The Points (-3, 101.5) And (3, -3.5) Are On The Graph Of A Linear Function K What Are The Coordinates

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Understanding how to determine the equation of a linear function given two points is a fundamental skill in algebra and coordinate geometry. When tasked with finding the linear function k that passes through the points $(-3, 101.5)$ and $(3, -3.5)$, the goal is to find the slope and the equation of the line, as well as verifying the coordinates that define this line. This process not only enhances problem-solving skills but also deepens the understanding of the relationship between points and lines on a Cartesian plane.

In this article, we will explore how to find the linear function k passing through the specified points, step-by-step. We will discuss the significance of the slope, how to derive the line's equation, and interpret the results in the context of coordinate geometry. Additionally, we'll cover the importance of the slope-intercept form, the methods to verify the points lie on the line, and common pitfalls to avoid.

Understanding the Basics: Points and Linear Functions

Before diving into calculations, it is crucial to understand the foundational concepts involved:

What Is a Linear Function?

A linear function, often expressed as $y = mx + b$, represents a straight line on the coordinate plane. Here:
- m is the slope of the line, indicating its steepness and direction.
- b is the y-intercept, the point where the line crosses the y-axis.

Given Points on a Line

When two points are known, such as $(-3, 101.5)$ and $(3, -3.5)$, they uniquely determine a straight line, provided these points are distinct. Our goal is to find the equation of this line, which involves calculating the slope and the y-intercept.

Calculating the Slope of the Line

The first step in deriving the linear function k is to compute the slope m. The slope formula between two points (x_1, y_1) and (x_2, y_2) is:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying the given points:

$$
x_1 = -3, \quad y_1 = 101.5
$$
$$
x_2 = 3, \quad y_2 = -3.5
$$

Calculate the numerator:

$$
y_2 - y_1 = -3.5 - 101.5 = -105
$$

Calculate the denominator:

$$
x_2 - x_1 = 3 - (-3) = 6
$$

Thus,

$$
m = \frac{-105}{6} = -17.5
$$

The slope of the line k is -17.5, indicating a steep downward trend as x increases.

Formulating the Equation of the Line

With the slope $m = -17.5$, the next step is to find the line's equation. The point-slope form is particularly useful here:

$$
y - y_1 = m(x - x_1)
$$

Substituting one of the known points, for instance $(-3, 101.5)$:

$$
y - 101.5 = -17.5(x + 3)
$$

Expanding the right side:

$$
y - 101.5 = -17.5x - 52.5
$$

Now, solve for y:

$$
y = -17.5x - 52.5 + 101.5
$$
$$
y = -17.5x + 49
$$

This is the slope-intercept form of the linear function k:

$$
\boxed{
k: y = -17.5x + 49
}
$$

Verifying the Equation with the Given Points

It's important to verify that both points satisfy the derived equation:

Check point $(-3, 101.5)$:

$$
y = -17.5(-3) + 49 = 52.5 + 49 = 101.5
$$

Matches the coordinate exactly.

Check point $(3, -3.5)$:

$$
y = -17.5(3) + 49 = -52.5 + 49 = -3.5
$$

Again, matches precisely.

This confirms that the linear function $k: y = -17.5x + 49$ passes through both points.

Interpreting the Line's Characteristics

Understanding the characteristics of the line helps in visualizing and applying this linear function:

Slope Interpretation

- The slope of -17.5 indicates the line falls sharply as x increases.
- For each increase of 1 in x, y decreases by 17.5 units.

Y-Intercept

- The y-intercept at 49 means the line crosses the y-axis at the point $(0, 49)$.

Graphical Representation

- Plotting the points $(-3, 101.5)$ and $(3, -3.5)$ on a coordinate plane will show a straight line passing through these points.
- The line extends infinitely in both directions, maintaining a constant slope.

Applications and Importance of Finding the Equation

Knowing the linear function k has practical applications across various fields:

1. Data Analysis: Modeling relationships between variables.
2. Physics: Describing constant velocity motion.
3. Economics: Understanding cost functions with linear relationships.
4. Engineering: Designing systems with predictable linear responses.

Furthermore, the ability to derive a line from two points is foundational in solving more complex problems involving systems of equations, inequalities, and transformations.

