 3x$$

For various x values between 6.5 and 9.75:

x	$y = 39 - 3x$	Longest side $2x$	Triangle Inequality Checks
6.5	$39 - 19.5 = 19.5$	13	$x + y = 6.5 + 19.5 = 26 > 13$ ✓ $x + 2x = 19.5 > y = 19.5$ (Equal, so not strictly greater; but for a valid triangle, sum must be strictly greater, so at $x=6.5$, equality occurs, so it's degenerate - excluded.)
7	$39 - 21 = 18$	14	$x + y = 7 + 18 = 25 > 14$ ✓ $3x = 21 > y = 18$ ✓
8	$39 - 24 = 15$	16	$x + y = 8 + 15 = 23 > 16$ ✓ $3x = 24 > 15$ ✓
9	$39 - 27 = 12$	18	$x + y = 9 + 12 = 21 > 18$ ✓ $3x = 27 > 12$ ✓
9.75	$39 - 29.25 = 9.75$	19.5	$x + y = 9.75 + 19.5 = 29.25 > 19.5$ ✓ $3x = 29.25$ equals y , so triangle inequality is no longer strictly satisfied, leading to a degenerate triangle.

Note: When the sum of two sides equals the third, the triangle is degenerate (a straight line). To have a valid triangle, the sum must be strictly greater.

Conclusion:

- Valid side lengths are those where x is strictly greater than 6.5 and less than approximately 9.75.
- Corresponding y values decrease as x increases but remain positive within this range.

Summary of Possible Side Lengths

Based on the calculations and triangle inequality constraints, the possible side lengths are:

- Shortest side (x): Any value in the interval $(6.5, 9.75)$
- Remaining side (y): Calculated as $y = 39 - 3x$, which decreases from approximately 19.5 to just above 9.75

- Longest side ($2x$): Twice the shortest side, ranging from approximately 13 to 19.5

Expressed explicitly, the range of side lengths:

```
\[
\boxed{
\begin{aligned}
& 6.5 < x < 9.75 \\
& y = 39 - 3x, \quad 9.75 < y < 19.5 \\
& \text{Longest side} = 2x, \quad 13 < 2x < 19.5
\end{aligned}
}
```

Practical Implications and Applications

Understanding the relationships between side lengths in triangles is crucial across various fields:

- Engineering and Construction: Ensuring structural stability by adhering to triangle inequality principles.
- Design and Manufacturing: Creating components with precise dimensions, especially where triangular parts are involved.
- Mathematical Education: Developing problem-solving skills and reinforcing algebraic and geometric concepts.

This problem exemplifies how ratios and constraints shape the possible dimensions of a triangle, emphasizing the importance of inequalities and algebraic manipulation in real-world applications.

Conclusion: The Beauty of Triangle Ratios

By dissecting the problem — where the perimeter is known, and the ratio between the shortest and longest sides is specified — we've uncovered a nuanced range of side lengths that satisfy all the conditions for a valid triangle. This exploration highlights the interconnectedness of geometry and algebra, illustrating how simple ratios can lead to a spectrum of solutions within defined bounds.

Whether you're a student mastering geometric principles or a professional designing structural elements, understanding these relationships ensures precision, stability, and adherence to fundamental mathematical laws. The elegance of triangles lies in their simplicity and the deep insights they offer into the fabric of shapes and spaces.

Disclaimer: Always verify triangle inequalities when designing or analyzing physical structures to ensure stability and safety.

The Perimeter Of A Triangle Is 39 Inches If The Length Of The Shortest Side Is $\frac{1}{2}$ The Length Of The

The Perimeter Of A Triangle Is 39 Inches If The Length Of The Shortest Side Is $\frac{1}{2}$ The Length Of The

Related Articles

- [The Value Of A Painting When Purchased Was Represented By V. If Over A Three Year Period The Painting](#)
- [System Reliability Theory: Models, Statistical Methods, And Applications By M. Rausand, A. Barros, And](#)
- [The 1950s BE SURE TO FINSH YOUR DINNER. THERE ARE CHILDREN STARVING IN CHINA AND INDIA! Today BE SURE](#)

[Back to Home](#)