The Points (1, 5) And (0, 2) Fall On A Particular Line. What Is Its Equation In Slope-intercept Form?

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Understanding how to find the equation of a line passing through specific points is a fundamental concept in algebra and coordinate geometry. When given two points, such as (1, 5) and (0, 2), mathematicians and students alike can determine the line's equation in slope-intercept form ($y = mx + b$). This process involves calculating the slope of the line and then using one of the points to solve for the y-intercept. In this comprehensive guide, we will walk through each step to find the equation of the line, explore key concepts involved, and provide tips to master this essential skill.

What Is the Slope-Intercept Form of a Line?

The slope-intercept form of a line is a straightforward way to express a straight line on the coordinate plane. It is written as:

$$y = mx + b$$

Where:
- m is the slope of the line, indicating its steepness and direction.
- b is the y-intercept, the point where the line crosses the y-axis.

Knowing how to derive the equation in this form from two points is crucial for graphing lines, solving systems of equations, and understanding the relationships between variables.

Step-by-Step Process to Find the Equation of the Line

To find the equation of the line passing through (1, 5) and (0, 2), follow these key steps:

1. Calculate the Slope (m)

The slope between two points (x_1, y_1) and (x_2, y_2) is given by:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

Applying this formula:

- $x_1 = 1$, $y_1 = 5$
- $x_2 = 0$, $y_2 = 2$

Calculate:

$$m = (2 - 5) / (0 - 1) = (-3) / (-1) = 3$$

Result: The slope of the line is 3.

2. Use the Slope and One Point to Find the y-Intercept (b)

The slope-intercept form is $y = mx + b$. To find b, substitute the slope and the coordinates of one of the points into the equation and solve for b.

Using point (1, 5):

$$5 = 3(1) + b$$

Solve for b:

$$5 = 3 + b \rightarrow b = 5 - 3 \rightarrow b = 2$$

Result: The y-intercept is 2.

3. Write the Equation of the Line

Now that we have the slope ($m = 3$) and the y-intercept ($b = 2$), the equation in slope-intercept form is:

$$y = 3x + 2$$

This is the equation of the line passing through the points (1, 5) and (0, 2).

Key Points to Remember

When working with points and lines, keep in mind these essential concepts:

- **Slope calculation:** Always subtract the y-values and x-values of the points carefully to find the slope.
- **Using a point to find b:** Substitute the coordinates into $y = mx + b$ to solve for the y-intercept.

- **Line equation:** The slope-intercept form is ideal for graphing and understanding the behavior of the line.
- **Verifying the equation:** Plug the second point into the derived equation to verify correctness.

Additional Tips for Finding Line Equations

Tip 1: Always check your calculations

Before finalizing the equation, verify your slope calculation and substitution steps. Small errors can lead to incorrect equations.

Tip 2: Use both points to confirm your work

After finding the equation, substitute the other point into your formula to ensure it satisfies the equation.

Tip 3: Practice with different points

Try deriving equations from various pairs of points to strengthen your understanding.

Applications of Finding Line Equations

Knowing how to find the equation of a line from two points has numerous real-world applications, including:

1. **Graphing data trends:** Connecting data points with lines for visualization.
2. **Physics:** Calculating the relationship between variables such as distance and time.
3. **Economics:** Modeling cost or revenue functions.
4. **Engineering:** Designing components and analyzing systems.

Common Mistakes to Avoid

While deriving the equation of a line is straightforward, some common pitfalls include:

- **Mixing up the points**: Always double-check which point is which before calculations.
- **Incorrect slope calculation**: Remember the order of subtraction matters; always subtract y-values and x-values in the same order.
- **Forgetting to verify**: After deriving the equation, test it against both points to confirm accuracy.

Summary

To summarize, the process of finding the line equation passing through points (1, 5) and (0, 2) involves:

1. Calculating the slope: **m = 3**.
2. Using one point to solve for the y-intercept: **b = 2**.
3. Formulating the line equation: **y = 3x + 2**.

This method can be applied universally to any two points to determine the equation of the line in slope-intercept form, making it an essential skill in algebra.

Conclusion

Understanding how to find the equation of a line from two points is fundamental for students and professionals working with graphs, data, or mathematical models. The key steps—calculating the slope, using a point to find the y-intercept, and writing the final equation—are straightforward once mastered. Remember to verify your results by plugging in the second point, and practice with various examples to build confidence. Mastery of this skill enhances your ability to analyze and interpret linear relationships across different disciplines.

By following this comprehensive guide, you now know exactly how to derive the line equation for points (1, 5) and (0, 2), and how to apply these principles to other problem-solving scenarios in algebra and beyond.

Frequently Asked Questions

How do you find the equation of a line passing through points (1, 5) and (0, 2)?

First, calculate the slope using m = (y2 – y1) / (x2 – x1), then use point-slope form and convert to slope-intercept form y = mx + b.

What is the slope of the line passing through points (1, 5) and (0, 2)?

The slope is (2 - 5) / (0 - 1) = (-3) / (-1) = 3.

How do you determine the y-intercept of the line passing through (1, 5) and (0, 2)?

Since the point (0, 2) has an x-coordinate of 0, its y-coordinate 2 is the y-intercept, so b = 2.