Common Challenges and How to Avoid Them

While calculating the equation of a line is straightforward, certain pitfalls can lead to errors:

- Incorrect slope calculation: Always double-check numerator and denominator.
- Sign errors: Pay attention to negative signs in points.
- Using the wrong point: Either point can be used to derive the line, but consistency is key.
- Misapplication of formulas: Remember the slope formula and the algebra involved in expanding and simplifying expressions.

To avoid these issues:
- Write down each step clearly.
- Verify calculations with substitution.
- Use graphing tools or software for visual confirmation.

Conclusion: Final Equation and Key Takeaways

The linear function k that passes through the points $(-3, 101.5)$ and $(3, -3.5)$ is:

$$
\boxed{
k: y = -17.5x + 49
}
$$

This equation encapsulates the relationship between x and y for all points lying on this line. It highlights the importance of calculating the slope accurately and using point-slope or slope-intercept forms to quickly derive the line's equation.

Understanding how to perform these calculations reinforces fundamental algebraic skills and prepares students and professionals to analyze real-world data effectively. Whether for academic purposes, engineering, economics, or data science, mastering this process is essential for interpreting linear relationships on the coordinate plane.

Additional Resources for Learning:

- Interactive graphing calculators (e.g., Desmos)
- Algebra textbooks covering linear equations
- Online tutorials on coordinate geometry
- Practice problems involving points and lines

By practicing these concepts, you will strengthen your ability to analyze and interpret linear functions, a skill that is invaluable across many disciplines.

Frequently Asked Questions

How do I find the linear function K passing through the points (-3, 101.5) and (3, -3.5)?

To find the linear function K, first calculate the slope $m = (y_2 - y_1) / (x_2 - x_1) = (-3.5 - 101.5) / (3 - (-3)) = (-105) / 6 = -17.5$. Then, use point-slope form with one of the points, say (-3, 101.5): $y - 101.5 = -17.5(x + 3)$. Simplifying gives the equation of K.

What is the slope of the line passing through the points (-3, 101.5) and (3, -3.5)?

The slope m is $(-3.5 - 101.5) / (3 - (-3)) = -105 / 6 = -17.5$.

What are the coordinates of the two points used to determine the linear function K?

The points are (-3, 101.5) and (3, -3.5).

Can I find the equation of the line K directly from the points?

Yes, by calculating the slope and using point-slope form, you can derive the equation of the line passing through these points.

What is the y-intercept of the line K passing through the given points?

First, find the slope $m = -17.5$. Then, use one point, for example (-3, 101.5), to find the y-intercept (b) using $y = mx + b$: $101.5 = -17.5(-3) + b \rightarrow 101.5 = 52.5 + b \rightarrow b = 49$. Thus, the y-intercept is 49.

What is the equation of the linear function K passing through points (-3, 101.5) and (3, -3.5)?

The equation is $y = -17.5x + 49$.

How do I verify that these points lie on the line y = -17.5x + 49?

Substitute each x-value into the equation and check if the y-value matches: For $x = -3$, $y = -17.5(-3) + 49 = 52.5 + 49 = 101.5$; for $x = 3$, $y = -17.5(3) + 49 = -52.5 + 49 = -3.5$. Both points satisfy the equation.

What are the coordinates of the point where the line K crosses the y-axis?

The line crosses the y-axis at the y-intercept, which is at (0, 49).

Additional Resources

Linear Function Analysis

When delving into the fascinating world of mathematics, especially the study of linear functions, understanding how specific points relate to the graph of a function is fundamental. In this comprehensive exploration, we examine the points (-3, 101.5) and (3, -3.5), and determine the explicit form of the linear function k passing through these coordinates. This analysis not only reveals the underlying structure of the function but also demonstrates the step-by-step process used in deriving its equation, providing clarity for students, educators, and math enthusiasts alike.

Understanding the Problem: Points on a Linear Graph

When given two points, such as (-3, 101.5) and (3, -3.5), a common goal is to find the equation of the line that passes through both. This is a classic problem in algebra, often approached through the point-slope or slope-intercept forms of a line. The fundamental idea hinges on knowing that a straight line in the Cartesian plane can be described by a linear function:

$$
k(x) = mx + b
$$

where:
- m is the slope of the line,
- b is the y-intercept.