What is the slope-intercept form of the line passing through (1, 5) and (0, 2)?

Using slope m = 3 and y-intercept b = 2, the equation is y = 3x + 2.

Can the points (1, 5) and (0, 2) be on a horizontal line?

No, because the y-values are different; a horizontal line would have a constant y-value.

What method do you use to verify the line equation passes through both points?

Substitute the x-values into the equation y = 3x + 2 and check if the y-values match the points.

How does understanding the slope help in writing the line's equation?

The slope indicates the line's steepness and direction, essential for formulating its equation in slope-intercept form.

What is the significance of the point (0, 2) in the line's equation?

The point (0, 2) is the y-intercept, the point where the line crosses the y-axis, directly giving the value of b in y = mx + b.

Additional Resources

The points (1, 5) and (0, 2) fall on a particular line. What is its equation in slope-intercept form? This question is a classic problem in coordinate geometry, illustrating how to determine the equation of a line when given two points. Understanding how to find the line's equation is fundamental in mathematics, with applications spanning from graphing functions to solving real-world problems involving linear relationships. In this comprehensive guide, we'll walk through the process step-by-step, ensuring clarity whether you're a student, teacher, or anyone interested in mastering line equations.

Understanding the Basics: What Is the Slope-Intercept Form?

Before diving into the problem, it's essential to understand the slope-intercept form of a line. The slope-intercept form is written as:

$y = mx + b$

Where:
- m is the slope of the line, which indicates its steepness.
- b is the y-intercept, the point where the line crosses the y-axis.

Knowing two points on a line allows us to determine both m and b, leading us to the full equation.

Step 1: Identifying the Given Points

In our problem, we have:
- Point A: (1, 5)
- Point B: (0, 2)

Both points are provided on the same line, which simplifies our calculations.

Step 2: Calculating the Slope (m)

The slope m measures how much y changes for a unit change in x. It is calculated as:

$m = (y_2 - y_1) / (x_2 - x_1)$

Using the points:
- $(x_1, y_1) = (1, 5)$
- $(x_2, y_2) = (0, 2)$

Plugging in:
- $m = (2 - 5) / (0 - 1)$
- $m = (-3) / (-1)$
- $m = 3$

Interpretation: The line has a slope of 3, meaning for each increase of 1 in x, y increases by 3.

Step 3: Finding the Y-Intercept (b)

Now that we have the slope, we can find b in the slope-intercept form $y = mx + b$.

Method 1: Using the known point and slope

Choose either point; the more convenient is the point at x=0, because its x-coordinate is 0, which directly gives b:

- At (0, 2):

$y = mx + b$

$2 = (3)(0) + b$

$2 = 0 + b$

$b = 2$

Method 2: Verify with the other point

Alternatively, plug in (1, 5):

- $5 = 3(1) + b$

- $5 = 3 + b$

- $b = 2$

Both methods confirm that $b = 2$.

Step 4: Write the Equation in Slope-Intercept Form

Now, combine the values:

$y = 3x + 2$

This is the equation of the line in slope-intercept form that passes through points (1, 5) and (0, 2).

Additional Insights and Applications

Why is this important?

Understanding how to derive the equation of a line from two points is foundational in algebra and coordinate geometry. It allows you to:
- Graph lines accurately
- Find the intersection points of lines
- Model real-world relationships, such as speed vs. time or cost vs. quantity

Real-World Example

Suppose a car travels such that its distance (in miles) over time (in hours) follows a linear pattern. If after 0 hours the car is 2 miles away, and after 1 hour, it's 5 miles away, then the line describing this relationship has the equation $y = 3x + 2$. The slope indicates the car's speed (3 miles/hour), and the y-intercept indicates the initial distance when time is zero.

Tips for Solving Similar Problems

1. Always verify the points lie on the same line

- Confirm by plugging in the points into your derived equation.

2. Use the slope formula carefully

- Remember to subtract y-values from top to bottom and x-values from left to right.

3. Choose the simplest point to find b

- The point with x=0 simplifies calculations directly.

4. Practice with different point pairs

- For better mastery, try solving equations with points that have larger or negative coordinates.

Common Mistakes to Avoid

- Switching the points inadvertently: ensure you subtract correctly for the slope.
- Forgetting to verify the line: always check if both points satisfy the equation.
- Miscalculating the slope: watch out for signs and division errors.
- Using incorrect formulas: remember the slope formula is $(y_2 - y_1) / (x_2 - x_1)$.

Summary

To determine the equation of a line passing through the points (1, 5) and (0, 2), follow these steps:

1. Calculate the slope: m = 3
2. Find the y-intercept, b, using the point (0, 2): b = 2
3. Write the line's equation in slope-intercept form: y = 3x + 2

This process exemplifies the straightforward method of deriving a linear equation from two known points, a skill that is both fundamental and versatile across various fields of mathematics and applied sciences.

Final Thoughts

Mastering how to find the equation of a line from two points enhances your analytical capabilities and provides a solid foundation for exploring more advanced concepts like systems of equations and linear transformations. Keep practicing with different point combinations, and soon you'll be able to handle any line equation problem with confidence!

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