The process involves two key steps:
1. Calculating the slope m using the given points.
2. Using one of the points to solve for b.

Let’s explore how to accomplish each step explicitly.

Calculating the Slope m: The Heart of the Line

The slope of a line reflects its steepness and is calculated by the change in y over the change in x:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying the points:

$$
(x_1, y_1) = (-3, 101.5)
$$
$$
(x_2, y_2) = (3, -3.5)
$$

Calculating the differences:

$$
\Delta y = y_2 - y_1 = -3.5 - 101.5 = -105
$$
$$
\Delta x = x_2 - x_1 = 3 - (-3) = 6
$$

Thus, the slope is:

$$
m = \frac{-105}{6} = -17.5
$$

Key Insight: The slope is a constant value that quantifies the rate at which $k(x)$ decreases as x increases. Here, the negative slope indicates a descending line.

Determining the Equation of the Line: From Slope to Function

Having established that:

$$
m = -17.5
$$

the next step is to find the y-intercept b. This involves substituting one of the known points into the general line equation:

$$
k(x) = mx + b
$$

Choosing the point $(-3, 101.5)$:

$$
101.5 = (-17.5)(-3) + b
$$

Calculating:

$$
-17.5 \times -3 = 52.5
$$

So:

$$
101.5 = 52.5 + b
$$

Solving for b:

$$
b = 101.5 - 52.5 = 49
$$

Result: The y-intercept b is 49.

Final Equation of the Linear Function k

Putting it all together, the explicit form of the linear function is:

$$
k(x) = -17.5x + 49
$$

This function is now fully characterized by its slope and intercept, and it passes through both points as verified below.

Verification: Confirming the Points Lie on the Graph

To ensure accuracy, plug in the x-values from the original points:

- For $x = -3$:

$$
k(-3) = -17.5 \times -3 + 49 = 52.5 + 49 = 101.5
$$

which matches the given point $(-3, 101.5)$.

- For $x = 3$:

$$
k(3) = -17.5 \times 3 + 49 = -52.5 + 49 = -3.5
$$

which matches the second point $(3, -3.5)$.

This confirms the correctness of the derived function.

Interpreting the Result: What Does the Function Tell Us?

The linear function $k(x) = -17.5x + 49$ embodies a precise relationship between the independent variable x and the dependent variable $k(x)$. Its key features include:

- Slope ($m = -17.5$): For each unit increase in x, the value of $k(x)$ decreases by 17.5 units. This steep negative slope indicates a rapid decline.
- Y-Intercept ($b = 49$): When $x=0$, the function crosses the y-axis at 49.

Understanding these components allows for a comprehensive grasp of the line's behavior across the Cartesian plane.

Practical Applications and Educational Significance

Understanding how to derive the equation of a line from two points is a foundational skill with broad applications:

- Data Analysis: In fields like economics and engineering, linear models approximate relationships between variables.
- Graphical Interpretation: Visualizing the line aids in predicting values and understanding trends.
- Problem Solving: This process enhances algebraic reasoning and reinforces the importance of systematic calculations.

For educators, illustrating this derivation step-by-step fosters conceptual clarity, helping students internalize the mechanics of linear functions.

Summary and Key Takeaways

- The two points provided, (-3, 101.5) and (3, -3.5), define a unique straight line.
- The slope \(m\) is calculated as \(-17.5\), indicating a steep downward trend.
- Using the slope and one point, the y-intercept \(b\) is found to be 49.
- The resulting linear function:

\[
\boxed{
k(x) = -17.5x + 49
}
\]

- Both given points satisfy this equation, confirming its validity.

Final Note: Mastery of this process enhances one’s ability to analyze and interpret linear relationships, a crucial skill across numerous scientific and mathematical disciplines.

In conclusion, the points (-3, 101.5) and (3, -3.5) lie on the graph of the linear function \(k(x) = -17.5x + 49\). This exploration exemplifies the power of algebraic techniques in decoding the geometric relationships embedded in Cartesian coordinates, offering both theoretical insight and practical utility.

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